

Techniques to Solve linear 1st order ODE. : Integrating factor.

A first order equation contains only y' and no higher derivatives. A linear first-order equation means one which can be written in the following form,

$$y' + Py = Q. \quad (1)$$

P & Q are functions of x .

To begin with take $Q=0$

$$y' + Py = 0 \Rightarrow \frac{dy}{dx} = -Py \quad (2)$$

$$\frac{dy}{y} = -P dx \Rightarrow \ln y = -\int P dx + C$$

$$y = e^{-\int P dx + C} = A e^{-\int P dx}, \quad A = e^C : \text{a const.}$$

$$I = \int P dx.$$

$$\frac{dI}{dx} = P.$$

$$y = A e^{-I} \quad \text{or} \quad y e^I = A.$$

$$\begin{aligned} \frac{d}{dx}(y e^I) &= y' e^I + y e^I \frac{dI}{dx} = y' e^I + y e^I P \\ &= e^I (\underbrace{y' + Py}_{\text{rhs of (1)}}) = Q e^I \end{aligned}$$

$$y e^I = \int Q e^I dx + C.$$

$$y = e^{-I} \int Q e^I dx + C e^{-I}.$$

} with $I = \int P dx.$