PH101

Saurabh Basu

Class timings (Group II): 9 am-10 am (Wednesdays)

10 am -11 am (Thursdays)

Class timings (Group IV): 4 pm- 5 pm (Wednesdays)

3 pm – 4 pm (Thursdays)

Special: 18th August and 15th September (Friday)

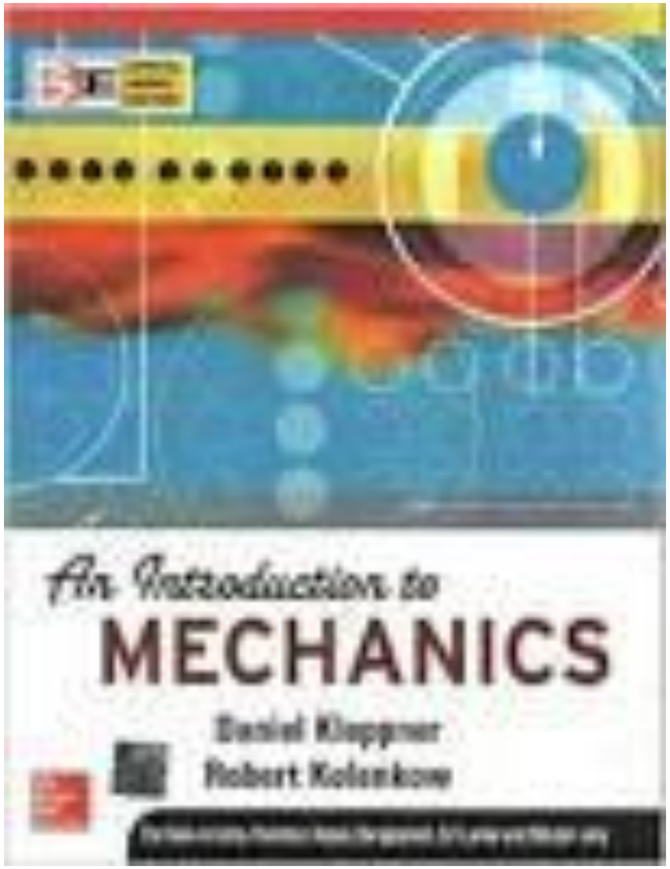
Class timings (Group II): 11 am-12 noon

Class timings (Group IV): 2 pm- 3 pm

SYLLABUS upto Mid-Sem.

Classical Mechanics: Review of Newtonian Mechanics in rectilinear coordinate system. Motion in plane polar coordinates. Conservation principles. Collision problem in laboratory and centre of mass frame. Rotation about fixed axis. Non-inertial frames and pseudo forces. Rigid body dynamics.

TEXT BOOK


An Introduction to
MECHANICS
Daniel Kleppner
Robert Kolenkow

- **Acknowledgement:**

Some slides and contents are taken from Prof. S.B. Santra & Prof. C.Y. Kadolkar

http://www.iitg.ernet.in/aksarma/PH101.html

PH 101 Tutorial Groups

Groups	Room No.	Tutors
Time: 8.00-8.55 hrs. On Mondays for All Students		
TG1	L1	
TG2	L2	
TG3	L3	
TG4	L4	
TG5	1006	
TG6	1G1	
TG7	1G2	
TG8	1207	
TG9	2101	
TG10	2102	
TG11	4001	
TG12	4G3	
TG13	4G4	
TG14	4005	

http://shilloi.iitg.ernet.in/~acad/intranet/tt/Groupings2017.htm

VECTORS

MATHEMATICAL PRILIMINARIES

- **Definition of vector:**

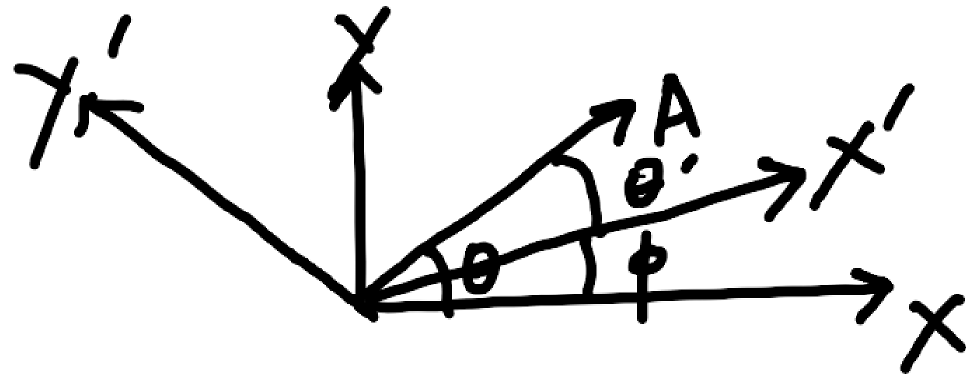
A vector is defined by its invariance properties under certain operations --

- **Translation**
- **Rotation**
- **Inversion etc**

Invariance Under Rotation

$A_x = A\cos\theta$

$A_y = A\sin\theta$



$A_x' = A\cos\theta'$

$A_y' = A\sin\theta'$

$$A_x' = A\cos(\theta - \phi) = A\cos\theta\cos\phi + A\sin\theta\sin\phi$$

$$A_y' = A\sin(\theta - \phi) = A\sin\theta\cos\phi - A\cos\theta\sin\phi$$

Simplifying

$$A_x' = A_x\cos\phi + A_y\sin\phi$$

$$A_y' = -A_x\sin\phi + A_y\cos\phi$$

- **In a compact form**

$$\begin{pmatrix} A_x' \\ A_y' \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

Transformation equations for the components of a vector can be written as,

$$\overline{A'} = R\overline{A}$$

GENERALISATION TO 3 DIMENSIONS

- Consider the Rotation Matrix in 3D,

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation about *z*-axis by an angle θ

With the help of this we shall prove that $\vec{A} \times \vec{B}$ is a vector i.e. it is invariant under rotation.

- Since $\vec{A}$ is a vector its component transform as,

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$A_x{}' = A_x\cos\theta + A_y\sin\theta$

$A_y{}' = - A_x\sin\theta + A_y\cos\theta$

$A_z{}' = A_z$

(because of rotation about ***z***-axis, the ***z***-component remains invariant.)

Similarly

$B_x' = B_x \cos\theta + B_y \sin\theta,$

$B_y' = - B_x \sin\theta + B_y \cos\theta$

$B_Z' = B_Z$

Now, consider the vector,

$$\bar{C}' = \bar{A}' \times \bar{B}'$$

$$\bar{A}' \times \bar{B}' = (\bar{A}' \times \bar{B}')_x + (\bar{A}' \times \bar{B}')_y + (\bar{A}' \times \bar{B}')_z$$

Consider only x- component (for a moment)

$(\overrightarrow{A} \times \overrightarrow{B})_X = (-\sin\theta A_x + \cos\theta A_y) B_Z' - (-\sin\theta\, B_x + \cos\theta B_y)\, A_Z'$

Since

$$A_Z' = A_Z$$
$$B_Z' = B_Z$$

$$(\overrightarrow{A} \times \overrightarrow{B})_X = \sin\theta(B_x\, A_z - A_z\, B_x) + \cos\theta(A_y\, B_z - B_y\, A_z)$$
$$(\overrightarrow{A}' \times \overrightarrow{B}')_X = R_x\, (\overrightarrow{A} \times \overrightarrow{B})_X$$

Similarly we can prove it for the other components also.

$$(\overrightarrow{A}' \times \overrightarrow{B}')_y = R_y\, (\overrightarrow{A} \times \overrightarrow{B})_y\, ;\ (\overrightarrow{A}' \times \overrightarrow{B}')_z = R_z\, (\overrightarrow{A} \times \overrightarrow{B})_z$$

Hence, $(\overrightarrow{A} \times \overrightarrow{B})$ is invariant under rotation and transforms like a vector.

Vector Multiplication

- **Scalar product or Dot product**

$$\bar{A}.\bar{B} = |A||B|\cos\theta$$

[Remember W = $\vec{F}.\vec{s}$]

- **Vector Product or Cross Product**

$$\bar{A} \times \bar{B} = |A||B|\sin\theta\hat{n}$$

[Remember $\overrightarrow{L}$= $\vec{r} \times \vec{p}$]

Vector Calculus

- Gradient: To know the direction along which a scalar function changes the fastest
- φ(x, y, z) is scalar function in cartesian coordinates

$$\bar{\nabla}\Phi = \hat{x}\frac{\partial \Phi}{\partial x} + \hat{y}\frac{\partial \Phi}{\partial y} + \hat{z}\frac{\partial \Phi}{\partial z}$$

Gradient operator

$$\bar{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Find $\bar{\nabla}$φ for φ $(x, y, z) = r = \sqrt{x^2 + y^2 + z^2}$

Divergence

- It quantifies how much a vector function diverges.It is scalar.

$$\bar{\nabla}.\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- Example: $\bar{A} = x\hat{x} + y\,\hat{y} + z\hat{z}$

$$\bar{\nabla}.\bar{A} = 3$$

Curl

- Circulation of a vector field,

$$\bar{\nabla} \times \bar{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

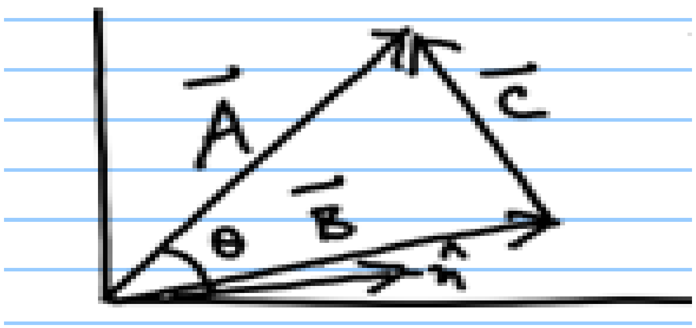
$$\bar{A} = \bar{r}$$

$$\bar{\nabla} \times \bar{r} = 0$$

Problem 1.11(K &K)

- Let $\bar{A}$ be an arbitrary vector and $\hat{n}$ be a unit vector in some fixed direction. Show $\bar{A} = (\bar{A}.\hat{n})\hat{n} + (\hat{n} \times \bar{A}) \times \hat{n}$

From fig $\bar{A} = \bar{B} + \bar{C}$



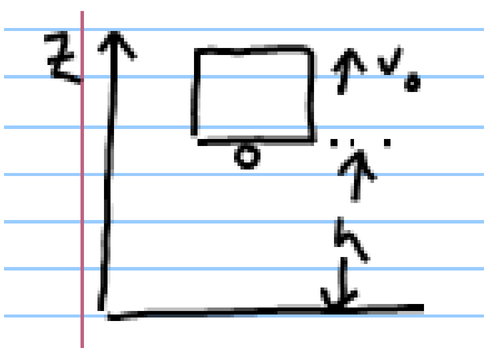
$$\bar{B} = A\,cos\theta = (\bar{A}.\hat{n})\hat{n}$$

$$\bar{C} = A\,sin\theta = (\hat{n} \times \bar{A}) \times \hat{n}$$

Hence proved.

Problem 1.13(K &K)

- An elevator ascends from the ground with uniform speed. At time T_1 a boy drops a marble through the floor. The marble falls with uniform acceleration g = 9.8 m/s^2 and hits the ground T_2 sec later. Find the height of the elevator at time T_1



$$Z = h + V_0(t - T_1) - \frac{1}{2}g(t - T_1)^2$$

At T_2 marble reaches ground

$$0 = h + V_0 T_2 - \frac{1}{2} g {T_2}^2 \text{ but } V_0 = \frac{h}{T_1}$$

$$h = \frac{\frac{1}{2}\, g T_2^2 T_1}{T_1 + T_2}$$

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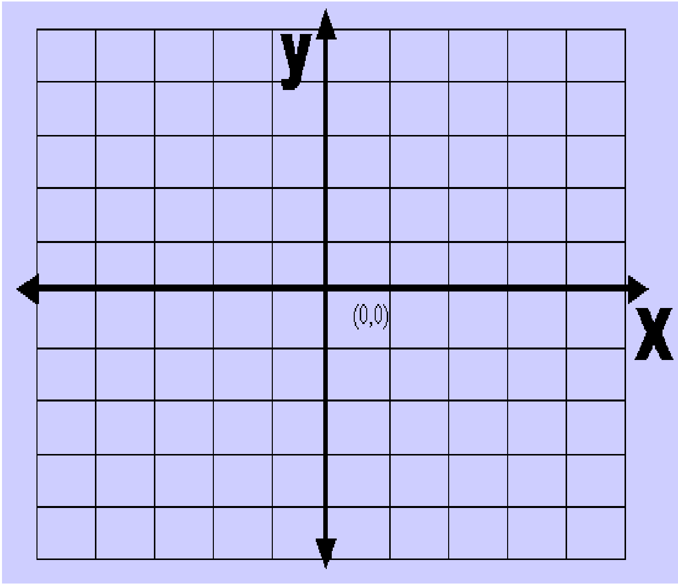
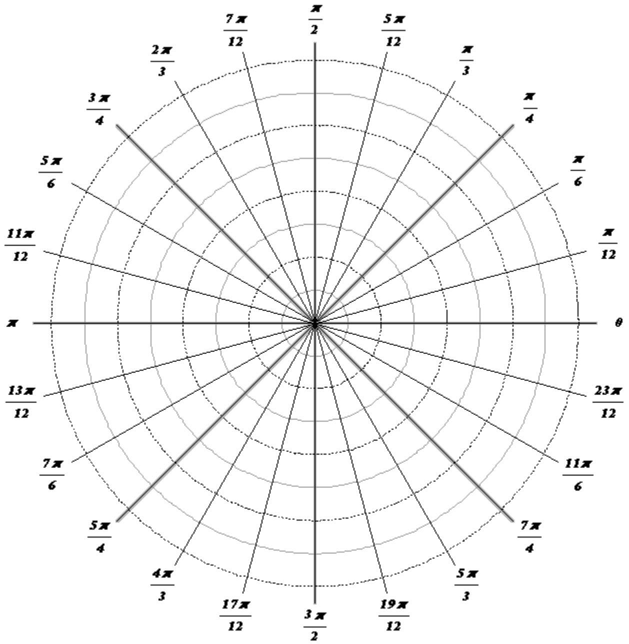
Polar Coordinates

You are familiar with plotting with a rectangular coordinate system.

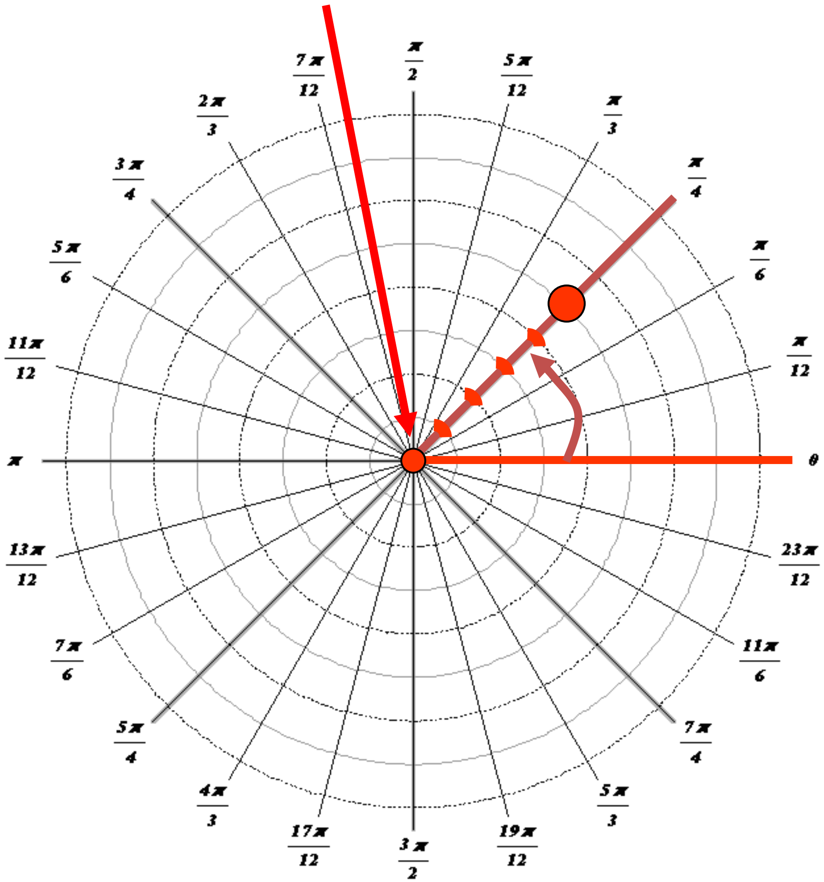


We are going to look at a new coordinate system called the polar coordinate system.



The center of the graph is called the pole.

Angles are measured from the positive *x* axis.



Points are represented by a radius and an angle

$$(r, \theta)$$

To plot the point

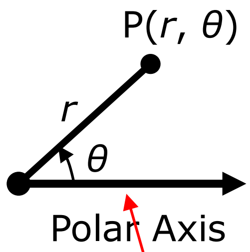
$$\left(5, \frac{\pi}{4}\right)$$

First find the angle $\pi/4$

Then move out along the terminal side 5

Polar Coordinates

To define the Polar Coordinates of a plane we need first to fix a point which will be called the Pole (or the origin) and a half-line starting from the pole. This half-line is called the Polar Axis.

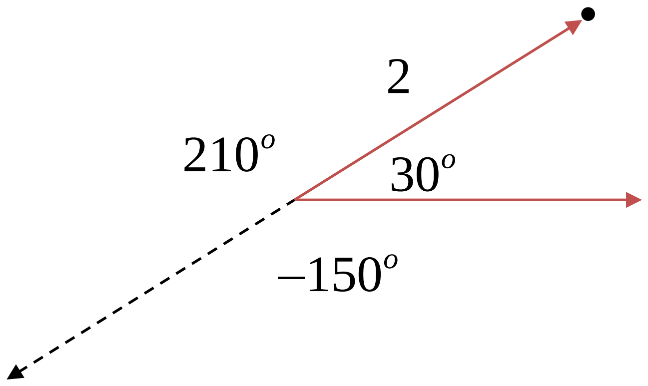


A positive angle.

Polar Angles

The Polar Angle θ of a point P, P $\neq$ pole, is the angle between the Polar Axis and the line connecting the point P to the pole. Positive values of the angle indicate angles measured in the counterclockwise direction from the Polar Axis.

More than one coordinate pair can refer to the same point.



$$\left(2, 30^o\right)$$

$$=\left(-2, 210^o\right)$$

$$=\left(-2, -150^o\right)$$

All of the polar coordinates of this point are:

$$\left(2, 30^o + n \cdot 360^o\right)$$

$$\left(-2, -150^o + n \cdot 360^o\right) \qquad n = 0,\ \pm 1,\ \pm 2\ \ldots$$