Region Elimination Method

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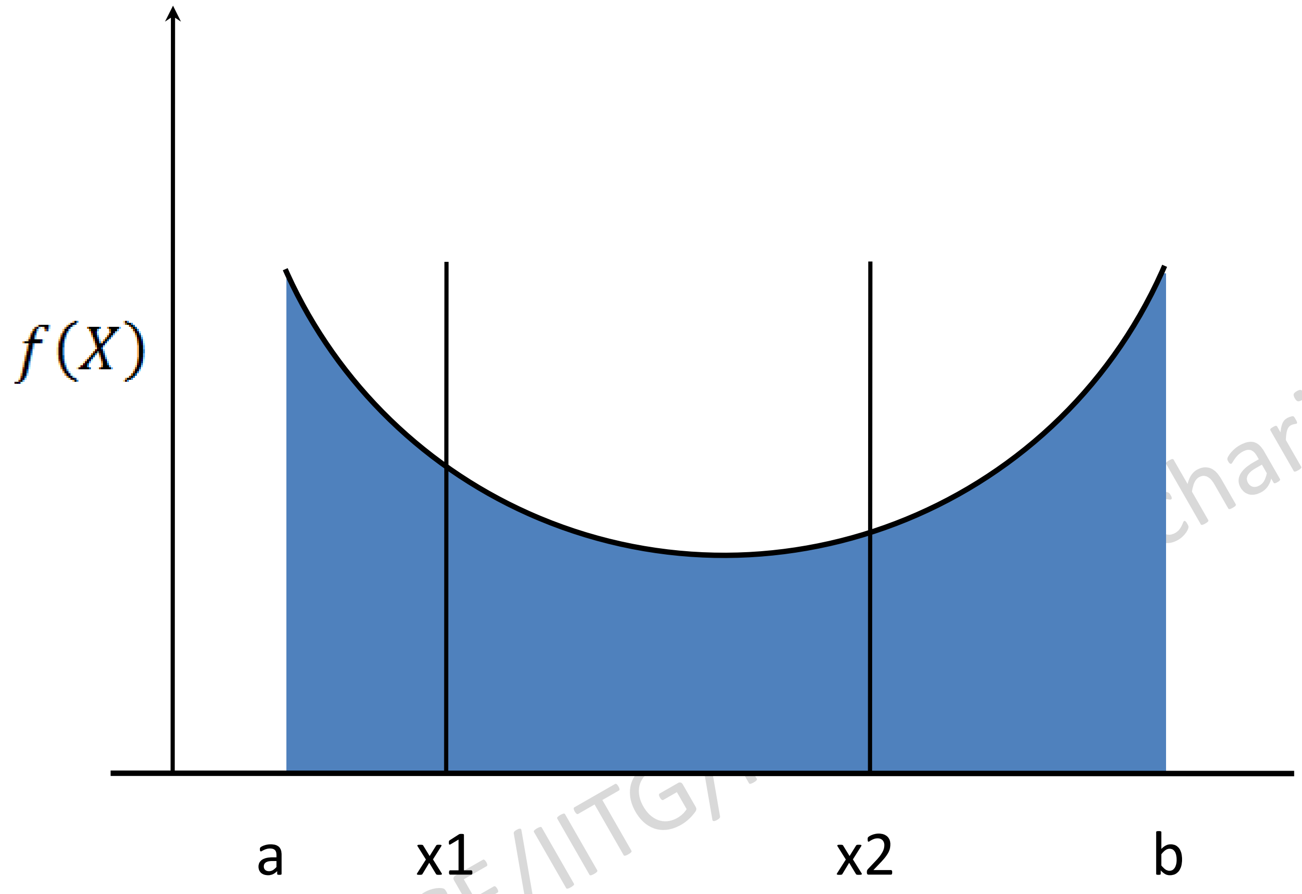
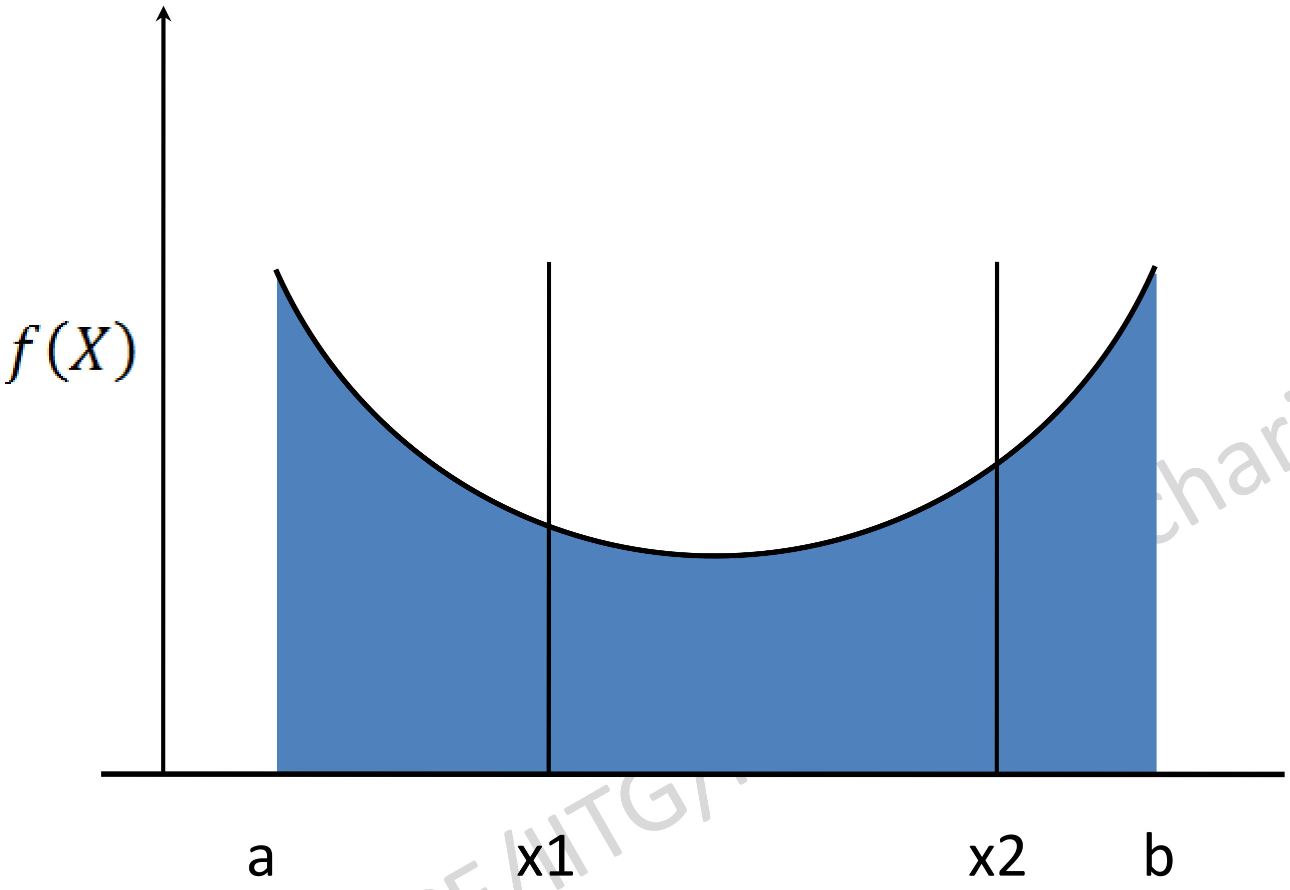
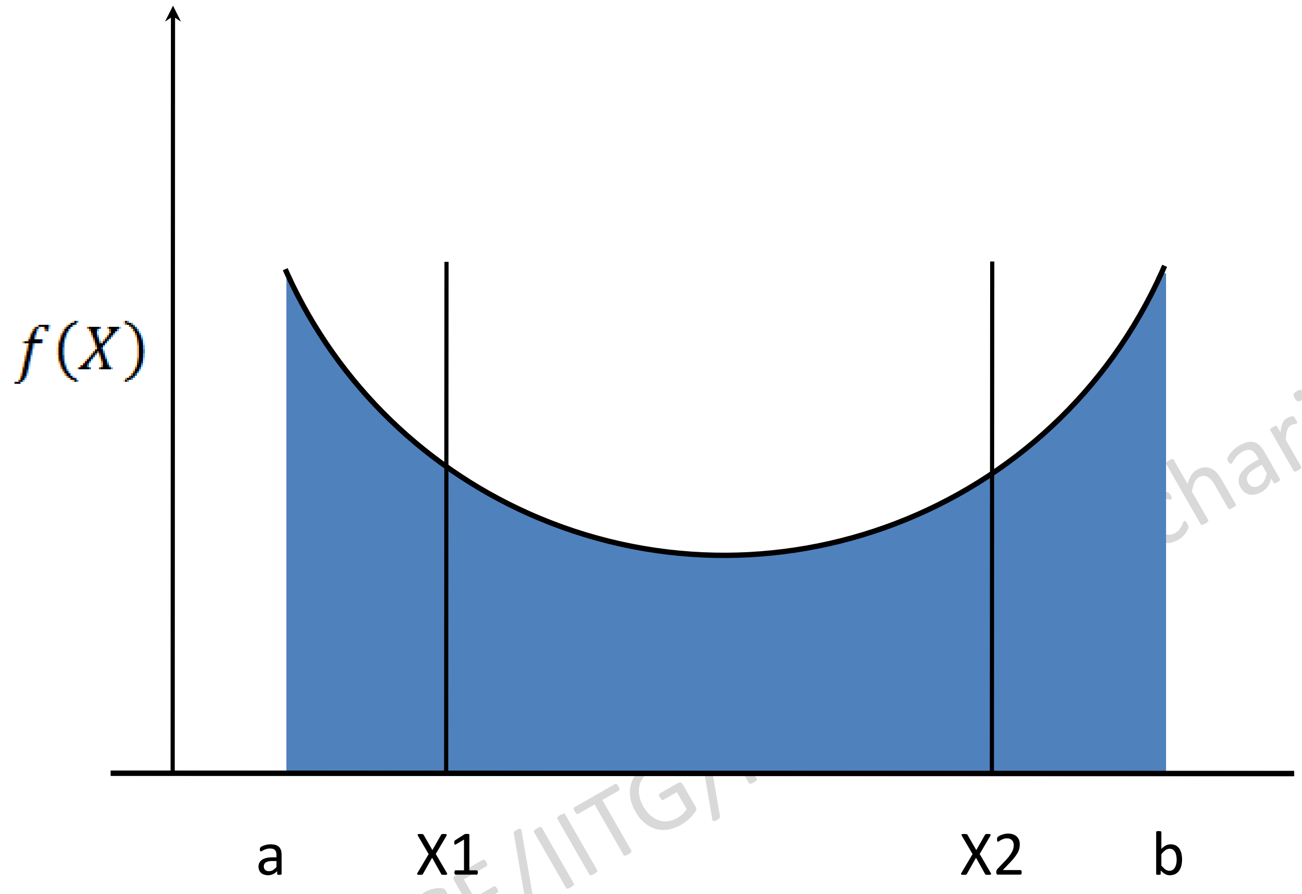
$$if\ f(X_1) > f(X_2)$$



$$if\ f(X_1) > f(X_2)$$

$$if\ f(X_2) > f(X_1)$$





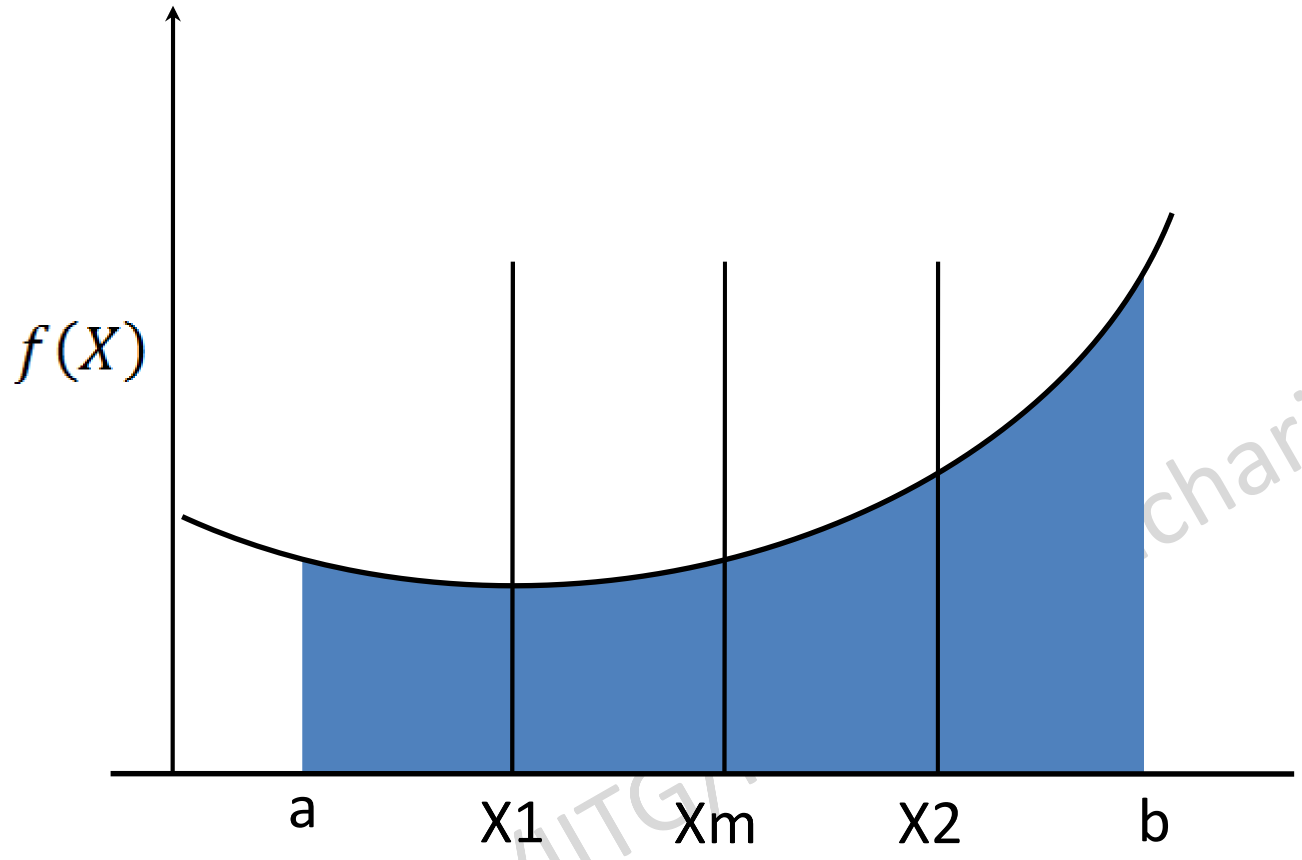
$if\ f(X_1) > f(X_2)$

$if\ f(X_2) > f(X_1)$

$if\ f(X_2) = f(X_1)$

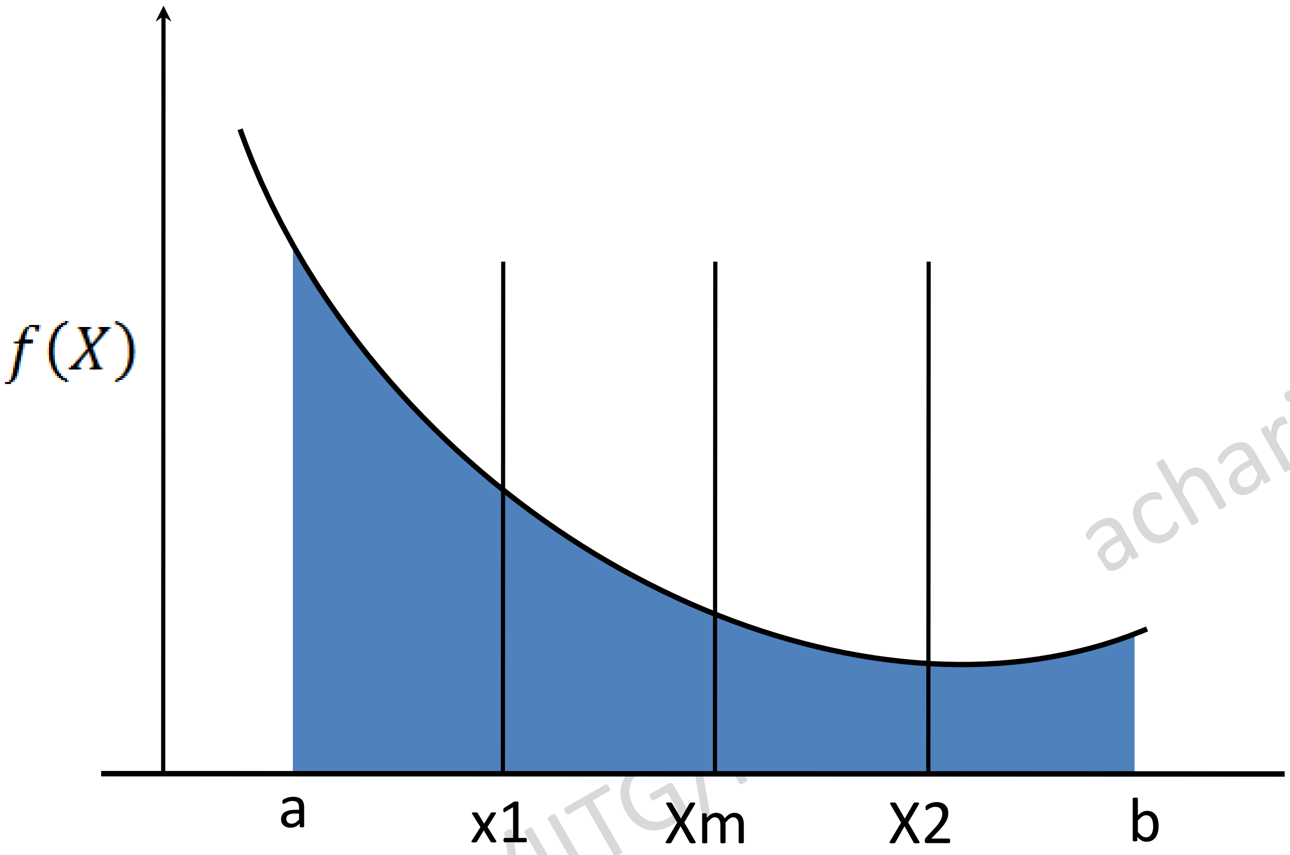
Interval halving method

$$if\ f(X_1) < f(X_m)$$



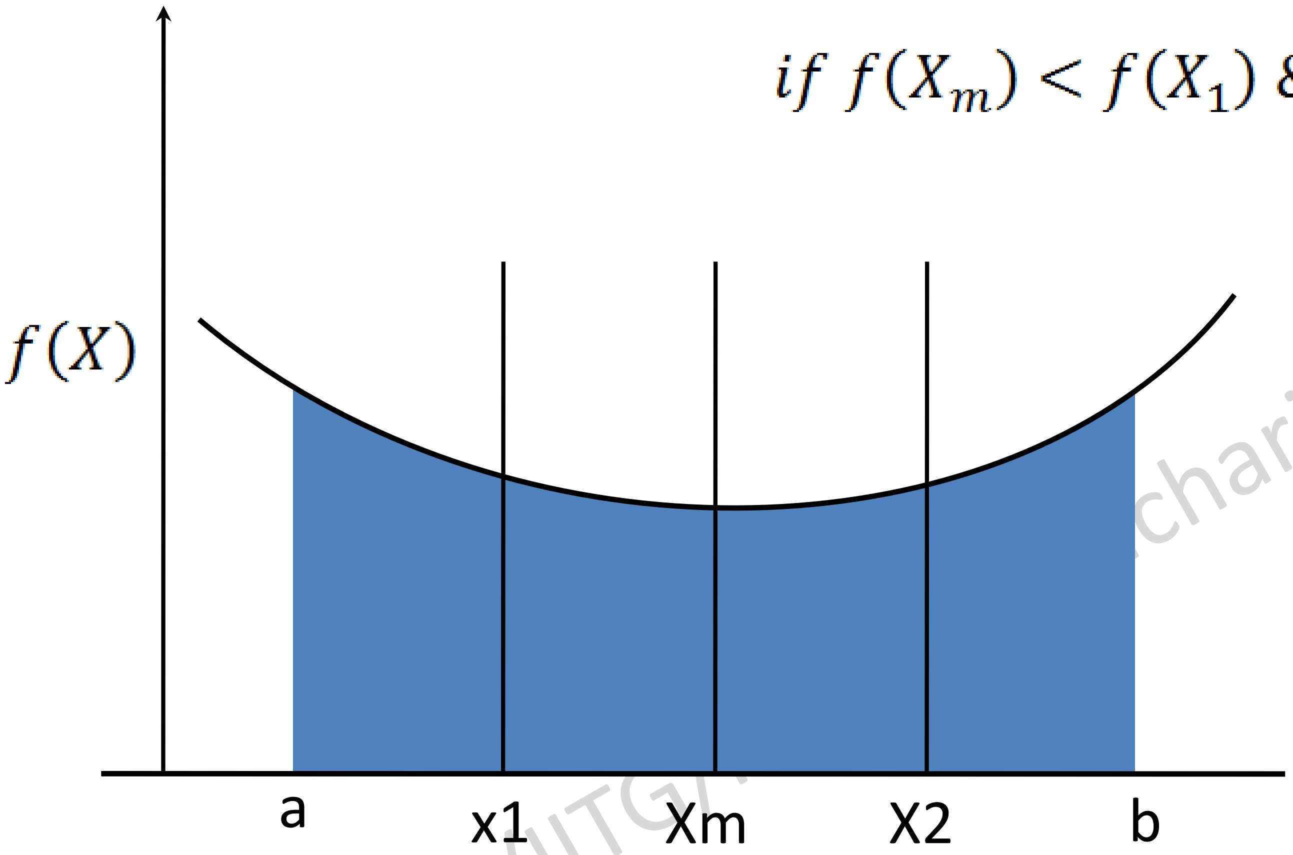
Interval halving method

$if\ f(X_2) < f(X_m)$



Interval halving method

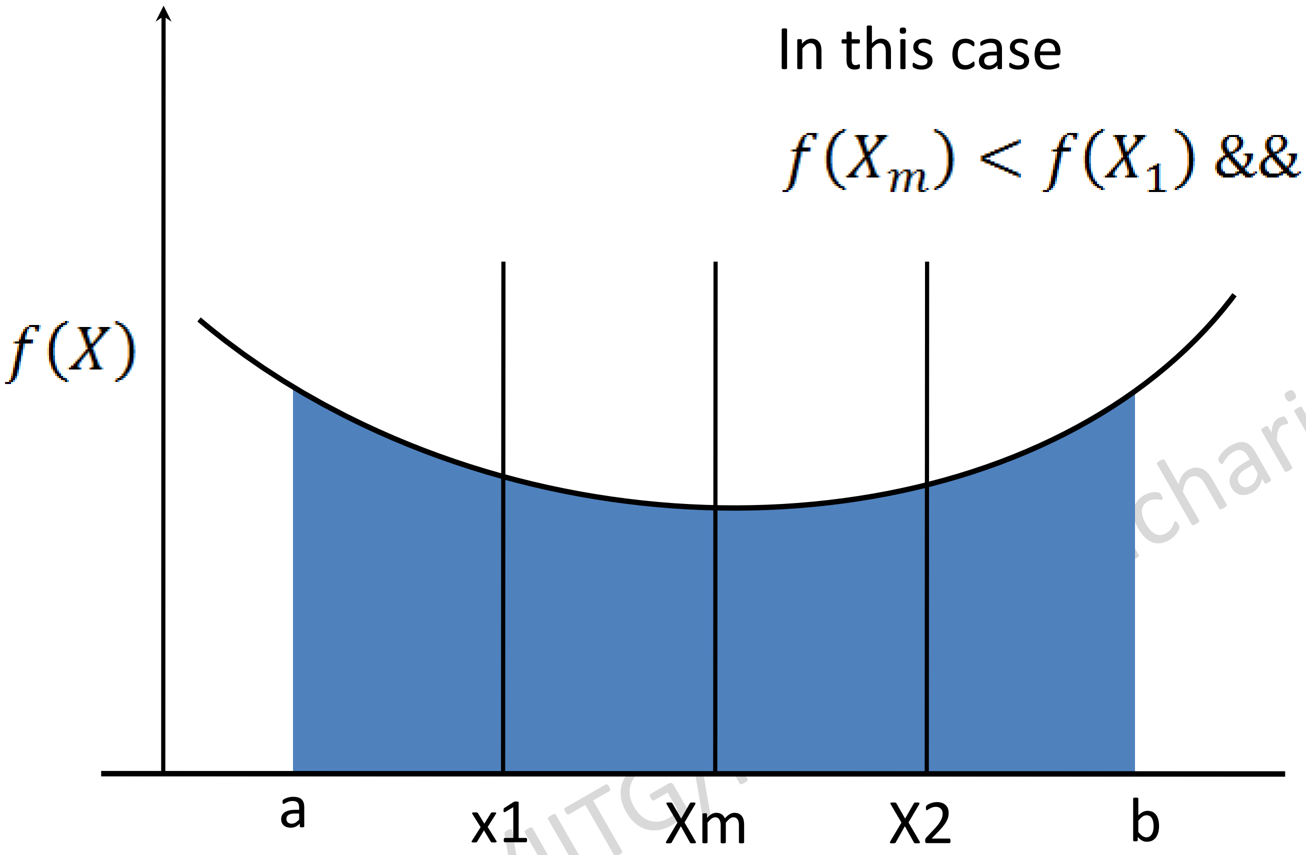
$$if\ f(X_m) < f(X_1)\ \&\&\ f(X_m) < f(X_2)$$



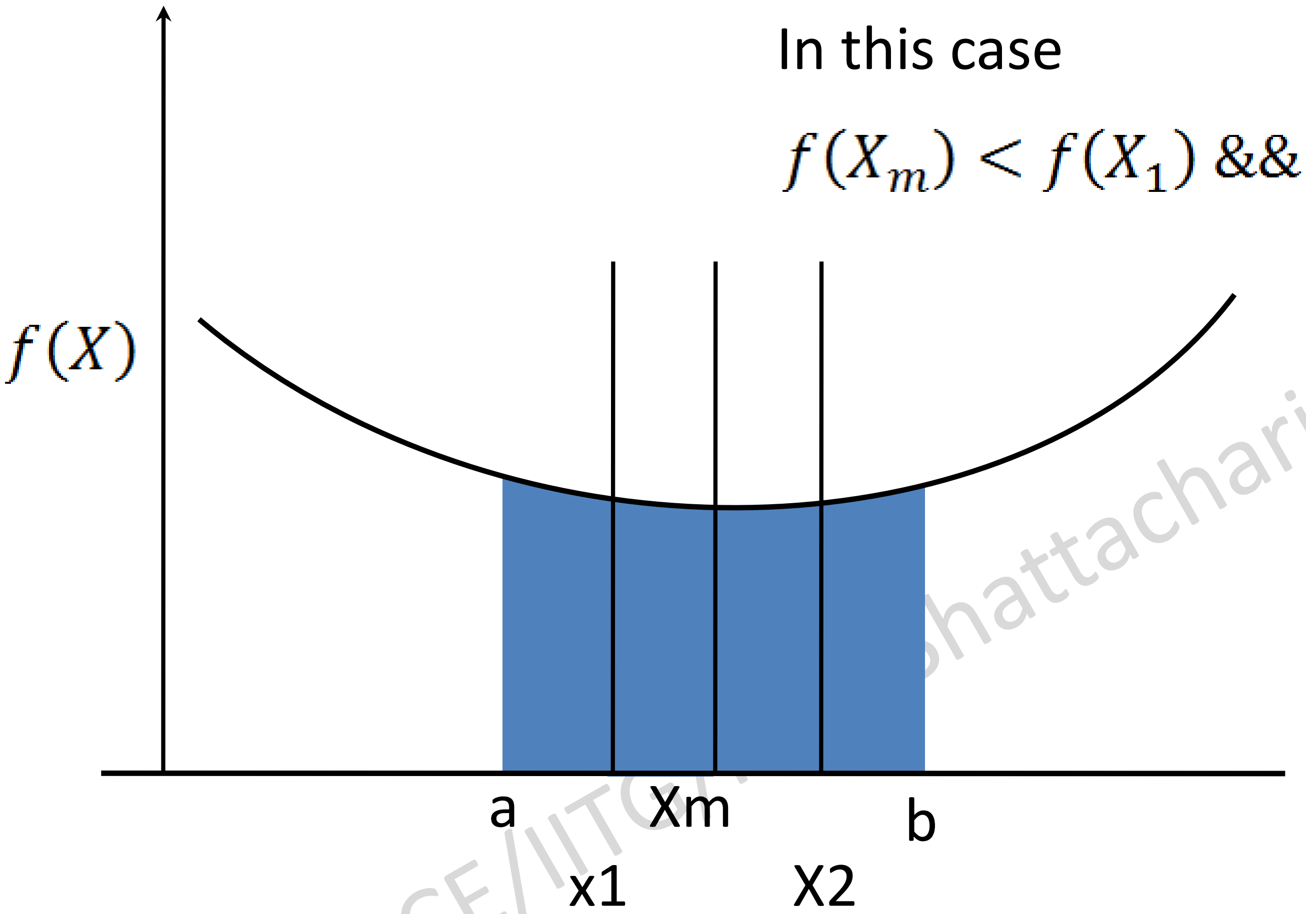
Interval halving method

In this case

$$f(X_m) < f(X_1) \;\&\&\; f(X_m) < f(X_2)$$



Interval halving method

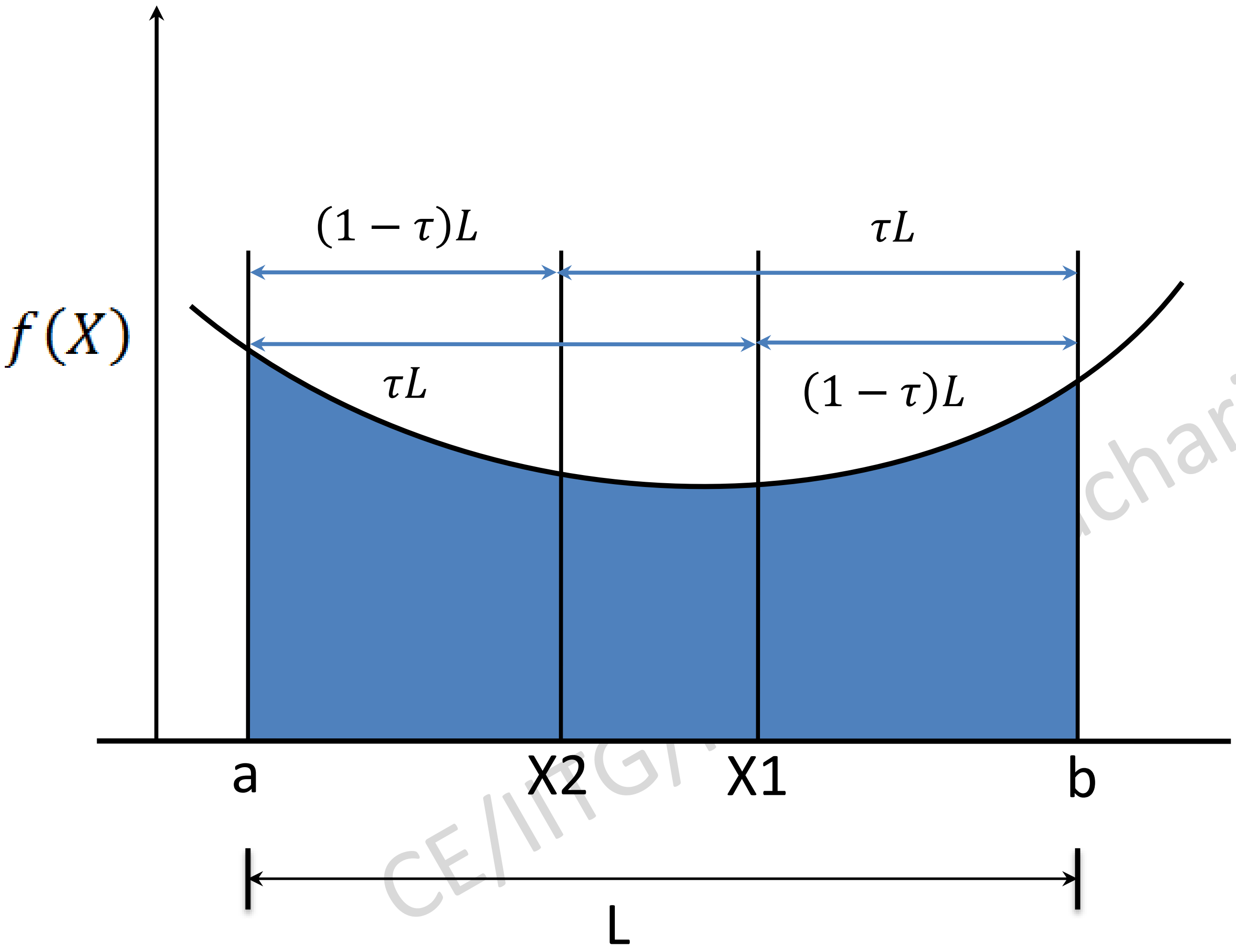


In this case

$$f(X_m) < f(X_1) \;\&\&\; f(X_m) < f(X_2)$$

CONTINUE

Golden Section Search Method

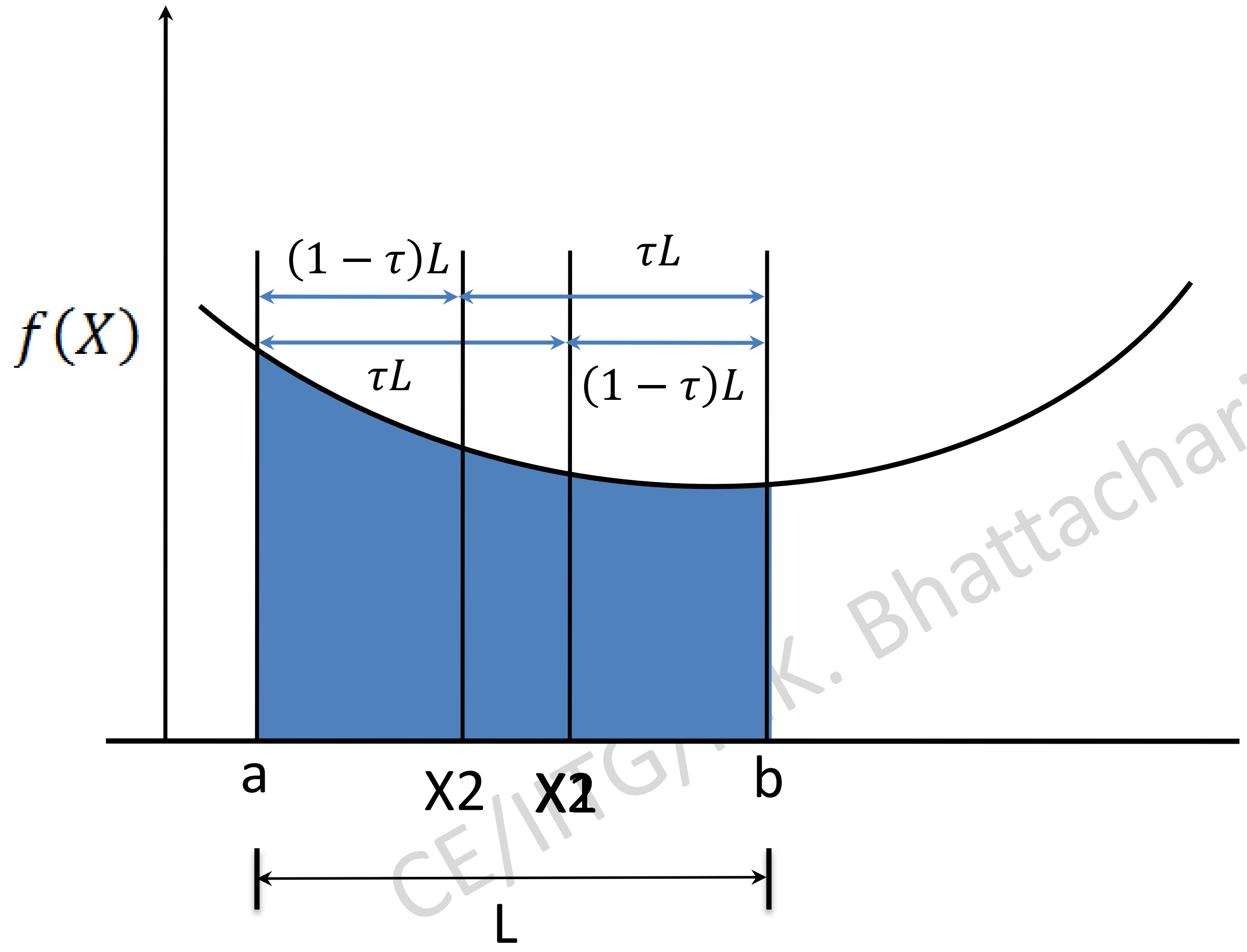


Apply region elimination rules

Suppose

$$f(X_1) > f(X_2)$$

Golden Section Search Method

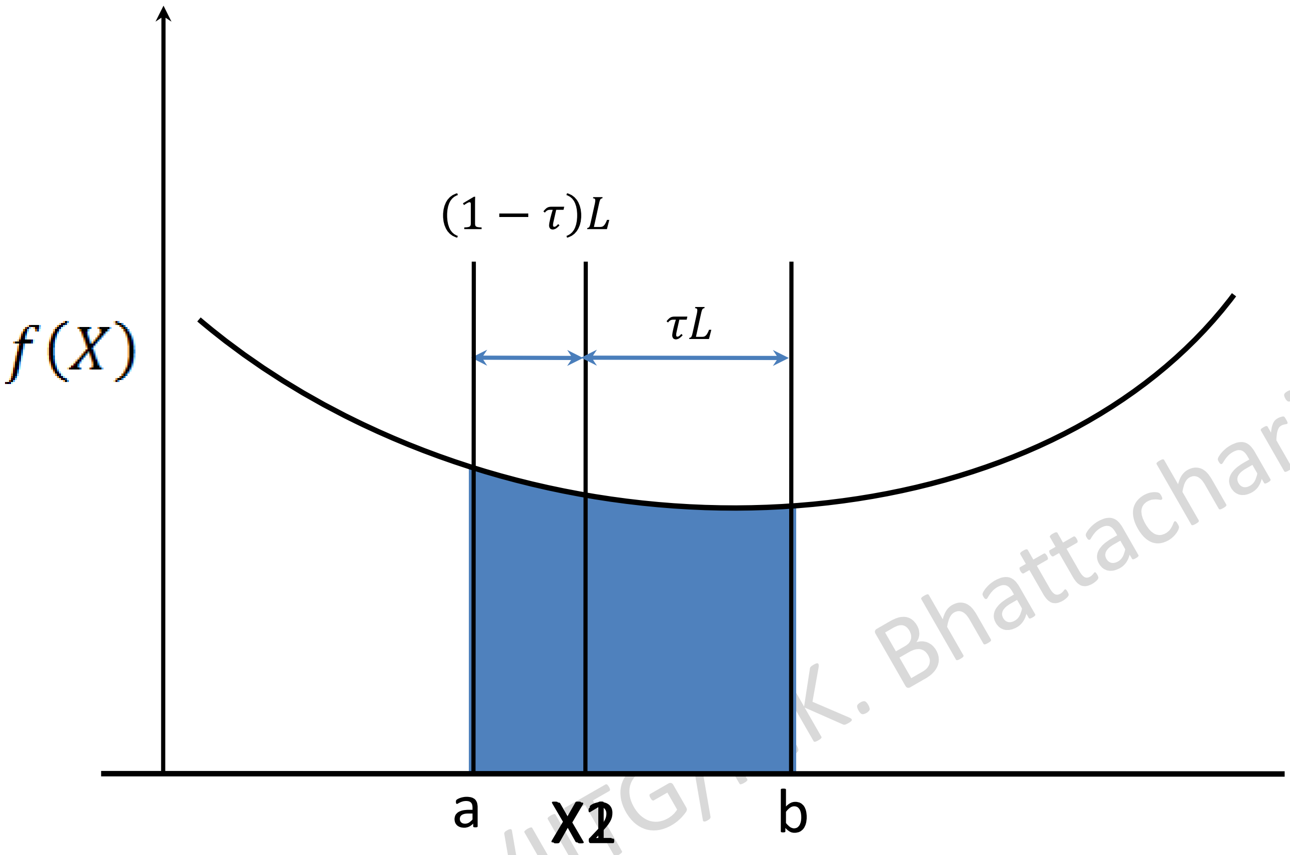


Apply region elimination rules

Suppose

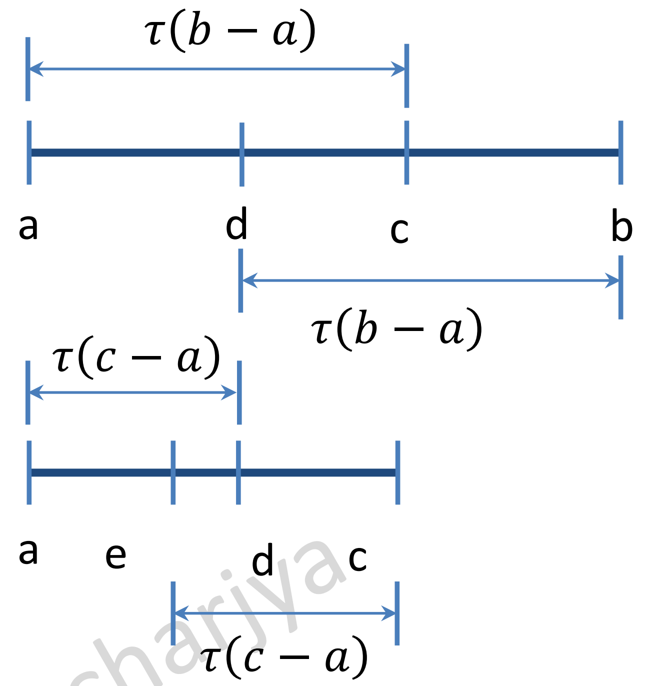
$$f(X_1) < f(X_2)$$

Golden Section Search Method



Continue iteration

Golden section search method



$c = a + \tau(b-a)$ (1)

$d = b - \tau(b-a)$ (2)

If $f(d) < f(c)$

$d = a + \tau(c-a)$ (3)

Putting (1) in (3), we have

$d = a + \tau(a + \tau(b-a) - a)$

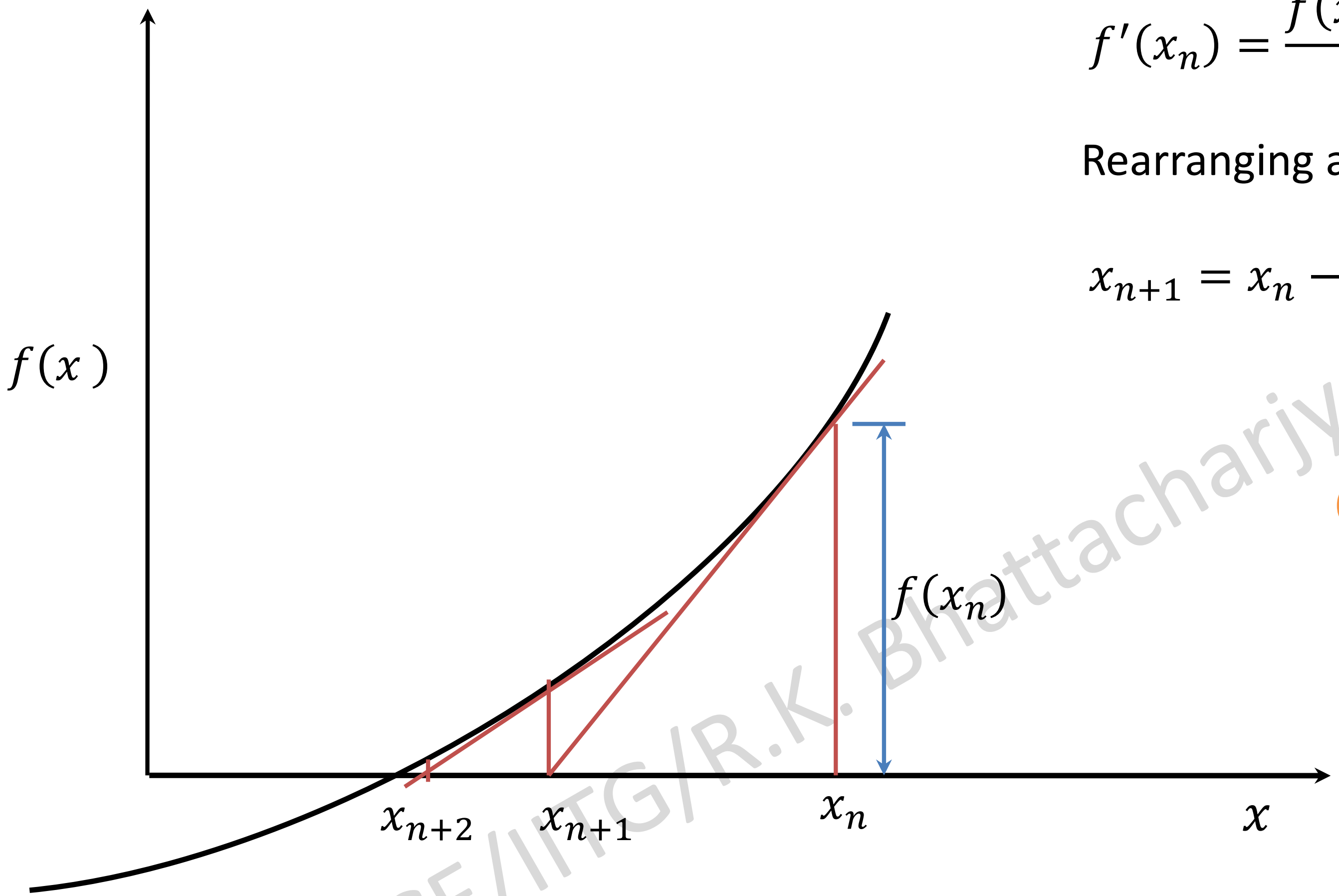
$d = a + \tau^2(b-a)$ (4)

Equating (4) and (2), we have

$b - \tau(b-a) = a + \tau^2(b-a)$

$\tau^2 + \tau - 1 = 0$ Solving τ=0.618, -1.618 0.618 is the golden

Newton-Raphson method



$$f'(x_n) = \frac{f(x_n) - f(x_{n+1})}{x_n - x_{n+1}}$$

Rearranging and putting $f(x_{n+1})$=0

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Continue iteration

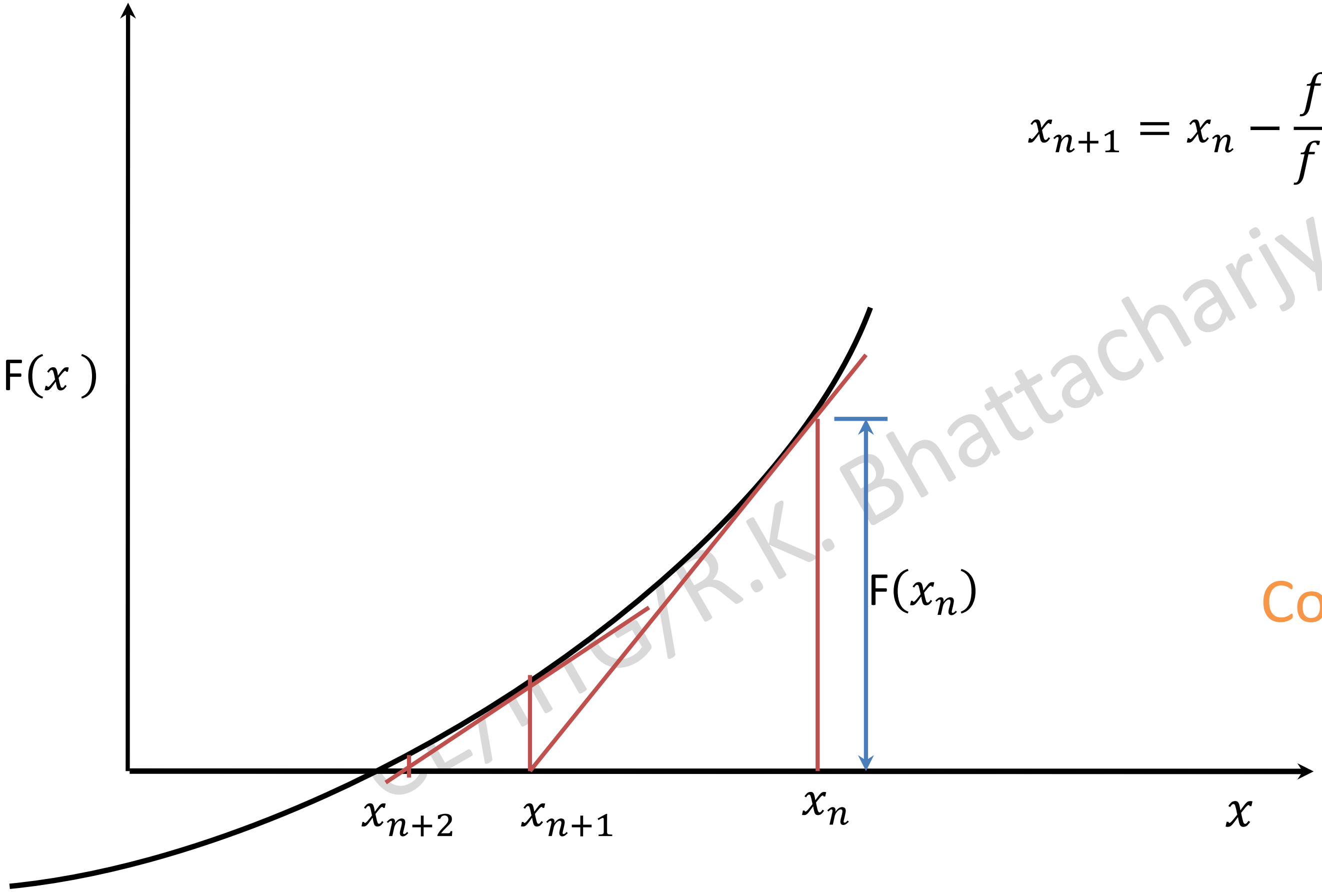
Newton-Raphson method

Incase optimization problem, $f'(x) = 0$

Considering $F(x) = f'(x)$

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$



Continue iteration

QUIZ

1. If $f(x)$ is an unimodal convex function in the interval $[a, b]$, then $f'(a) \times f'(b)$ is

a) Positive
b) Negative
c) It may be negative or may be positive
d) None of the above

2. For the same function, take any point c between $[a, b]$. If$f'(c)$ is less than 0, then minima does not lie in

a) $[a, c]$
b) $[c, b]$
c) $[a, b]$
d) None of the above

2. For the same function, take any point c between $[a, b]$. If$f'(c)$ is greater than 0, then minima does not lie in

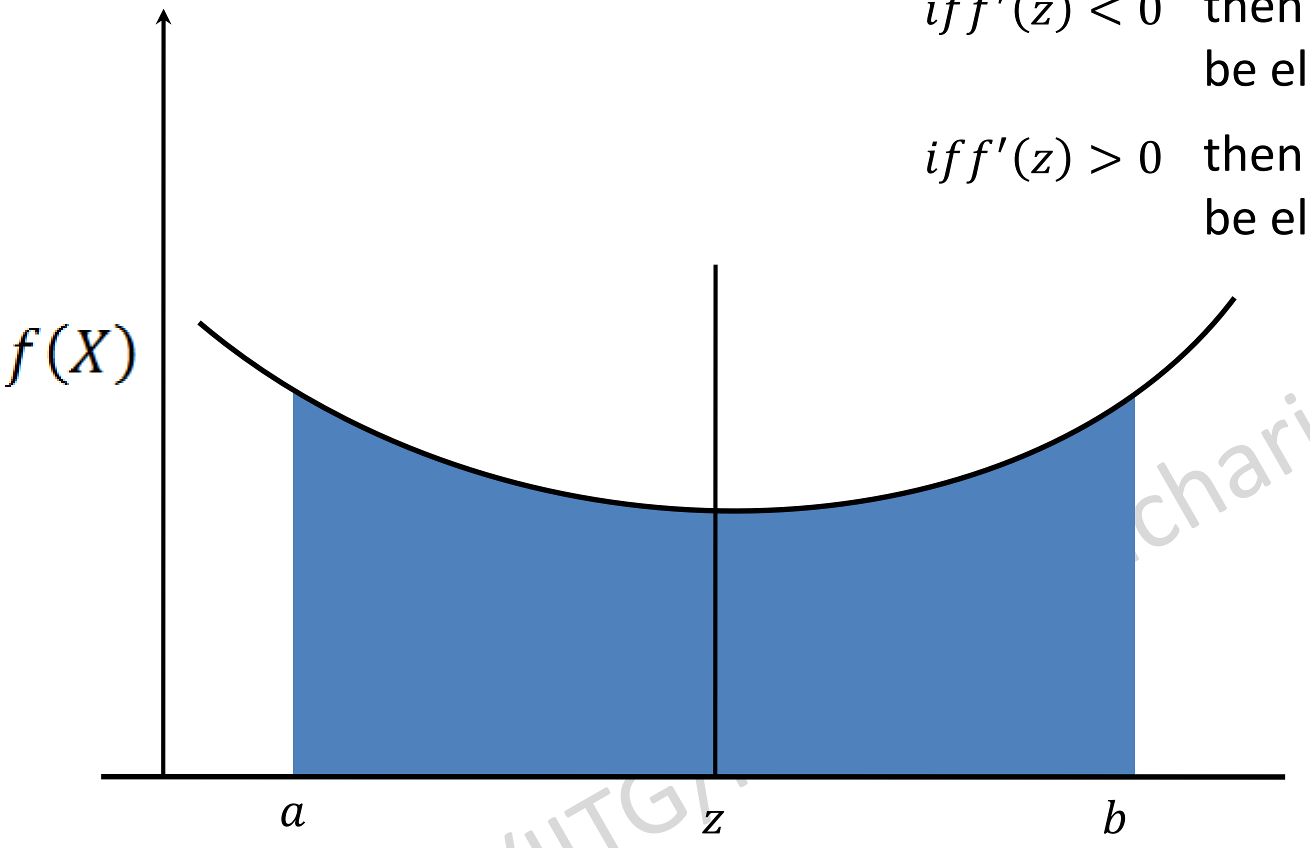
a) $[a, c]$
b) $[c, b]$
c) $[a, b]$
d) None of the above

Bisection method

Take a point $z = \frac{a+b}{2}$

$if\, f'(z) < 0$ then area between $[a, z]$ will be eliminated

$if\, f'(z) > 0$ then area between $[z, b]$ will be eliminated

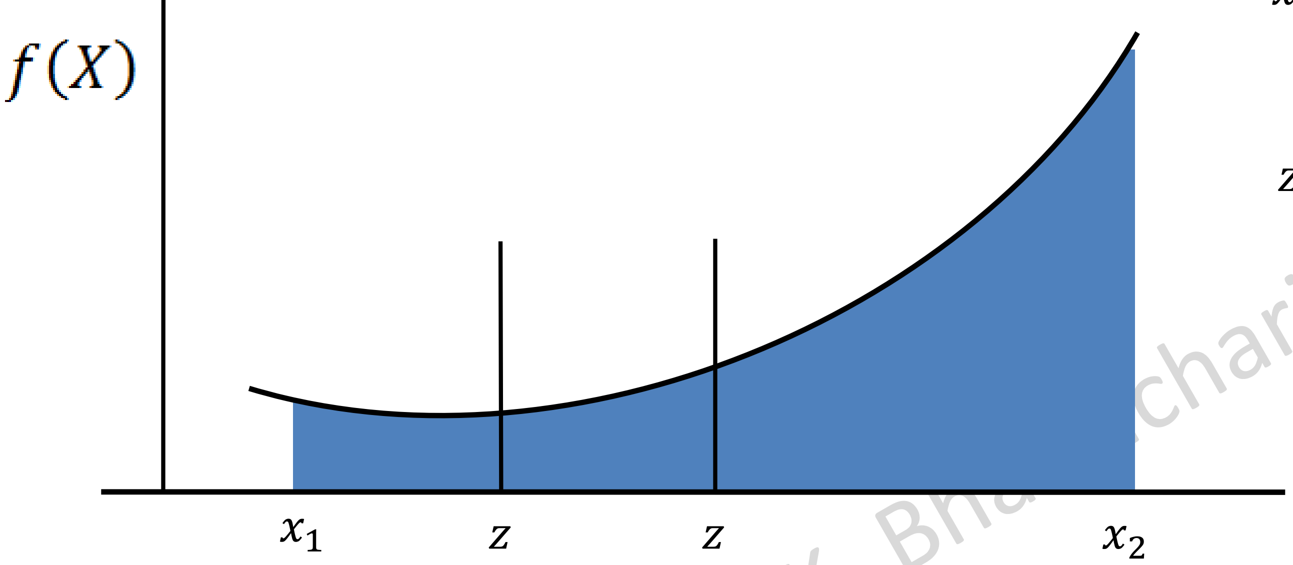


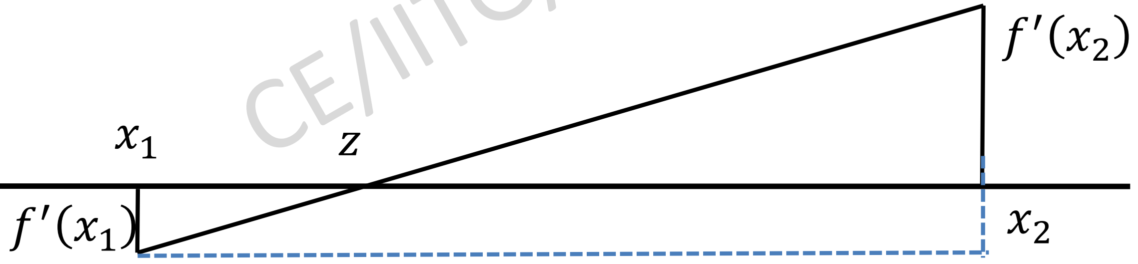
Disadvantage

- Magnitude of the derivatives is not considered

Apply region elimination technique

In this case $f'(z) > 0$ then area between $[z, x_2]$ will be eliminated





Considering similar triangle

$$\frac{f'(x_2)}{x_2 - z} = \frac{f'(x_2) - f'(x_1)}{x_2 - x_1}$$

$$z = x_2 - \frac{f'(x_2)}{\frac{f'(x_2) - f'(x_1)}{x_2 - x_1}}$$