DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA746: Fourier Analysis | Mid Semester Exam
Instructor: Rajesh Srivastava | February 24, 2019
Time duration: Two hours | Maximum Marks: 30

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let $f \in C_c^\infty(\mathbb{R})$ be a non-zero function and P be a polynomial of degree $n \geq 1$. Whether $P\hat{f}$ is a bounded function on $\mathbb{R}$? **[1]**

   (b) Does the space $\{f \in L^2(\mathbb{R}) : \text{supp}\hat{f} \text{ is compact}\}$ dense in $L^2(\mathbb{R})$? **[1]**

2. For $n \in \mathbb{N}$, define $F_n(x) = \chi_{[-1,1]} * \chi_{[-n,n]}(x)$. Verify that $F_2 \in C_c(\mathbb{R})$ and $\|F_2\|_u = 2$. Does $F_n(x) \to 2$ uniformaly? **[4]**

3. Let $f \in C^1(S^1)$. Show that there exists $M > 0$ such that $\sum_{n=-\infty}^{\infty} |\hat{f}(n)| \leq \|f\|_1 + M\|f'\|_2$. **[2]**

4. Let $f \in L^1(S^1)$ and $S_n(f)$ denotes the nth partial sum of the Fourier series of f. Show that $\left\|\frac{S_n(f)}{n}\right\|_1 \to 0$ as $n \to \infty$. **[4]**

5. Let f be a Riemann integrable function on $[-\pi, \pi]$. If f is differentiable at $t_o \in [\pi, \pi]$ then show that $S_n(f; t_o) \to f(t_o)$ as $n \to \infty$. **[3]**

6. Suppose $f \in C^1(S^1)$ is satisfying $[f * (1 + f)](t) = f'(t)$ for all $t \in S^1$. Show that f is constant. **[4]**

7. Let $f \in L^1(\mathbb{R})$ and $f(x) > 0$ for all $x \in \mathbb{R}$. Prove that there exists $\delta > 0$ such that the strict inequality $|\hat{f}(\xi)| < \hat{f}(0)$ holds, whenever $|\xi| > \delta$. **[3]**

8. Let $f, g \in L^2(\mathbb{R})$. Show that $f * g$ is a bounded continuous function on $\mathbb{R}$. Further, prove that $\lim_{|x|\to\infty} f * g(x) = 0$. **[4]**

9. For $f \in L^1(\mathbb{R})$, let $g(t) = 2\pi \sum_{n=-\infty}^{\infty} f(t + 2\pi n)$, then show that g is periodic and satisfying $\|g\|_{L^1(S^1)} \leq \|f\|_{L^1(\mathbb{R})}$. **[4]**

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