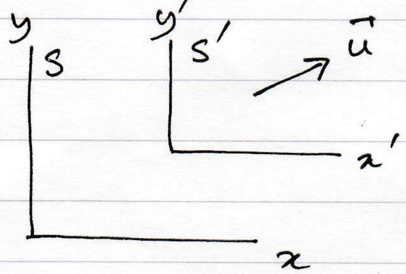
The center of mass frame

When we talk about momentum, it is understood that a certain reference frame has been chosen. Any noninertial reference frame will serve the purpose, but we shall see that there is one particular reference frame that is often advantageous to use



Consider a frame S and another S' which is moving with a constant velocity $\vec{u}$ wrt S.

Consider a system of particles, the velocity of the i^{th} particle in S is related to it's velocity in S' by

$$\vec{v}_i = \vec{v}_i' + \vec{u} \qquad (1).$$

This equation implies that if the momentum is conserved in S', then it is also conserved in S.

Thus let us consider an unique frame in which the total momentum of a system of particles is zero. This is called the CM frame. If the total momentum is $\vec{P} = \sum m_i \vec{v}_i$ in frame S, then the CM frame is the frame S' that moves with the velocity,

$$\vec{u} = \frac{\vec{P}}{M} = \frac{\sum_i m_i \vec{v}_i}{M} \quad \text{wrt } S. \qquad (2)$$

This follows from eq. (1)

$$\vec{P}' = \sum m_i \vec{v}_i' = \sum m_i \left(\vec{v}_i - \frac{\vec{P}}{M} \right) = \vec{P} - \vec{P} = 0. \qquad (3).$$

where $M = \sum m_i$ is used.