Non-inertial systems - Pseudo Forces

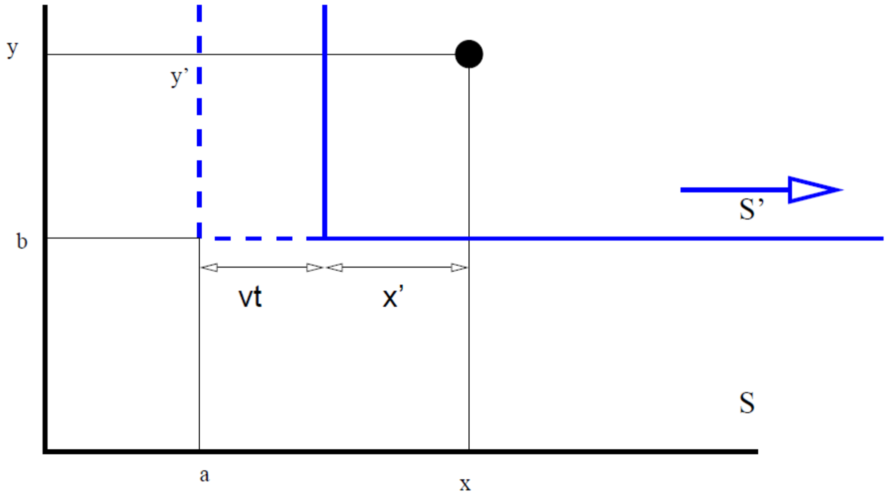
Frame of Reference

- An observer with a coordinate system and a clock etc.
- Could be moving!
- Coordinate transformations may be time dependent

Example

Coordinate Transformations

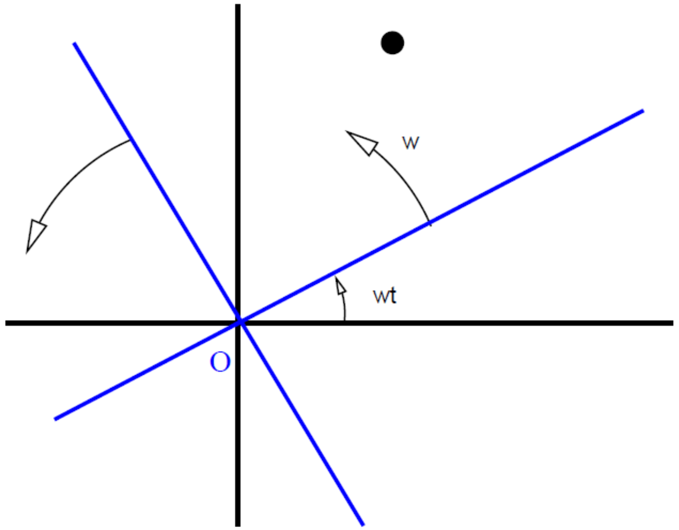
$$
\begin{aligned}
x' &= x - a - vt \\
y' &= y - b
\end{aligned}
$$



Example

Coordinate Transformations

$$
\begin{aligned}
x' &= x\cos(\omega t) + y\sin(\omega t) \\
y' &= -x\sin(\omega t) + y\cos(\omega t)
\end{aligned}
$$



Inertial Frames

If S' is moving wrt S with velocity V. In S frame a particle is moving under influence of a force F. The coordinate transformations are

$$\vec{r}' = \vec{r} - \vec{V}t$$

The velocity and acceleration of the particle

$$\begin{aligned} \vec{u}' &= \vec{u} - \vec{V} \\ \vec{a}' &= \vec{a} \end{aligned}$$

Accelerations measured in two frames are same!

If S is inertial, then

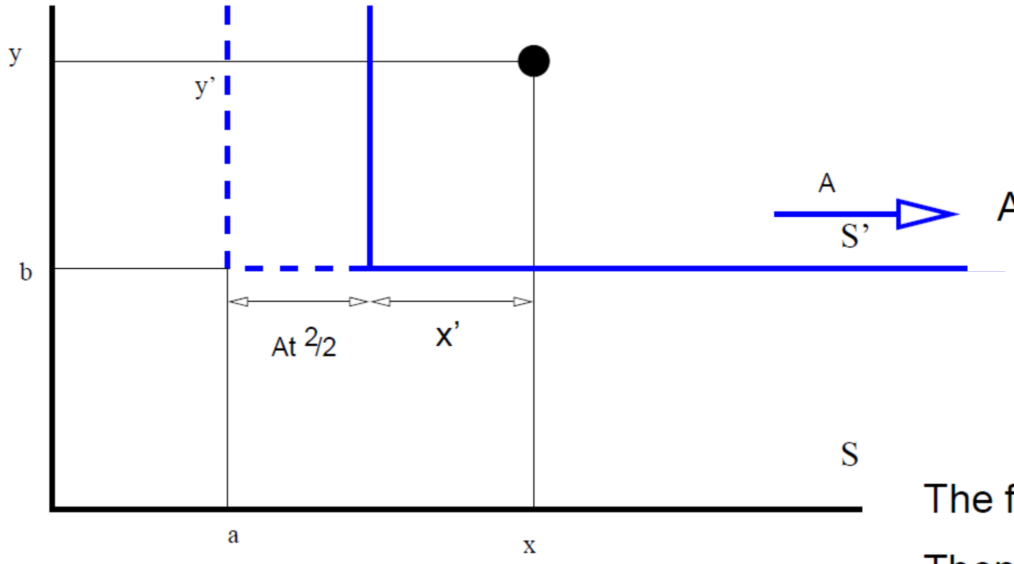
$$\begin{aligned} m\vec{a} &= \vec{F} \\ \Rightarrow m\vec{a}' &= \vec{F}' \end{aligned}$$

S' must be inertial! (Galilean Relativity)

Example

Coordinate Transformations

$$x' = x - a - \frac{1}{2}At^2$$
$$y' = y - b$$



If S' is moving wrt S with acceleration A. In S frame a particle is moving under influence of a force F. The coordinate transformations are

$$\vec{r'} = \vec{r} - \frac{1}{2}\vec{A}t^2$$

The velocity and acceleration of the particle

$$\vec{u'} = \vec{u} - \vec{A}t$$
$$\vec{a'} = \vec{a} - \vec{A}$$

Accelerations measured in two frames are not same!

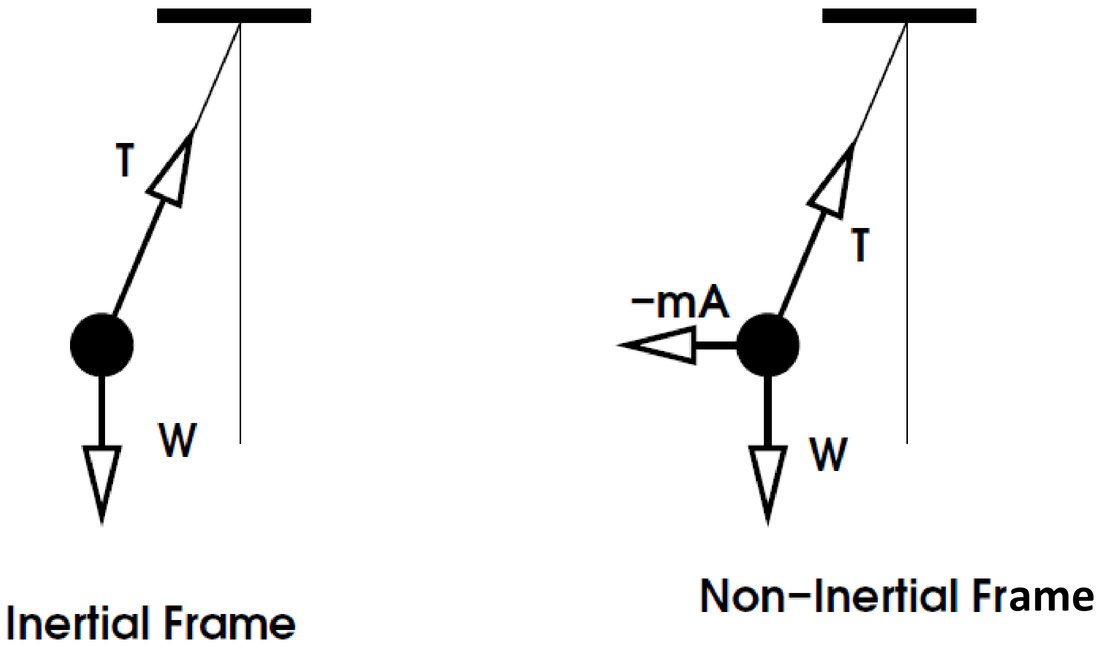
The force measured in S' be F'.

Then, if S is inertial $m\vec{a} = F$. This implies

$$m\vec{a'} = \vec{F} - mA$$

S' is not inertial. Everything would seem alright if a "Fictitious" force $-mA$ is considered. principle of equivalence.

Example



A car is moving with an acceleration A to the right. A pendulum is hung from the roof of the car. In inertial frame the bob is moving with an acceleration A. Passenger in the car sees the bob hanging steadily at an angle to the vertical.

Accelerating frame

Newton's second law $\boldsymbol{F} = m\boldsymbol{a}$ holds true only in inertial coordinate systems.

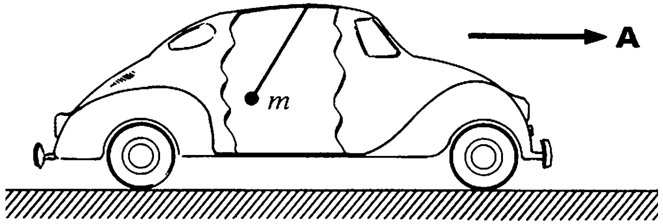
However, there are many noninertial (that is, accelerating) frames that one needs to consider, such as elevators, merry-go-rounds, and so on.

Is there any possible way to modify Newton's laws so that they hold in noninertial frames, or do we have to give up entirely on **F** = *ma*? It turns out that we can in fact hold on to **F** = *ma*, provided that we introduce some new "fictitious" forces. These are forces that a person in the accelerating frame thinks exist.

Consideration of noninertial systems will enable us to explore some of the conceptual difficulties of classical mechanics, and secondly it will provide deeper insight into Newton's laws, the properties of space, and the meaning of inertia.

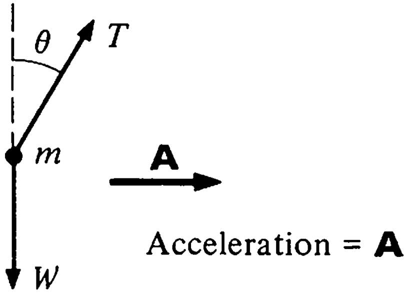
The Apparent Force of Gravity

A small weight of mass *m* hangs from a string in an automobile which accelerates at rate ***A***. What is the static angle of the string from the vertical, and what is its tension?



Let us analyze the problem both in an inertial frame and in a frame accelerating with the car.

Inertial system



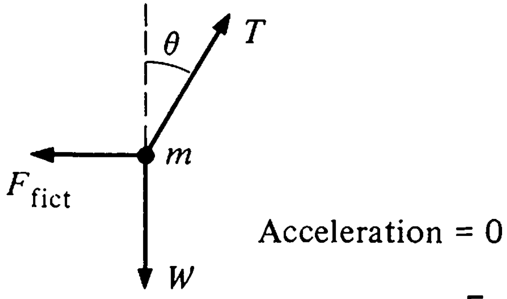
$$T\cos\theta - W = 0$$

$$T\sin\theta = MA$$

$$\tan\theta = \frac{MA}{W} = \frac{A}{g}$$

$$T = M(g^2 + A^2)^{1/2}$$

System accelerating with auto



$$T\cos\theta - W = 0$$

$$T\sin\theta - F_{\text{fict}} = 0$$

$$F_{\text{fict}} = -MA$$

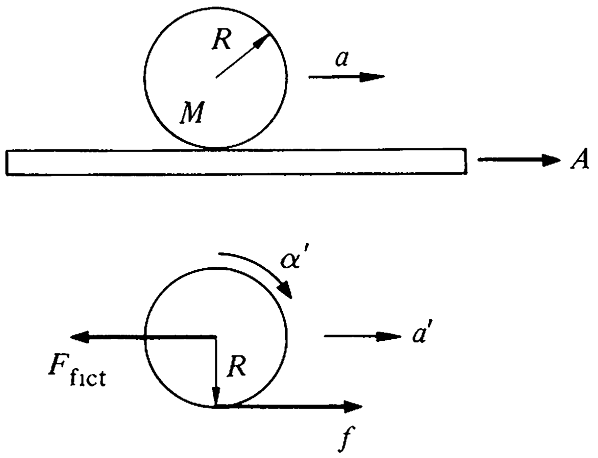
$$\tan\theta = \frac{A}{g}$$

$$T = M(g^2 + A^2)^{1/2}$$

From the point of view of a passenger in the accelerating car, the fictitious force acts like a horizontal gravitational force. The effective gravitational force is the vector sum of the real and fictitious forces.

Cylinder on an Accelerating Plank

A cylinder of mass M and radius R rolls without slipping on a plank which is accelerated at the rate A. Find the acceleration of the cylinder.



The force diagram for the horizontal force on the cylinder as viewed in a system accelerating with the plank is shown in the figure. $\boldsymbol{a'}$ is the acceleration of the cylinder as observed in a system fixed to the plank. f is the friction, and $\boldsymbol{F}_{\text{fict}}$ = $M\boldsymbol{A}$. with the direction shown. The equations of motion in the system fixed to the accelerating plank are

$$f - F_{\text{fict}} = Ma' \qquad \text{and} \qquad Rf = -I_0\alpha'.$$

The cylinder rolls on the plank without slipping, so $\alpha' R = a'$.

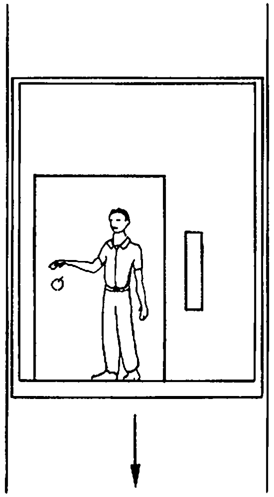
These yield $Ma' = -I_0 \dfrac{a'}{R^2} - F_{\text{fict}}$ or $a' = -\dfrac{F_{\text{fict}}}{M + I_0/R^2}$.

Since $I_0 = MR^2/2$, and $F_{\text{fict}} = MA$, we have $a' = -\frac{2}{3}A$.

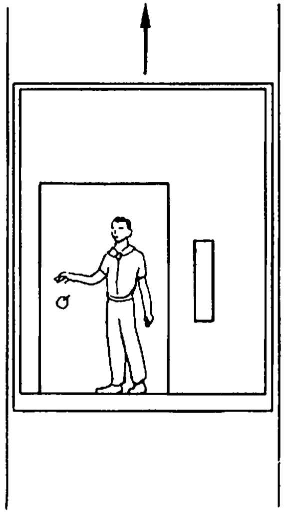
The acceleration of the cylinder in an inertial system is

$$a = A + a' = \tfrac{1}{3}A.$$

The Principle of Equivalence



A man is holding an apple in an elevator at rest in a gravitational field g. He lets go of the apple, and it falls with a downward acceleration $a = g$. Now consider the same man in the same elevator, but let the elevator be in free space accelerating upward at rate $a = g$. The man again lets go of the apple, and it again appears to him to accelerate down at rate g. From his point of view the two situations are identical. He cannot distinguish between acceleration of the elevator and a gravitational field.



There is no way to distinguish locally between a uniform gravitational acceleration ***g*** and an acceleration of the coordinate system ***A*** = -***g***. This is known as the principle of equivalence. However, such indistinguishable nature of two forces is valid only for point objects .

Gravitational field does not extend uniformly through all of space. Real forces arise from interactions between bodies, and for sufficiently large separations the forces always decrease. Real forces are then local. An accelerating coordinate system is nonlocal; the acceleration extends uniformly throughout space.

The tides on the earth exist because the gravitational force from a point mass like the moon or the sun is not uniform.

- By 1905, Albert Einstein had created a new framework for the laws of physics - his special theory of relativity. However, one aspect of physics appeared to be incompatible with his new ideas: the gravitational force as described by Newton's law of gravity. Special relativity provides a new framework for physics only when gravity is excluded. Years later, Einstein managed to unify gravity and his relativistic ideas of space and time. The result was another revolutionary new theory, general relativity.
- Einstein's first step towards that theory was the realization that, even in a gravitational field, there are reference frames in which gravity is nearly absent; in consequence, physics is governed by the laws of gravity-free special relativity - at least to a certain approximation, and only if one confines any observations to a sufficiently small region of space and time. This follows from what Einstein formulated as his *equivalence principle* which, in turn, is inspired by the consequences of free fall.

Rotating Frames

Consider a system moving about z axis

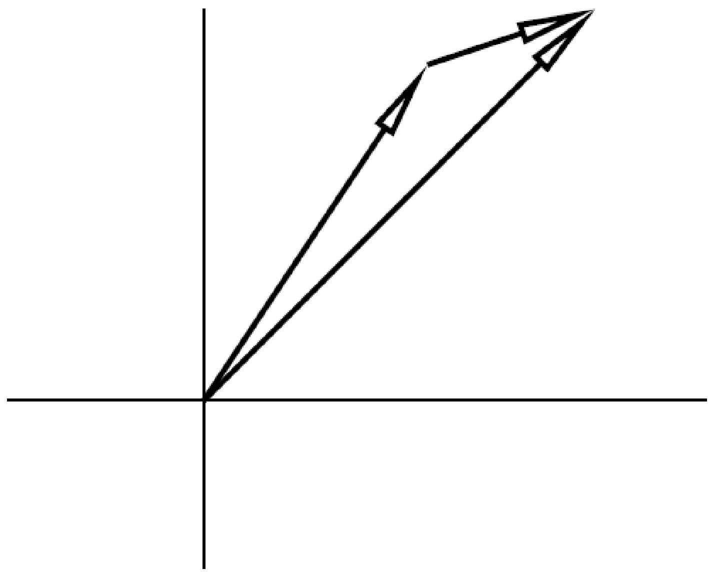
Coordinate Transformations

$$\begin{aligned} x' &= x\cos(\omega t) + y\sin(\omega t) \\ y' &= -x\sin(\omega t) + y\cos(\omega t) \\ z' &= z \end{aligned}$$

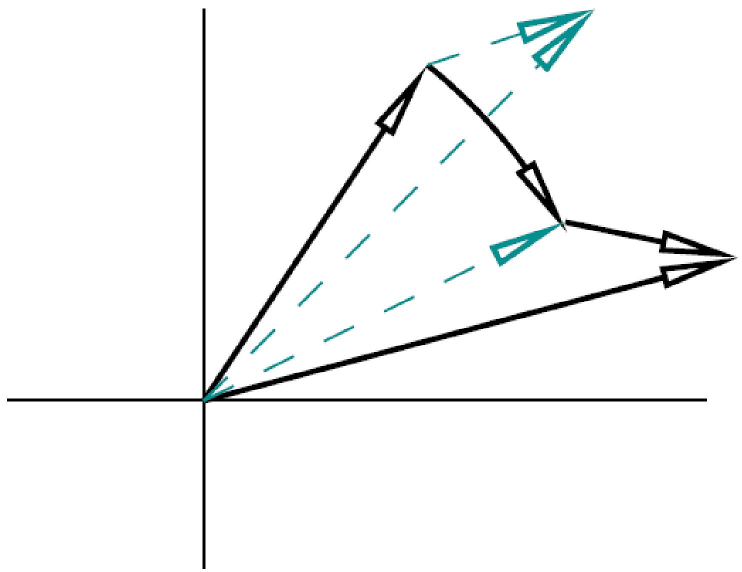
If seen from fixed system, the coordinate axes $\mathbf{i}'$ and $\mathbf{j}'$ would appear to be rotating. These vector relate to $\mathbf{i}$ and $\mathbf{j}$,

$$\begin{aligned} \mathbf{i}' &= \mathbf{i}\cos(\omega t) + \mathbf{j}\sin(\omega t) \\ \mathbf{j}' &= -\mathbf{i}\sin(\omega t) + \mathbf{j}\cos(\omega t) \\ \mathbf{k}' &= \mathbf{k} \end{aligned}$$

Suppose a vector $\vec{A}$ changes in fixed frame by amount $\Delta\vec{A}$ in time Δt.



Fixed Frame



Rotating Frame

In rotating frame the change would be same, if it had occurred instantaneously. But in time Δt, the frame has turned.

$$
\begin{aligned}
(\Delta\vec{A})' &= \Delta\vec{A} - \vec{\omega}\times\vec{A}(\Delta t) \\
\frac{d\vec{A}}{dt} &= \left(\frac{d\vec{A}}{dt}\right)_{rot} + \vec{\omega}\times\vec{A}
\end{aligned}
$$

If a particle is moving in fixed frame, the velocity is given by

$$\begin{aligned} \frac{d\vec{r}}{dt} &= \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega} \times \vec{r} \\ \vec{v} &= \vec{v}' + (\omega \times \vec{r}) \end{aligned}$$

The acceleration is given by

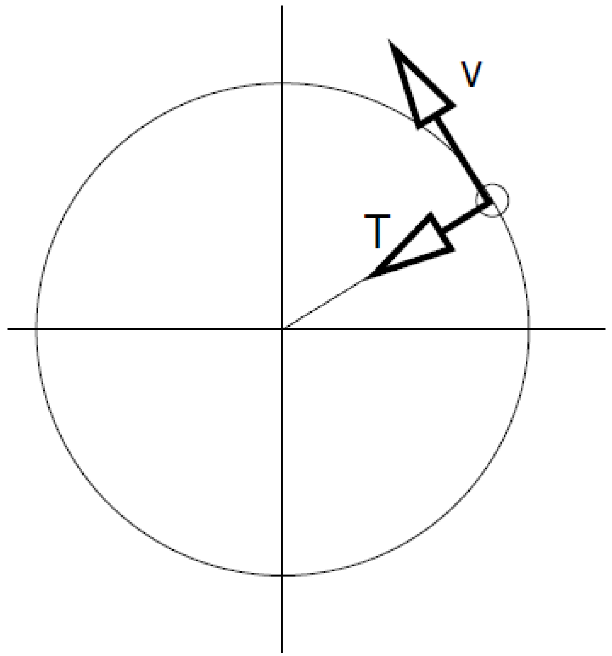
$$\begin{aligned} \frac{d\vec{v}}{dt} &= \left(\frac{d\vec{v}}{dt}\right)_{rot} + \vec{\omega} \times \vec{v} \\ \frac{d\vec{v}}{dt} &= \left(\frac{d\vec{v}'}{dt}\right)_{rot} + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \end{aligned}$$

$$\begin{aligned} m\vec{a}_{rot} &= m\vec{a} - 2\vec{\omega} \times \vec{v}' - \vec{\omega} \times (\vec{\omega} \times \vec{r}) \\ m\vec{a}_{rot} &= F - 2\vec{\omega} \times \vec{v}' - \vec{\omega} \times (\vec{\omega} \times \vec{r}) \end{aligned}$$

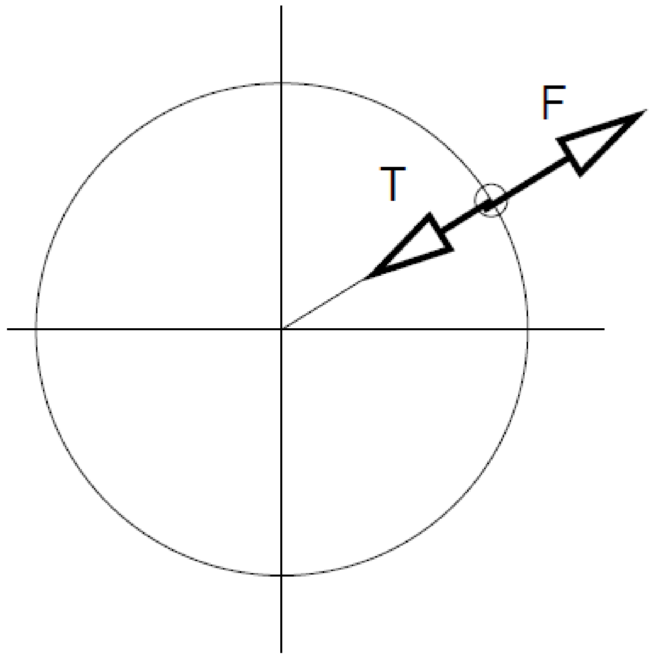
The two terms are called Coriolis Force and Centrifugal Forces.

Example

Consider a particle that is performing uniform circular motion in fixed frame with angular speed ω in a plane. A frame rotating with same angular speed will see the particle at rest. The free body diagrams are



Fixed Frame



Rotating Frame

Rotating Coordinate System

The transformation from an inertial coordinate system to a rotating system is fundamentally different from the transformation to a translating system.

A uniformly rotating system is intrinsically noninertial.

If a particle of mass *m* is accelerating at rate $\boldsymbol{a}$ with respect to inertial coordinates and at rate $\boldsymbol{a}_{\text{rot}}$ with respect to a rotating coordinate system, then the equation of motion in the two systems are given by

$$\mathbf{F} = m\mathbf{a} \quad \text{and} \quad \mathbf{F}_{\text{rot}} = m\mathbf{a}_{\text{rot}}.$$

If the accelerations of *m* in the two systems are related by $\mathbf{a} = \mathbf{a}_{\text{rot}} + \mathbf{A}$, where $\boldsymbol{A}$ is the relative acceleration, then

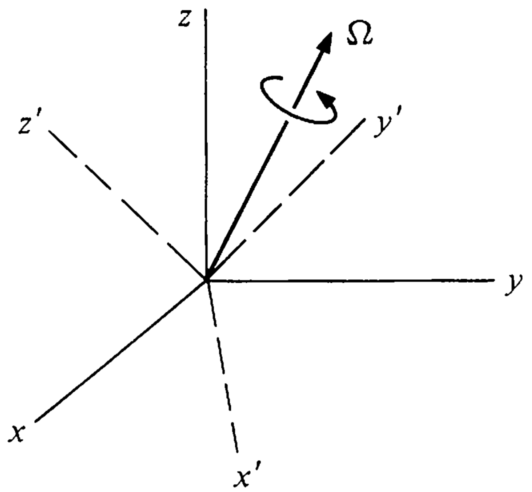
$$\begin{aligned}\mathbf{F}_{\text{rot}} &= m(\mathbf{a} - \mathbf{A}) \\ &= \mathbf{F} + \mathbf{F}_{\text{fict}}, \qquad \text{where } \mathbf{F}_{\text{fict}} = -m\mathbf{A}.\end{aligned}$$

Thus the argument is identical to that in a translating system. Our task now is to find $\boldsymbol{A}$ for a rotating system.

Time Derivatives and Rotating Coordinates

Consider an inertial coordinate system x, y, z and a coordinate system x', y', z' which rotates with respect to the inertial system at angular velocity Ω. The origins coincide. An arbitrary vector ***B*** can be described by components along base vectors of either coordinate system as

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \quad \text{or,} \quad \mathbf{B} = B_x'\hat{\mathbf{i}}' + B_y'\hat{\mathbf{j}}' + B_z'\hat{\mathbf{k}}',$$



where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ and $\hat{\mathbf{i}}', \hat{\mathbf{j}}', \hat{\mathbf{k}}'$ are the base vectors along the inertial axes and the rotating axes respectively.

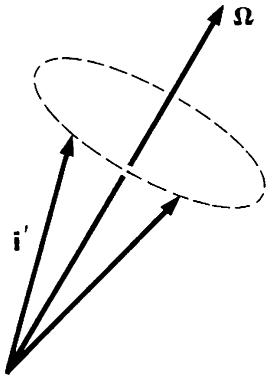
The x, y, z system is inertial so that $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are fixed in space. We have

$$\frac{d\mathbf{B}}{dt} = \frac{dB_x}{dt}\hat{\mathbf{i}} + \frac{dB_y}{dt}\hat{\mathbf{j}} + \frac{dB_z}{dt}\hat{\mathbf{k}}, \equiv (d\mathbf{B}/dt)_{\text{in}}$$

In rotating system,

$$\left(\frac{d\mathbf{B}}{dt}\right) = \left(\frac{dB_x'}{dt}\hat{\mathbf{i}}' + \frac{dB_y'}{dt}\hat{\mathbf{j}}' + \frac{dB_z'}{dt}\hat{\mathbf{k}}'\right) + \left(B_x'\frac{d\hat{\mathbf{i}}'}{dt} + B_y'\frac{d\hat{\mathbf{j}}'}{dt} + B_z'\frac{d\hat{\mathbf{k}}'}{dt}\right)$$

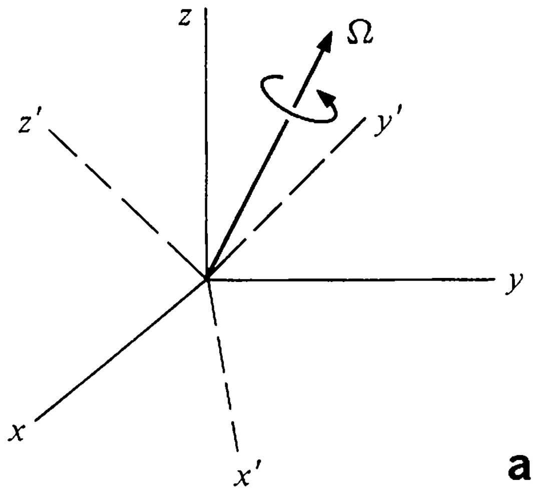
$$= \left(\frac{d\mathbf{B}}{dt}\right)_{\text{rot}} + \left(B_x'\frac{d\hat{\mathbf{i}}'}{dt} + B_y'\frac{d\hat{\mathbf{j}}'}{dt} + B_z'\frac{d\hat{\mathbf{k}}'}{dt}\right)$$



$d\mathbf{r}/dt = \boldsymbol{\omega} \times \mathbf{r}.$

$$\frac{d\hat{\mathbf{i}}'}{dt} = \boldsymbol{\Omega} \times \hat{\mathbf{i}}', \quad \text{and hence,} \quad \left(\frac{d\mathbf{B}}{dt}\right)_{\text{in}} = \left(\frac{d\mathbf{B}}{dt}\right)_{\text{rot}} + \boldsymbol{\Omega} \times \mathbf{B}.$$

Velocity and Acceleration in Rotating Coordinates



Since, $\left(\frac{d\mathbf{B}}{dt}\right)_{\text{in}} = \left(\frac{d\mathbf{B}}{dt}\right)_{\text{rot}} + \mathbf{\Omega} \times \mathbf{B}.$

$$\left(\frac{d\mathbf{r}}{dt}\right)_{\text{in}} = \left(\frac{d\mathbf{r}}{dt}\right)_{\text{rot}} + \mathbf{\Omega} \times \mathbf{r}. \quad \text{or} \quad \mathbf{v}_{\text{in}} = \mathbf{v}_{\text{rot}} + \mathbf{\Omega} \times \mathbf{r}.$$

$$\mathbf{a}_{\text{in}} = \left(\frac{d\mathbf{v}_{\text{in}}}{dt}\right)_{\text{in}} = \left(\frac{d\mathbf{v}_{\text{in}}}{dt}\right)_{\text{rot}} + \mathbf{\Omega} \times \mathbf{v}_{\text{in}}.$$

$$= \left[\frac{d}{dt}(\mathbf{v}_{\text{rot}} + \mathbf{\Omega} \times \mathbf{r})\right]_{\text{rot}} + \mathbf{\Omega} \times \mathbf{v}_{\text{rot}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}).$$

We shall assume that $\mathbf{\Omega}$ is constant, since this is the case generally needed in practice. Hence

$$\mathbf{a}_{\text{in}} = \mathbf{a}_{\text{rot}} + \mathbf{\Omega} \times \left(\frac{d\mathbf{r}}{dt}\right)_{\text{rot}} + \mathbf{\Omega} \times \mathbf{v}_{\text{rot}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}),$$

or $\mathbf{a}_{\text{rot}} = \mathbf{a}_{\text{in}} - 2\mathbf{\Omega} \times \mathbf{v}_{\text{rot}} - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}).$

Apparent Force in Rotating Coordinates

The force observed in the rotating system is

$$\mathbf{F}_{\text{rot}} = m\mathbf{a}_{\text{rot}} = m\mathbf{a}_{\text{in}} - m[2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rot}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})]$$

$$= \mathbf{F} + \mathbf{F}_{\text{fict}}, \qquad \text{where} \qquad \mathbf{F}_{\text{fict}} = -2m\boldsymbol{\Omega} \times \mathbf{v}_{\text{rot}} - m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$$

The first term is called the Coriolis force, a velocity dependent force and the second term, radially outward from the axis of rotation, is called the centrifugal force.

These are nonphysical forces; they arise from kinematics and are not due to physical interactions. Centrifugal force increases with distance *r*, whereas real forces always decrease with distance.

Coriolis and centrifugal forces seem quite real to an observer in a rotating frame. Driving a car too fast around a curve, it skids outward as if pushed by the centrifugal force. For an observer in an inertial frame, however, the sideward force exerted by the road on the tires is not adequate to keep the car turning with the road.
A rock whirling on a string, centrifugal force is pulling the rock outward. In a coordinate system rotating with the rock, this is correct; the rock is stationary and the centrifugal force is in balance with the tension in the string. In an inertial system there is no centrifugal force; the rock is accelerating radially due to the force exerted by the string.

Either system is valid for analyzing the problem.

Apparent Force in Rotating Coordinates

The force in the rotating system is

$$\begin{aligned}\mathbf{F}_{\text{rot}} &= m\mathbf{a}_{\text{rot}} = m\mathbf{a}_{\text{in}} - m[2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rot}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\ &= \mathbf{F} + \mathbf{F}_{\text{fict}},\end{aligned}$$

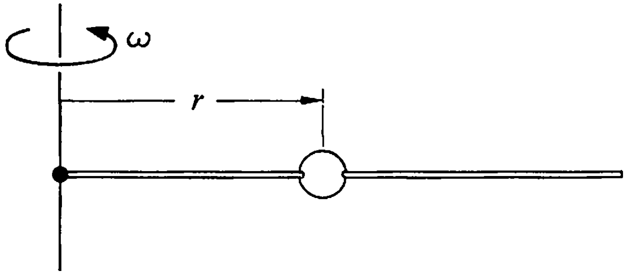
where $\mathbf{F}_{\text{fict}} = -2m\boldsymbol{\Omega} \times \mathbf{v}_{\text{rot}} - m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}).$

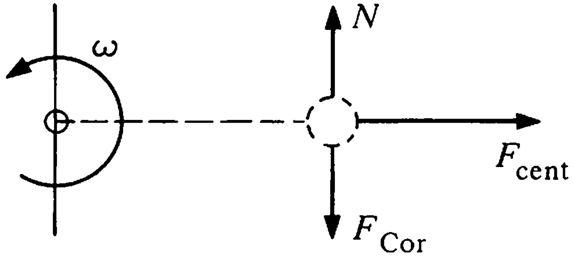
The first term is called the Coriolis force, a velocity dependent force and the second term, radially outward from the axis of rotation, is called the centrifugal force.

Now we will discuss a few examples.

The bead sliding on a stick

A bead slides without friction on a rigid wire rotating at constant angular speed ω. The problem is to find the force exerted by the wire on the bead.





In a coordinate system rotating with the wire the motion is purely radial. F_{cent} is the centrifugal force and F_{Cor} is the Coriolis force. Since the wire is frictionless, the contact force N is normal to the wire. In the rotating system the equations of motion are

$$F_{\text{cent}} = m\ddot{r} \quad \text{and} \quad N - F_{\text{Cor}} = 0.$$

Since, $F_{\text{cent}} = m\omega^2 r, \qquad m\ddot{r} - m\omega^2 r = 0,$

Hence, $r = Ae^{\omega t} + Be^{-\omega t},$

where A and B are constants depending on the initial conditions.

The other equation gives:

$$N = F_{\text{Cor}} = 2m\dot{r}\omega$$
$$= 2m\omega^2(Ae^{\omega t} - Be^{-\omega t}).$$

To complete the problem, apply the initial conditions which specify A and B.

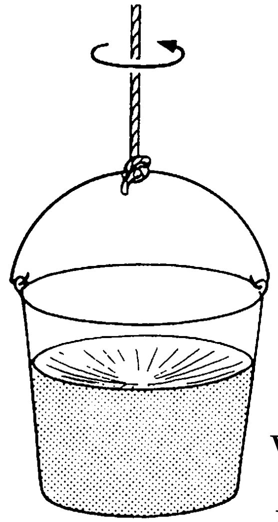
Consider the following initial conditions and find the final value of N.

$$(i)\ \text{at } t = 0, r = 0, \dot{r} = 0;\ (ii)\ \text{at } t = 0, r = 0, \dot{r} = v_0;\ (iii)\ \text{at } t = 0, r = a, \dot{r} = 0.$$

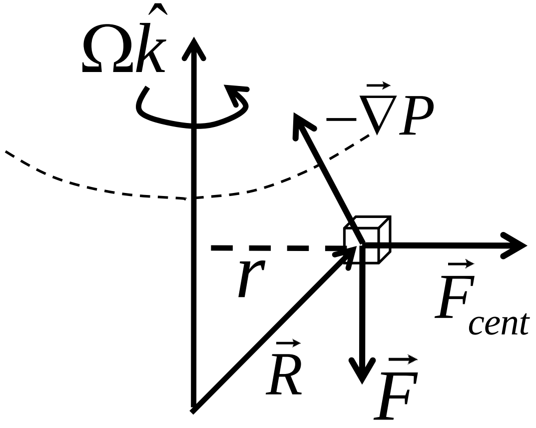
For (iii), $r = a\cosh\omega t$

Newton’s bucket

A bucket half full of water is made to rotate with angular speed Ω about its axis of symmetry, which is vertical. Find the pressure in the fluid. By considering the surfaces of constant pressure, find the shape of the free surface of the water.



In the rotating frame, the bucket is at rest. Suppose that the water has come to rest relative to the bucket. The equation of ‘hydrostatics’ in the rotating frame is



$$\vec{F} + \vec{F}_{cent} - \vec{\nabla}P = 0$$

where $\vec{F} = -\rho g\hat{k}$, the body force per unit volume,

$\vec{F}_{cent} = -\rho\vec{\Omega}\times\left(\vec{\Omega}\times\vec{R}\right)$, and $-\vec{\nabla}\vec{P}$ repesents the force due to pressure gradient.

Then, $\vec{\nabla}P = -\rho g\hat{k} - \rho\Omega^2\hat{k}\times\left(\hat{k}\times\vec{R}\right) = -\rho g\hat{k} + \rho\Omega^2 r\hat{r}$,

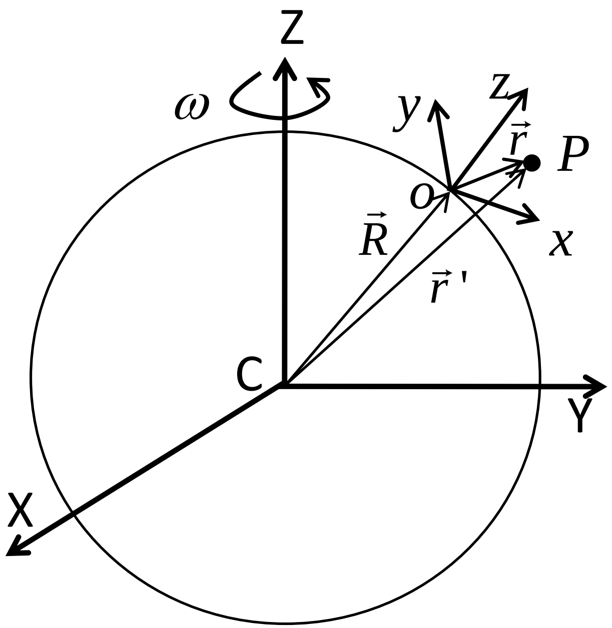
where *r* is the distance of the volume element from the rotation axis and $\hat{r}$ is the unit vector pointing in the direction of increasing *r*.

Then $\dfrac{\partial P}{\partial z} = -\rho g$ and $\dfrac{\partial P}{\partial r} = \rho\Omega^2 r$, after integration, $P = \dfrac{1}{2}\rho\Omega^2 r^2 - \rho g z + \text{constant}$

For pressure to be constant, $\dfrac{1}{2}\rho\Omega^2 r^2 - \rho g z = const.$ or $z = \dfrac{\Omega^2 r^2}{2g} + z_0$, a paraboloid.

Motion on the Rotating Earth

Fix an inertial frame at the center of the earth. Fix another coordinate system at some point on the surface of the earth but rotating along with the earth with angular velocity ω. A particle *P* of mass *m* is moving above the surface of the earth subject to a force ***F*** and acted upon by the gravitational force m***g***.



$$\vec{r}\,' = \vec{R} + \vec{r},$$

$$\left(\frac{d\vec{r}\,'}{dt}\right)_{in} = \left(\frac{d\vec{R}}{dt}\right)_{in} + \left(\frac{d\vec{r}}{dt}\right)_{in} = \left(\frac{d\vec{R}}{dt}\right)_{rot} + \vec{\omega}\times\vec{R} + \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\vec{r}$$

$$= \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\left(\vec{R}+\vec{r}\right)$$

$$\left(\frac{d^2\vec{r}\,'}{dt^2}\right)_{in} = \frac{d}{dt}\left[\left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\left(\vec{R}+\vec{r}\right)\right]_{rot} + \vec{\omega}\times\left[\left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\left(\vec{R}+\vec{r}\right)\right]$$

$$= \left(\frac{d^2\vec{r}}{dt^2}\right)_{rot} + 2\vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega}\times\left[\vec{\omega}\times\left(\vec{R}+\vec{r}\right)\right]$$

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{rot} = \vec{F} + m\vec{g} - m\vec{\omega}\times\left[\vec{\omega}\times\left(\vec{R}+\vec{r}\right)\right] - 2m\vec{\omega}\times\left(\frac{d\vec{r}}{dt}\right)_{rot}$$

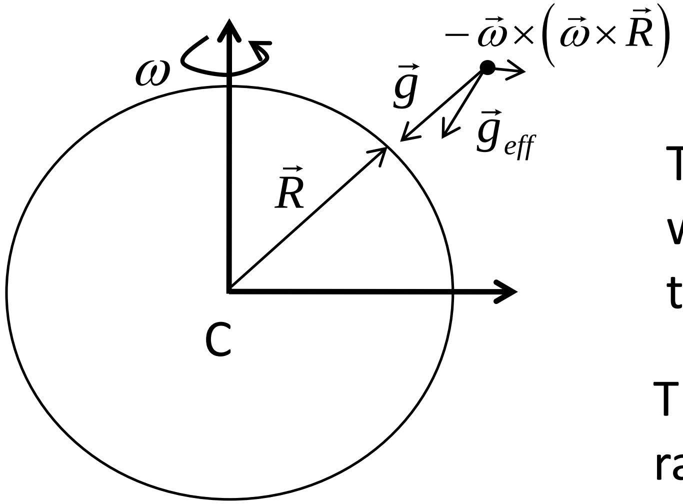
$$= \vec{F} + m\left[\vec{g} - \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\,'\right)\right] - 2m\vec{\omega}\times\vec{v}$$

$$= \vec{F} + m\vec{g}_{eff} - 2m\vec{\omega}\times\vec{v};\ \ \vec{g}_{eff} = \vec{g} - \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\,'\right)$$

v is the velocity of the particle in the rotating frame.

Effective *g*

$$\vec{g}_{eff} = \vec{g} - \vec{\omega}\times(\vec{\omega}\times\vec{r}\,') \qquad \vec{r}\,' = \vec{R} + \vec{r}$$



Since, r $\ll$ R, r' $\approx$ R, Hence, $\vec{g}_{eff} = \vec{g} - \vec{\omega}\times(\vec{\omega}\times\vec{R})$

The gravitational acceleration measured at any point will be this effective acceleration and it will be less than the acceleration due to the earth if it were not rotating.

The centrifugal acceleration, $-\vec{\omega}\times(\vec{\omega}\times\vec{R})$ always points radially outward. It is zero at the pole and maximum at the equator.

So the effective acceleration due to gravity does not act to the center of the earth. However, ***g***$_{\text{eff}}$ must be perpendicular to the surface of the earth. That is why the shape of the earth is an oblate ellipsoid, flattened at the poles.

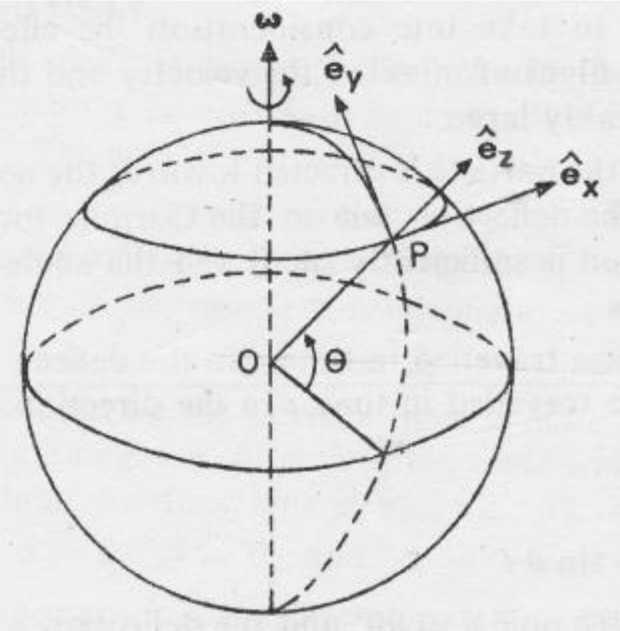
The maximum value of the centrifugal acceleration at the equator is: $\omega^2 R$.

$$\text{taking } R = 6.4\times10^{6}\,\text{m and } \omega = 7.29\times10^{-5} \text{ radian per sec,}$$

$$\omega^2 R = 3.4\times10^{-2}\ \text{m/s}^2$$

which is about 0.3 percent of the earth's gravitational acceleration.

Effect of Coriolis force



Consider a particle in the northern hemisphere at latitude θ moving with velocity ***v*** towards north, i.e. $\vec{v} = v\hat{j}$

So the Coriolis acceleration is $-2\omega\hat{k} \times v\hat{j} = 2\omega v \sin\theta \hat{i}$ toward the east. In the southern hemisphere it will toward west. It is maximum at the poles and zero at the equator.

At the north pole, for a particle moving with a velocity of 1 km/s, it is given by $2\omega v = 2 \times 7.29 \times 10^{-5} \times 10^{3} \approx 0.15 \mathrm{m/s}^2$

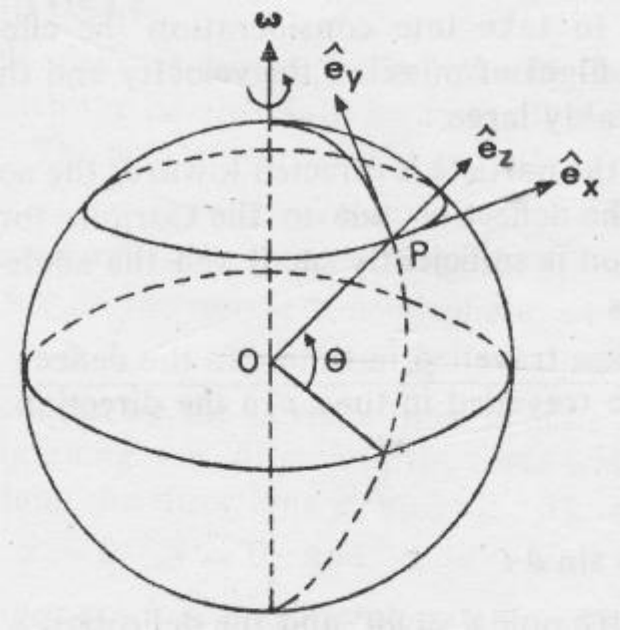
Although the magnitude of Coriolis acceleration is small, it plays important role in many phenomena on the earth.

1. It is important to consider the effect of Coriolis acceleration in the flight of missile, for which velocity and time of flight are considerably large. The equation of motion is given by $m\dfrac{d\vec{v}}{dt} = -mg\hat{k} - 2m\vec{\omega} \times \vec{v}$

2. The sense of wind whirling in a cyclone in the northern and southern hemisphere. In the northern hemisphere it is in the anticlockwise sense whereas in the southern hemisphere it is in the clockwise sense.

Freely falling particle

Find the horizontal deflection d from the plumb line caused by the Coriolis force acting on a particle in Earth's gravitational field from a height h above the Earth's surface.



Acceleration: $\mathbf{a} = \mathbf{g} - 2\boldsymbol{\omega} \times \mathbf{v}$

Components of **ω:**

$$\omega_x = 0,\ \omega_y = \omega \cos\theta \text{ and } \omega_z = \omega \sin\theta$$

Although, the Coriolis force produces small velocity components along x and y directions, they can be neglected in comparison to the vertical component.

$$\dot{x} \simeq 0, \quad \dot{y} \simeq 0 \quad \text{and} \quad \dot{z} = -gt$$

The components of acceleration are:

$$a_x = \ddot{x} = 2\omega g t \cos\theta$$
$$a_y = \ddot{y} = 0$$
$$a_z = \ddot{z} = -g$$

After integration:

$$x = \tfrac{1}{3}\omega g t^3 \cos\theta$$
$$z = z_0 - \tfrac{1}{2} g t^2$$

Since $z_0 = h$ $\quad t = \sqrt{\dfrac{2h}{g}}$ $\quad$ and $\quad d = \tfrac{1}{3}\omega \cos\theta \left(\dfrac{8h^3}{g}\right)^{1/2}$

Toward east

If h=100 m and θ=45° then the deflection would be 1.55 cm