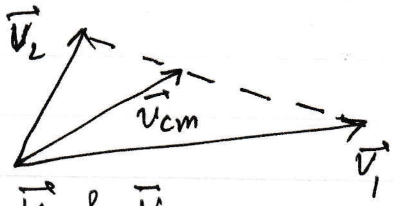
Collisions in 3 dimensions : CM frame analysis

In 3 dimensions it is always simpler to analyse the problem in CM coordinates.

Consider 2 particles with masses m_1, m_2 and initial velocities $\vec{v}_1$ & $\vec{v}_2$. The velocity of the CM,

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} \quad (1)$$

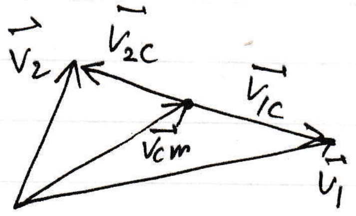


$\vec{v}_{cm}$ must lie in the line joining $\vec{v}_1$ & $\vec{v}_2$

In the ~~lab~~ CM frame,

$$\vec{v}_{1CM} = \vec{v}_1 - \vec{v}_{cm} = \frac{m_2}{m_1+m_2}\left(\vec{v}_1 - \vec{v}_2\right) \quad (2a)$$

$$\vec{v}_{2CM} = \vec{v}_2 - \vec{v}_{cm} = -\frac{m_1}{m_1+m_2}\left(\vec{v}_1 - \vec{v}_2\right) \quad (2b)$$



$\vec{v}_{1c}$ and $\vec{v}_{2c}$ lie back to back along the relative velocity vector $\vec{v}_{rel} = \vec{v}_1 - \vec{v}_2$

Likewise the momenta in the CM frame is

$$\vec{p}_{1CM} = m_1 \vec{v}_{1cm} = \frac{m_1 m_2}{m_1 + m_2}\left(\vec{v}_1 - \vec{v}_2\right) = \mu \vec{v}_{rel} \quad (3a)$$

and

$$\vec{p}_{2CM} = m_2 \vec{v}_{cm} = -\frac{m_1 m_2}{m_1+m_2}\left(\vec{v}_1 - \vec{v}_2\right) = -\mu \vec{v}_{rel}. \quad 3(b)$$

Thus the momenta are equal and opposite, ensuring that the total momenta in the CM frame is zero. Also $\mu = \frac{m_1 m_2}{m_1+m_2}$: reduced mass