

DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA642: Real Analysis -1
Instructor: Rajesh Srivastava
Time duration: 1.5 hours

Quiz - I
February 20, 2023
Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

1. (a) Does it imply for any sequence (x_n) in \mathbb{R} satisfies $0 \leq \inf x_n \sup x_n < \infty$ that x_n has a convergent subsequence? **1**
(b) Let E be the set of end points of the deleted open intervals in the construction of the Cantor set C . If g is a uniformly continuous function on E . How many uniformly continuous functions f on C exist with $f = g$ on E ? **1**
2. Let $f : (a, b) \rightarrow (c, d)$ be a monotone decreasing function. If f is surjective, then show that f is continuous. **2**
3. Let d be a metric on \mathbb{R} . Suppose $\varphi : [0, \infty) \rightarrow [0, \infty)$ be a monotone increasing function which vanishes only at zero. If φ is concave, then show that $\varphi \circ d$ is a metric on \mathbb{R} . **2**
4. Find a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, which is convex and satisfies $f(\alpha x) = |\alpha|f(x)$ for every $(\alpha, x) \in \mathbb{R} \times \mathbb{R}^n$, but f is not a norm on \mathbb{R}^n . **2**
5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \begin{cases} \frac{\sin x^2}{x} & \text{if } x \neq 0; \\ 0 & \text{otherwise.} \end{cases}$
Find non-negative constants A and B such that $f(x) \leq A|x| + B$ for every x in \mathbb{R} . Is it possible to find smallest A and B ? **2**

END