GIAN Course on Distributed Network Algorithms

Distributed Synchronization

«A man with one clock knows what time it is –
a man with two is never sure.»

Synchronization often very helpful to simplify design and analyze algorithms for distributed system and provide consistency. But also challenging....

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Distributed Synchronization

Remember BFS: easy and cheap in LOCAL model, more expensive (time and/or messages) in asynchronous model. E.g., D^2 runtime if we enforce synchronous rounds (Dijkstra algorithm).

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Distributed Synchronization

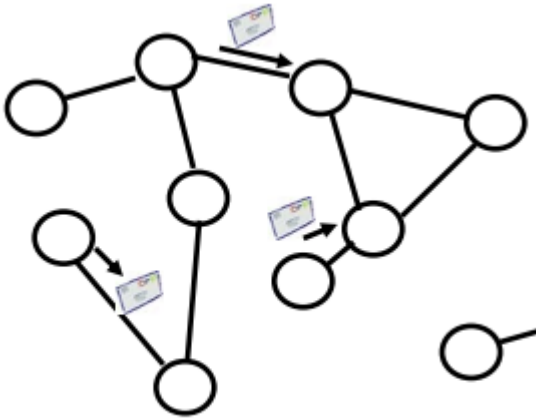
Remember BFS: easy and cheap in LOCAL model, more expensive (time and/or messages) in asynchronous model. E.g., D^2 runtime if we enforce synchronous rounds (Dijkstra algorithm).

Can we execute any LOCAL algorithm in asynchronous environment? Yes, with a synchronizer!

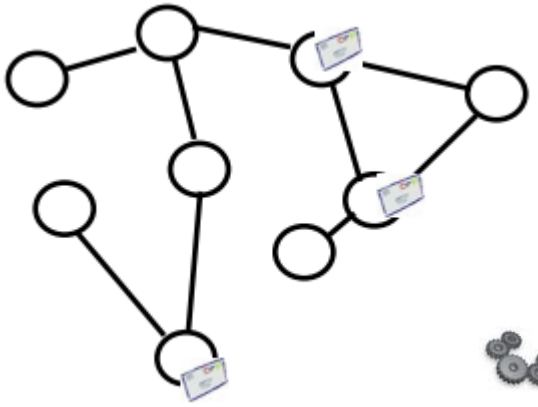
The LOCAL Model: A Synchronous Model

Synchronous LOCAL algorithms simple to design and reason about:

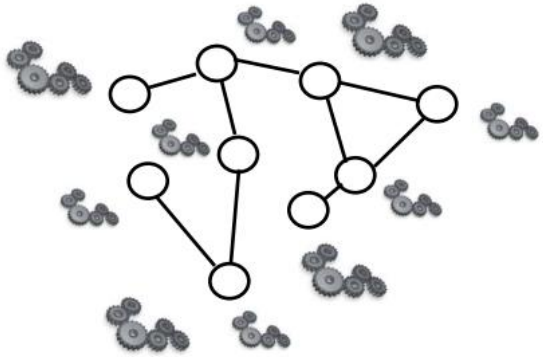
Send...



... receive...



... compute.



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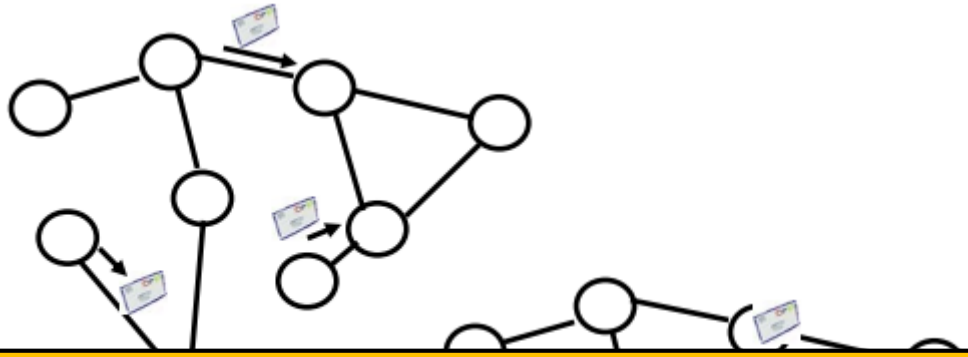

But how to render an asynchronous system synchronous? Run LOCAL algorithm in asynchronous environment: need a **distributed synchronizer**!

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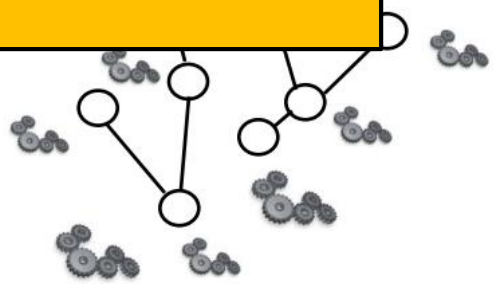
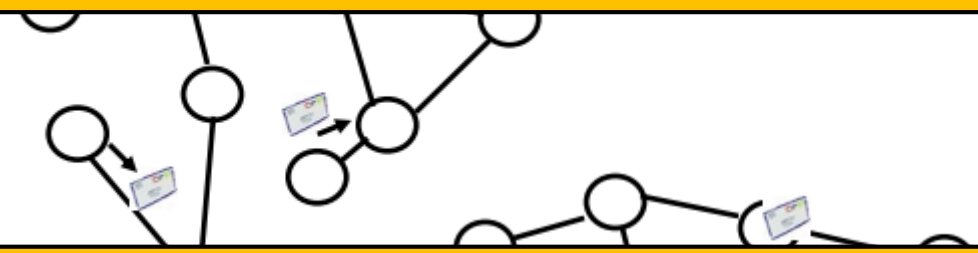


The LOCAL Model: A Synchronous Model

Synchrone[illegible]orithms simple to design and reason about:

Remember BFS: artificially synchronized protocol to make it use less messages in the worst case!

Send...



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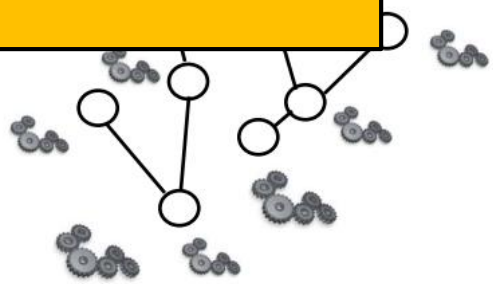
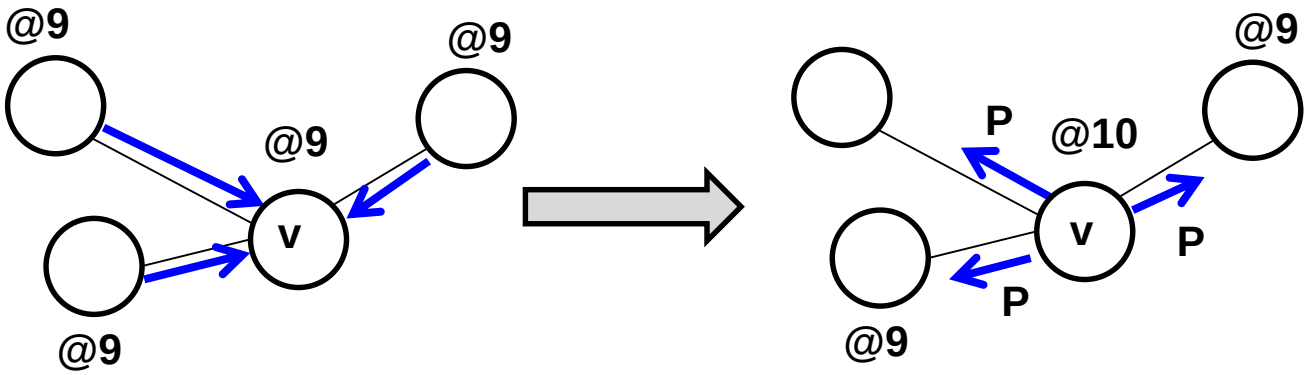


... compute.

Synchronizer

A synchronizer is a distributed algorithm which generates clock pulses (PULSE) at each node.



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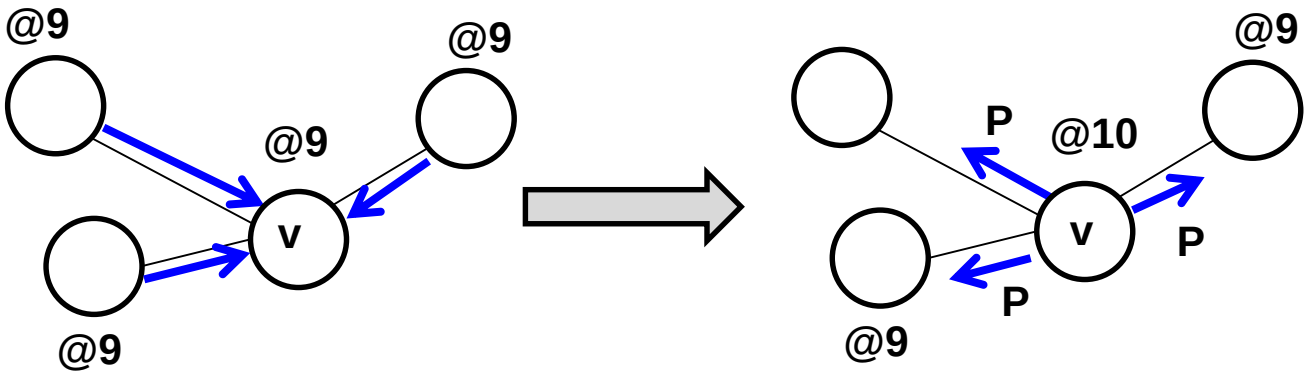

Valid Clock Pulse

A pulse generated at some node v is valid, iff it is generated after v received all the messages of the synchronous algorithm sent to v by its neighbors in the previous pulses.

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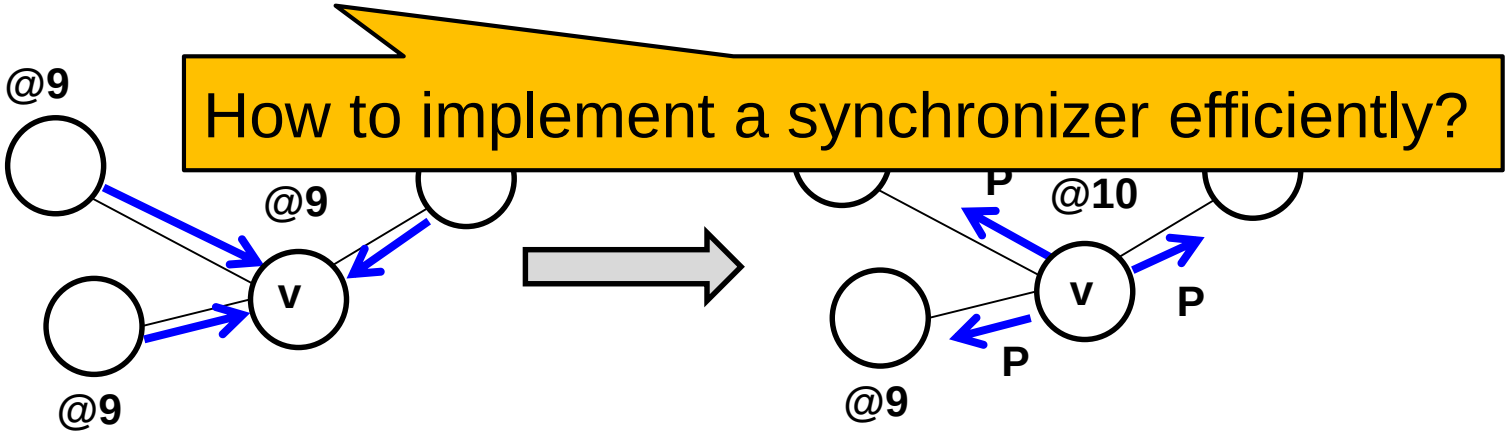

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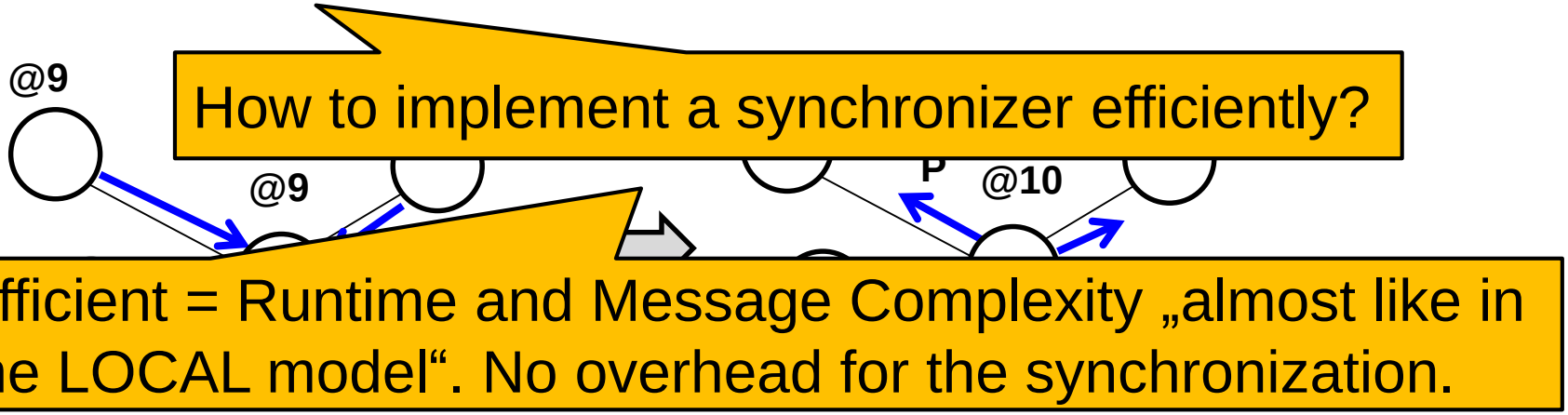

Valid Clock Pulse

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Formally: Overhead of Using Synchronizers

Overhead: Message needed for each pulse, independent of protocol data!

Overhead

Say T(A) and M(A) are time and message complexity of synchronous algorithm A, and T(S) and M(S) are complexities of a synchronizer for each pulse. Moreover, $T_{init}(S)$ and $M_{init}(S)$ to set up synchronizer.
Then:

$$T = T_{init}(S) + T(A)*(1+T(S)),\ M = M_{init}(S) + M(A) + T(A)*M(S)$$

Some setup costs.

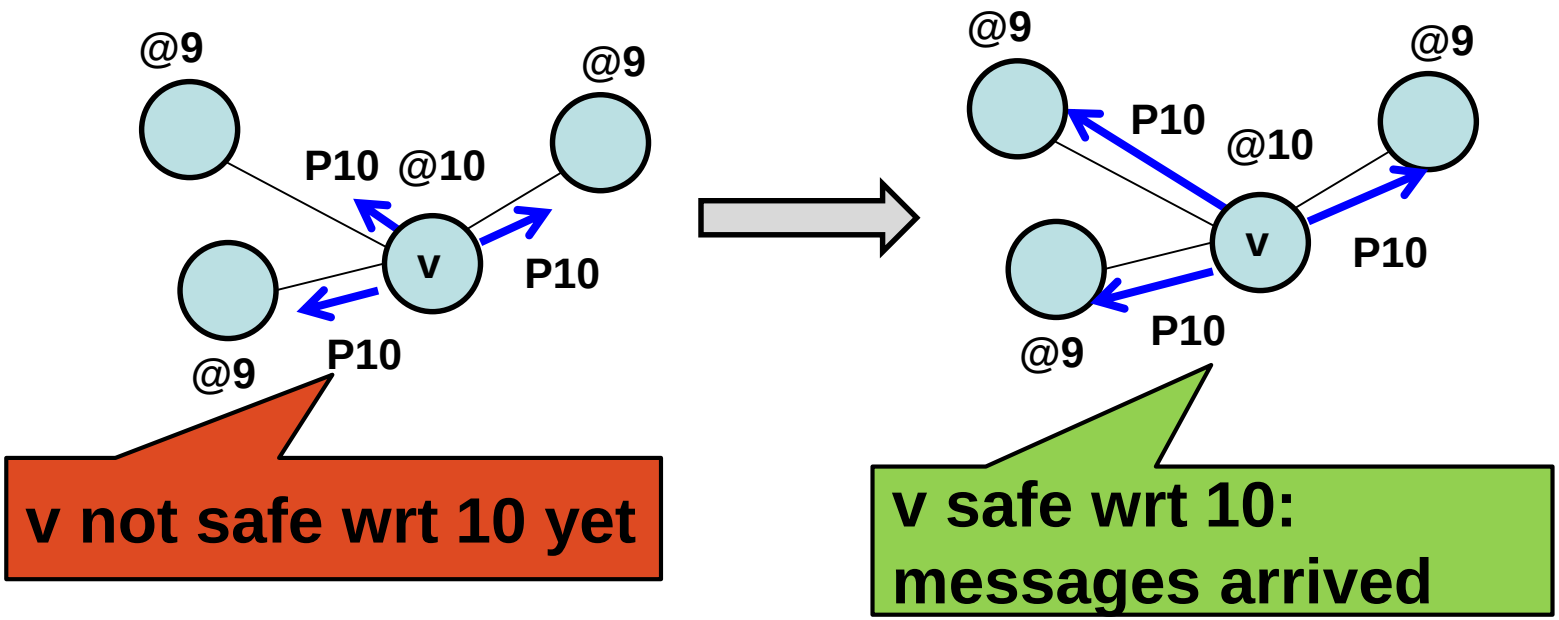
Extra rounds per LOCAL round.

Maybe additionally messages M(S) per each LOCAL round.

Safe Node

A node v is safe wrt certain clock pulse if all messages of the synchronous algorithm sent by v in that pulse have already arrived *at their destination*.

Idea: v at least knows what it sent itself!



Safe Node

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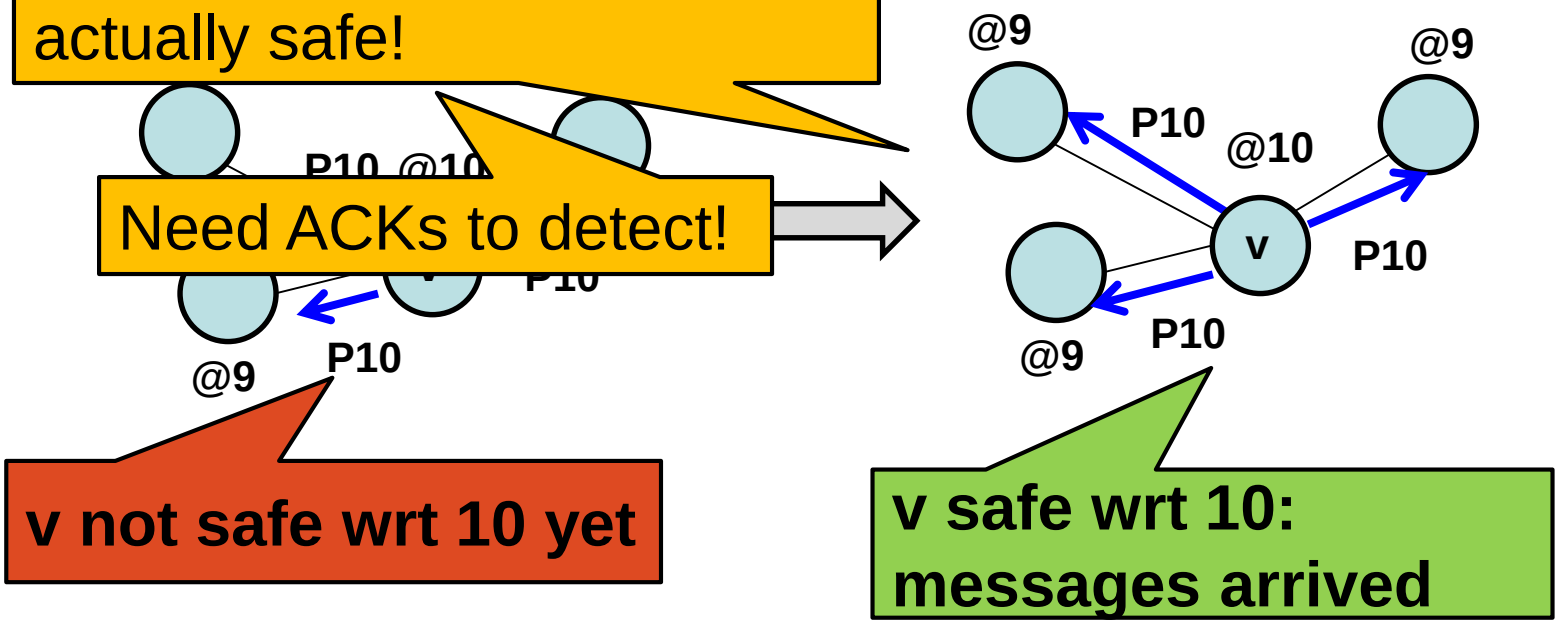
How to detect? At this point, v cannot know that it is actually safe!

sent itself!



Note: Once all neighbors of v are safe, v can generate the next pulse: it has received their messages for this round. This pulse must be valid.

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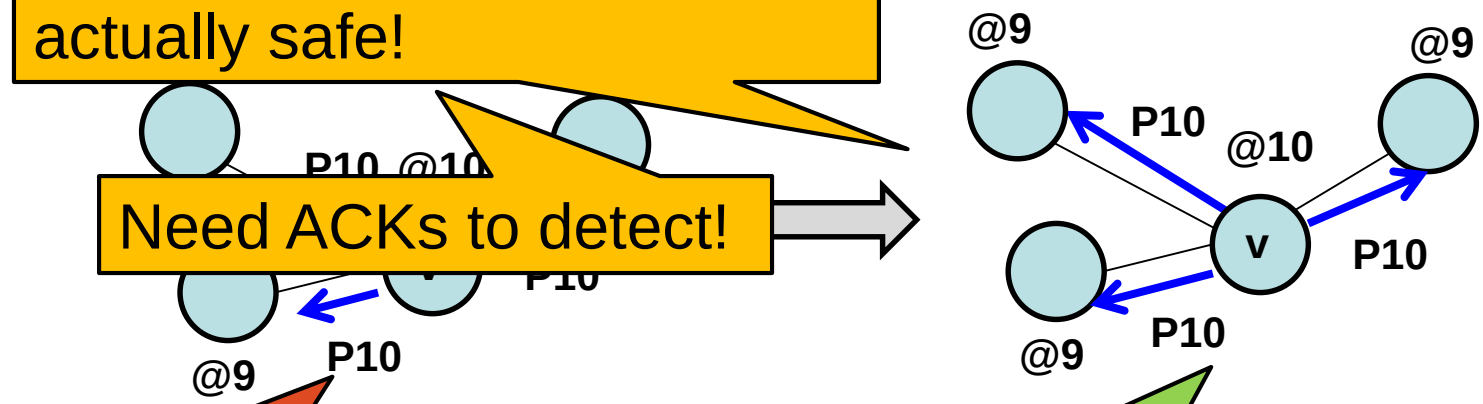
Need ACKs to detect!



v not safe wrt 10 yet

v safe wrt 10: messages arrived

AKA Local Synchronizer α!

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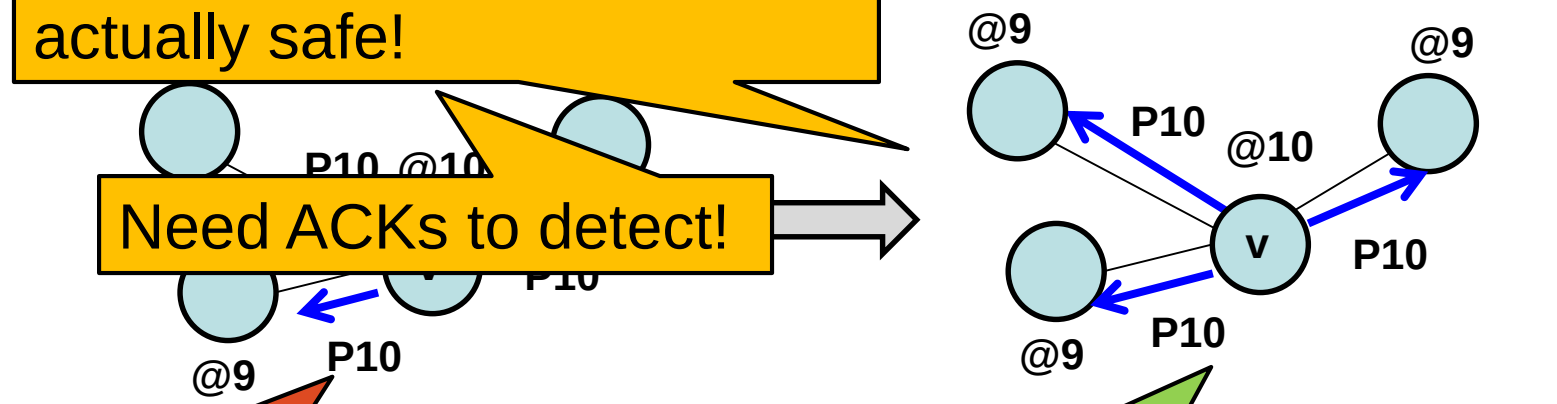

Need ACKs to detect!

v not safe wrt 10 yet

v safe wrt 10: messages arrived

The Local Synchronizer α

Synchronizer α

v's messages arrived!

At node v:

wait until v is safe (learn via ACKs)
send SAFE to all neighbors
wait until v receives SAFE messages from all neighbors
start new PULSE

v got their messages!

We do this for each regular LOCAL round!

The Local Synchronizer α

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Overhead?

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Time good: just O(1) interactions with neighbors: no initialization and local!

Synchronizer α

Overhead per synchronous round:

$T(\alpha) = O(1)$
$M(\alpha) = O(m)$

The Local Synchronizer α

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Overhead per synchronous round:

T(α) = O(1)
M(α) = O(m)

Messages not so good: *Every edge* sees 6 messages (PULSE, SAFE, ACK in both directions), in each round! Independently of whether LOCAL used this edge or not.

The Local Synchronizer α

Synch

At node v:

wai

send SAFE to all neighbors

wait until v receives SAFE messages from all neighbors

start new PULSE

The global synchronizer β provides the opposite tradeoff!

Time good: just O(1) interactions with neighbors: no initialization and local!

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Overhead per synchron

T(α) = O(1)

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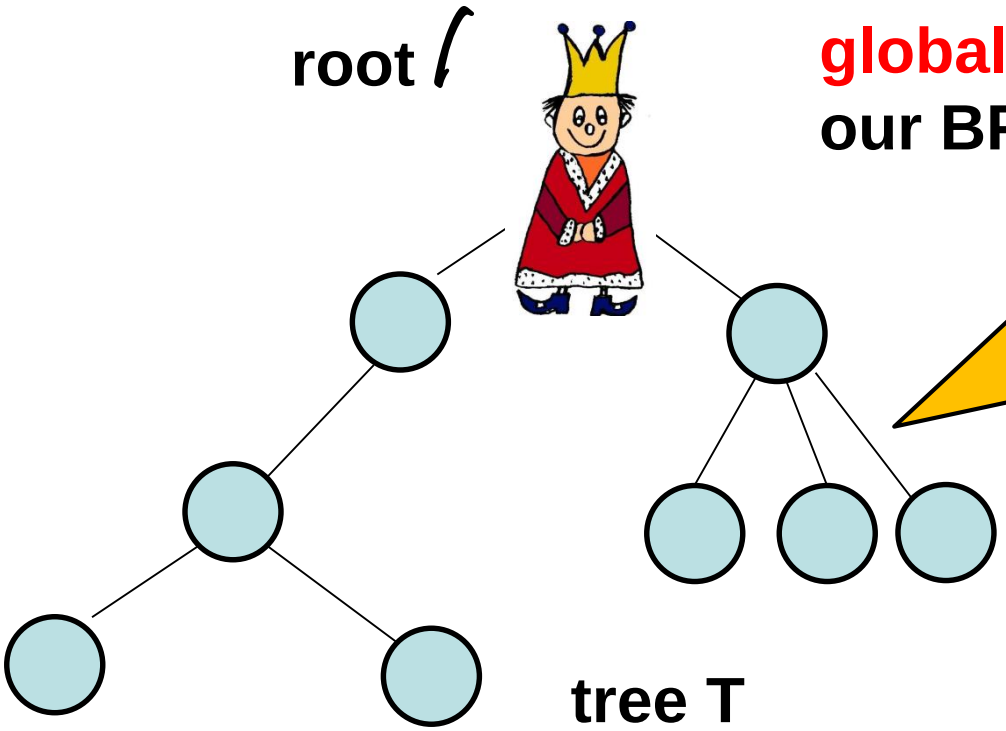
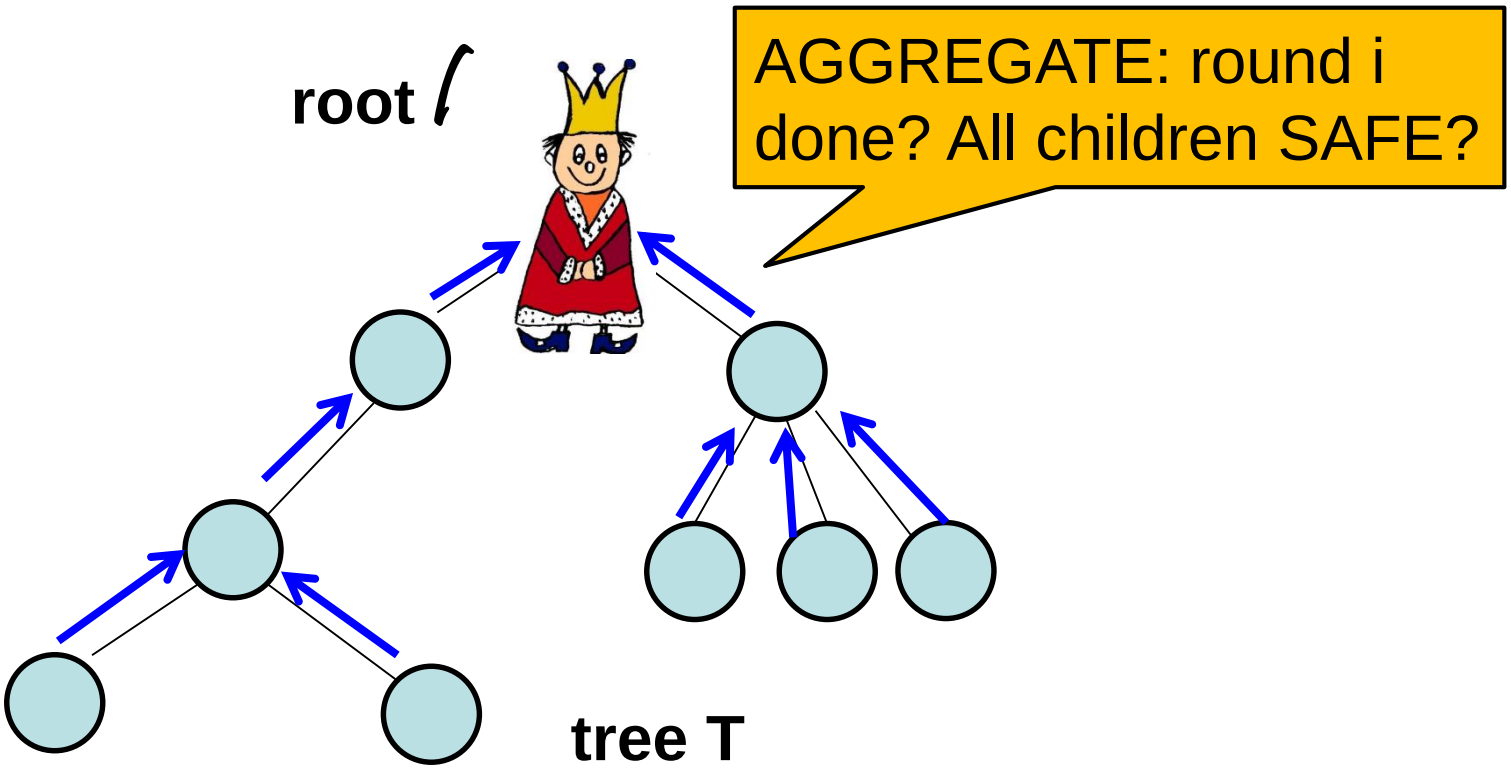
The Global Synchronizer β



Idea: leader coordinates global phases: Remember our BFS algorithm!

In each round essentially a ConvergeCast: broadcast PULSE and aggregate SAFE subtrees!

The Global Synchronizer β



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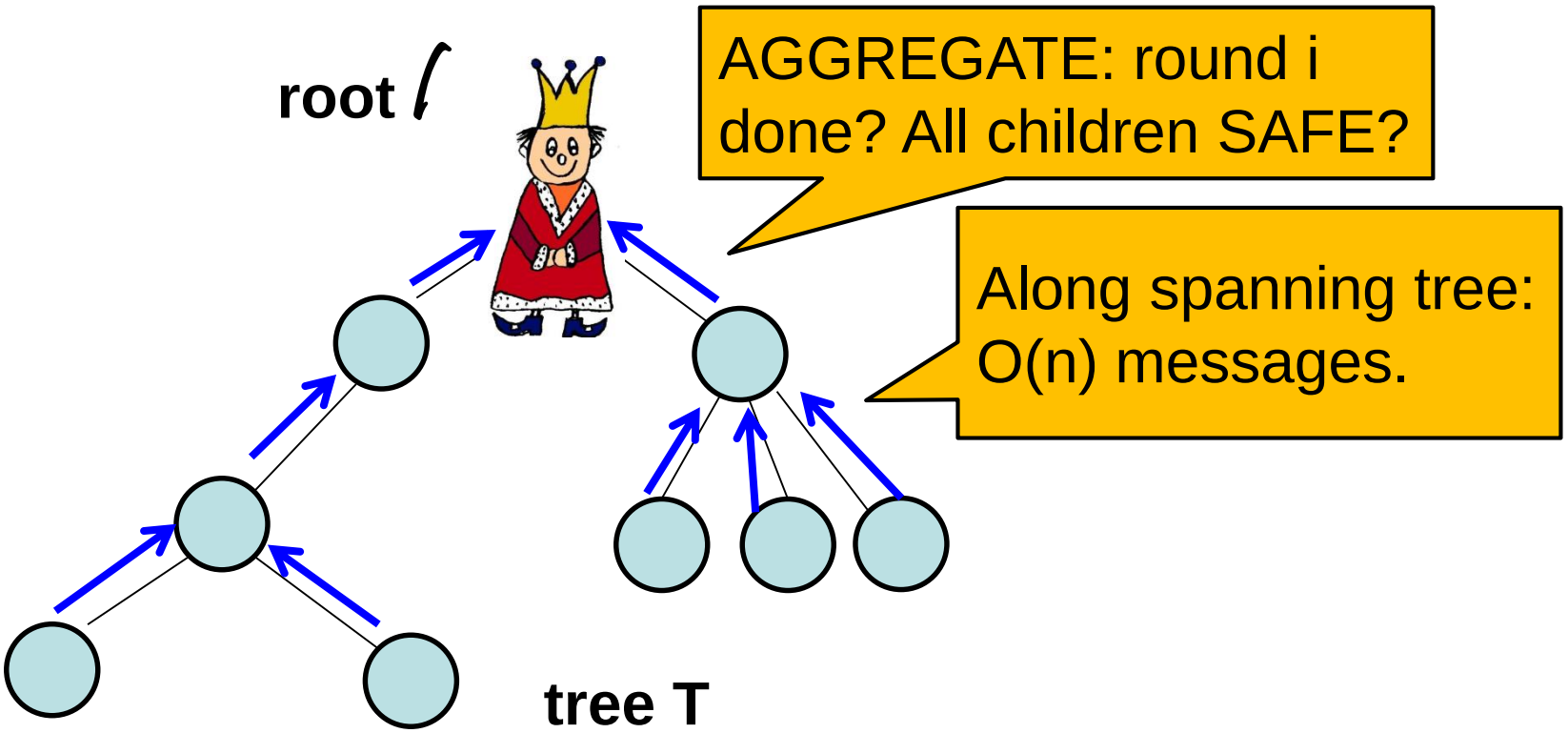

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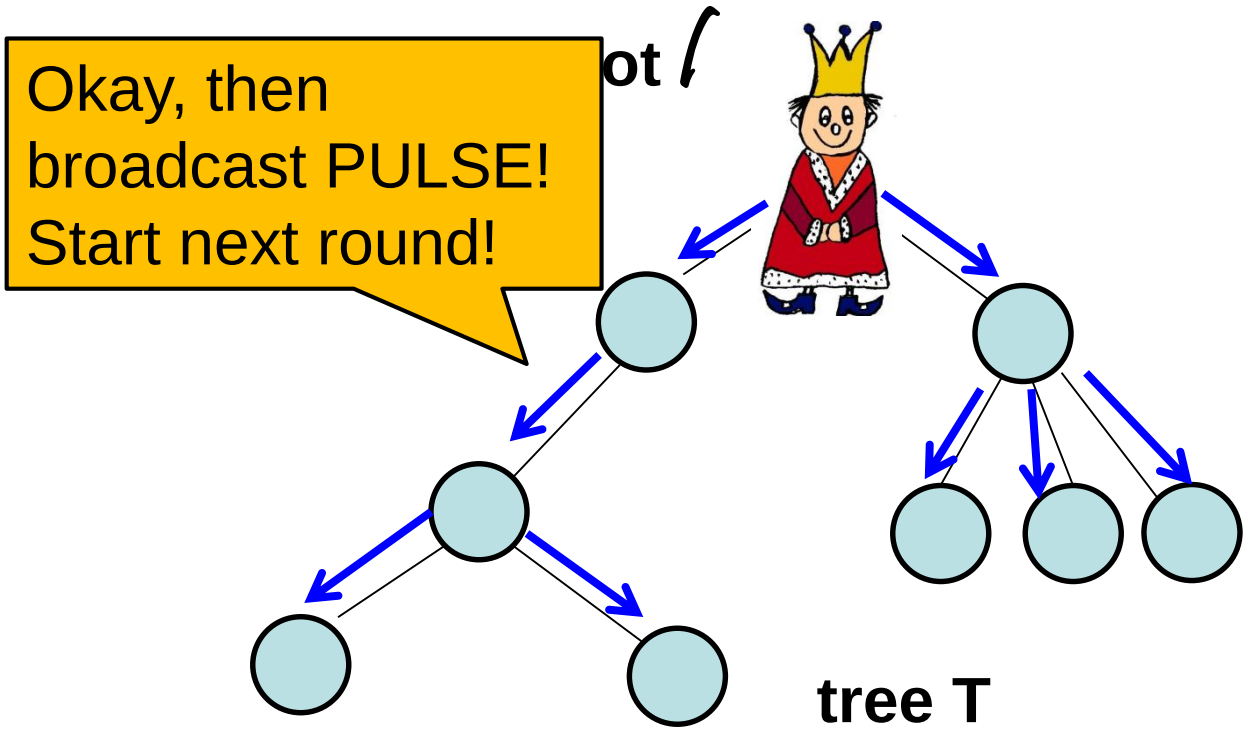

Synchronizer β

At node v:

wait until v is safe
wait until v receives SAFE message from all its children in tree
only then send SAFE message to parent in T
wait until PULSE received from parent
send PULSE to children
start PULSE

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Synchronizer β

Complexities per synchronous round:

$T(\beta) = O(\text{diam } T) = O(n)$
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Message cheap: convergecasts along spanning tree edges only!

Plus initialization: leader election!

A Tradeoff:

Synchronizer α

Overhead per synchronous round:

$T(\alpha) = O(1)$

$M(\alpha) = O(m)$

Fast but many messages.

Slow but message efficient.

Synchronizer β

Complexities per synchronous round:

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Can we get the best of both worlds?

A Tradeoff:

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Not so bad in sparse graphs: m small!

Slow but message efficient.

Not so bad in low diameter graphs! Diam small.

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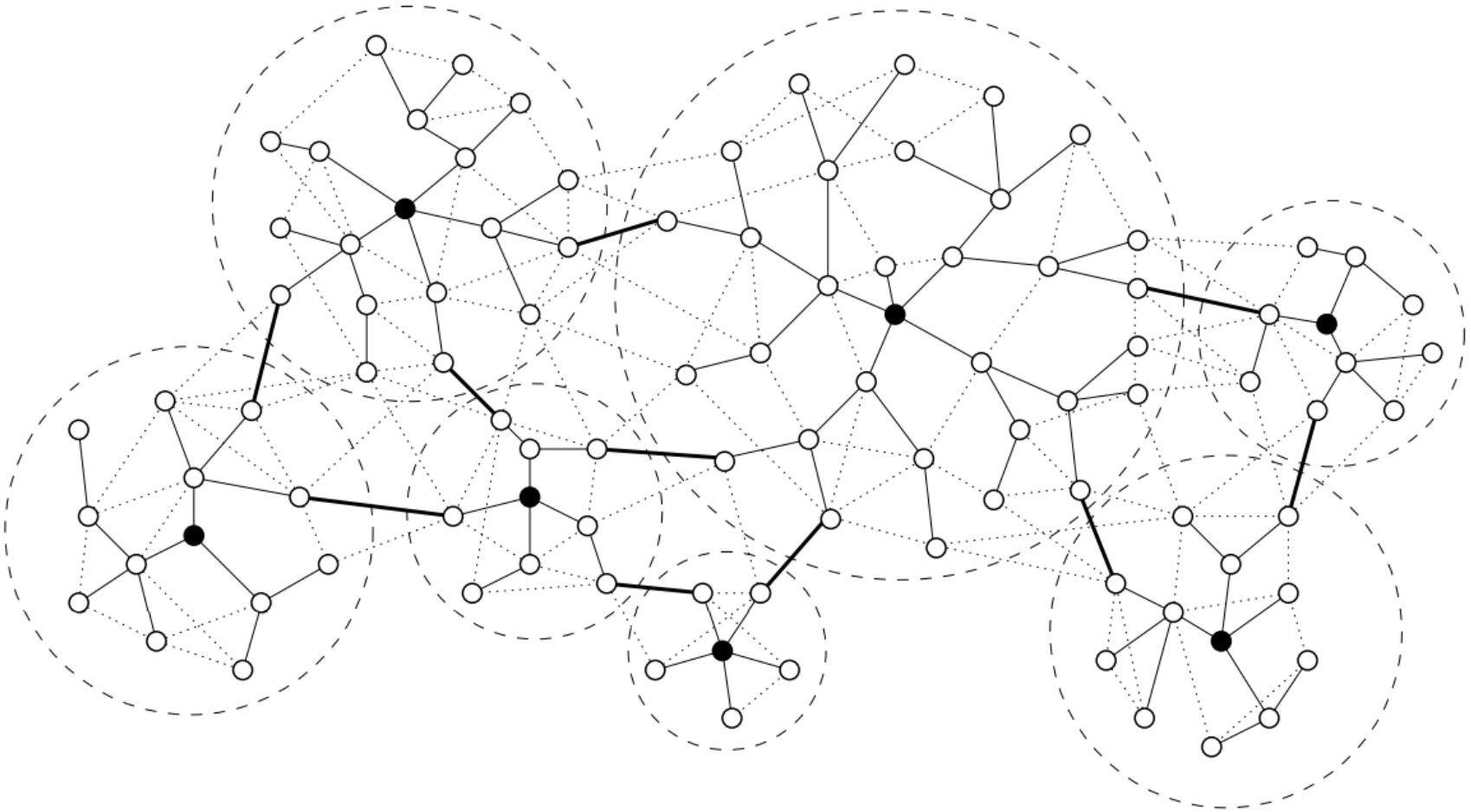
Idea: partition the network into low-diameter clusters with sparse interconnections! Inside: can use **β** synchronizer (small diameter), across cluster can use **α** synchronizer (number of messages m small)!

Can we get the best of both worlds?

The Hybrid Synchronizer γ

Idea: Execute β intra- and α inter-cluster

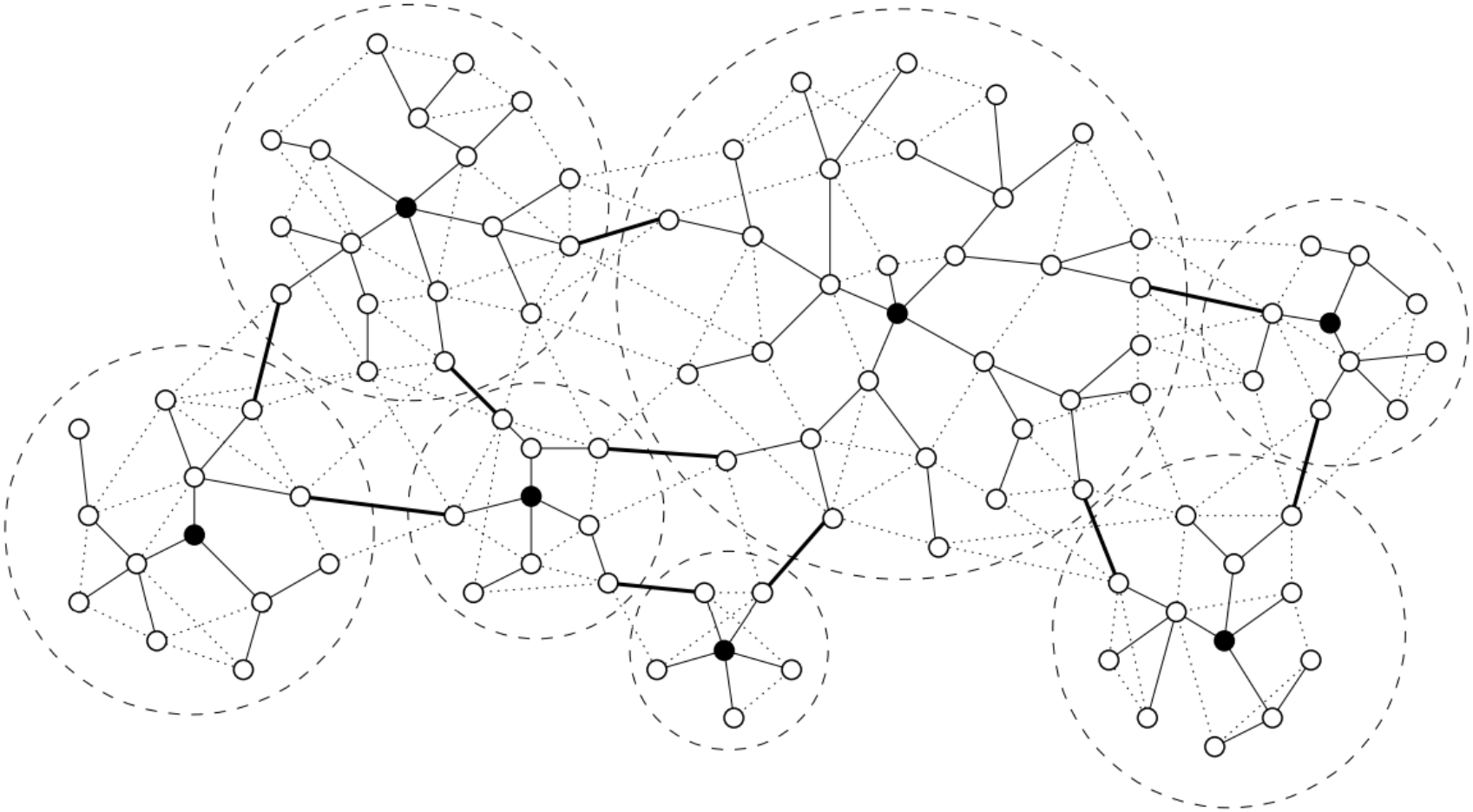


The … er γ

Small diameter: convergecast fast.

Number of edges m: small.

Idea: Execute β intra- and α inter-cluster



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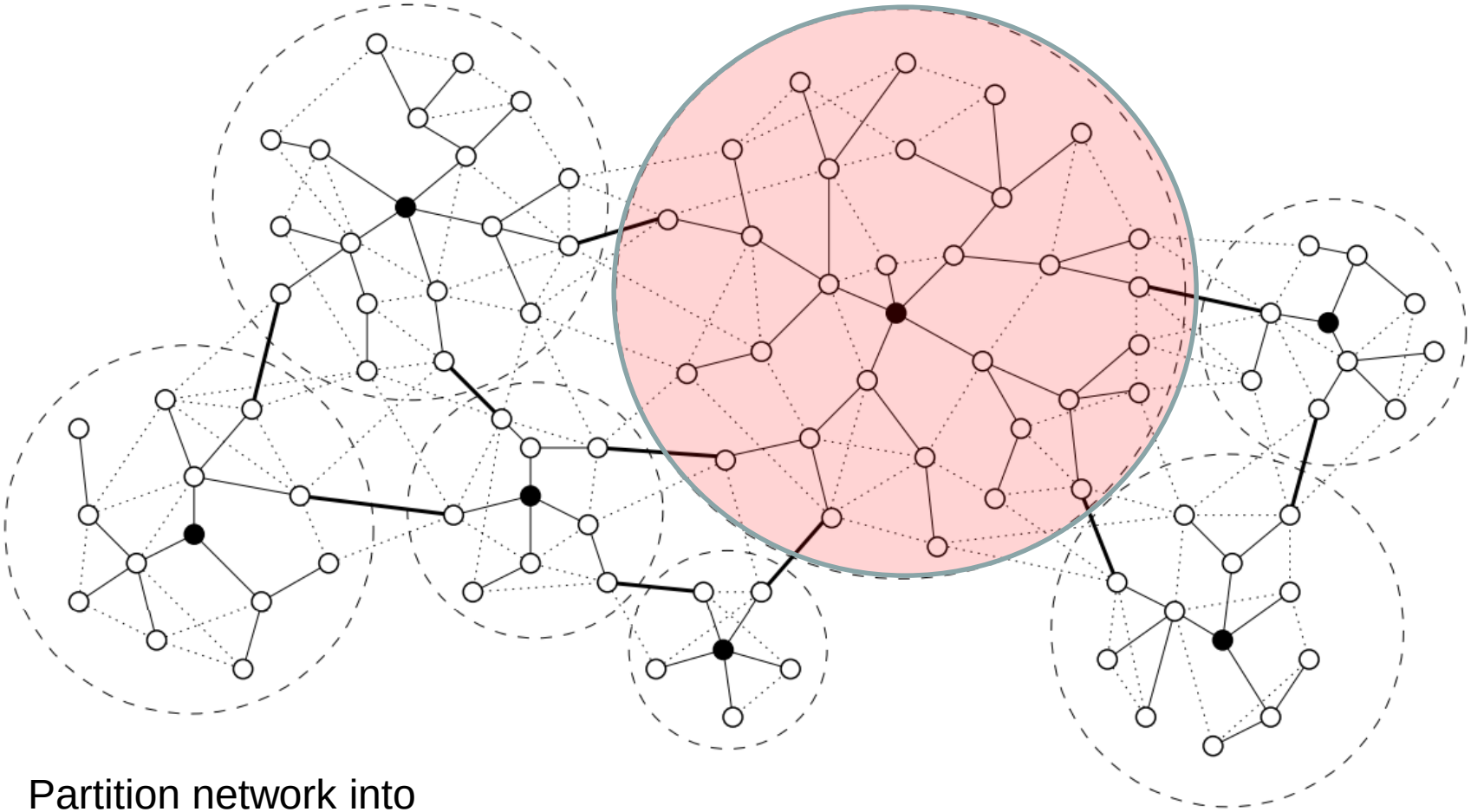

Partition network into
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The Hybrid Synchronizer γ

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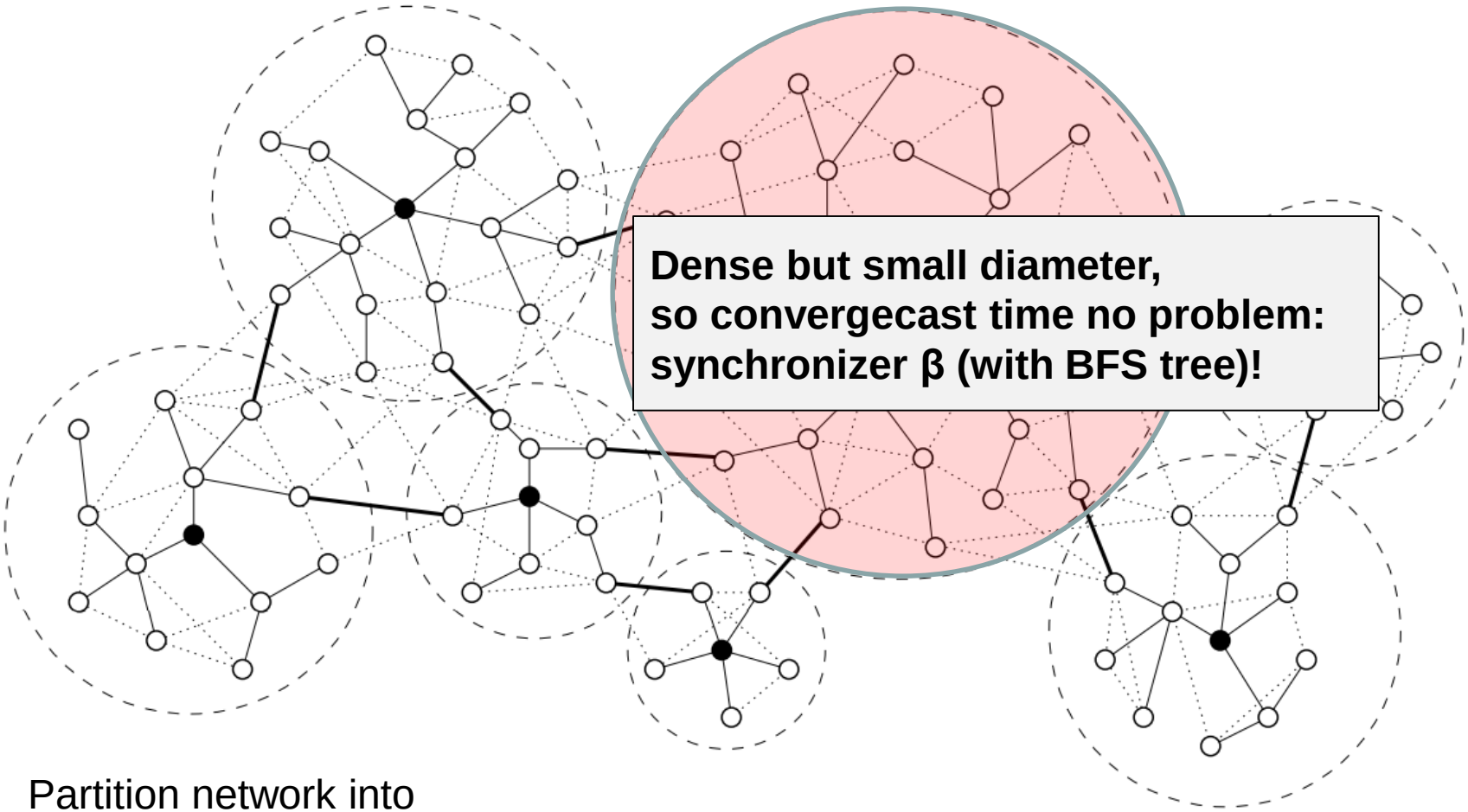

Partition network into
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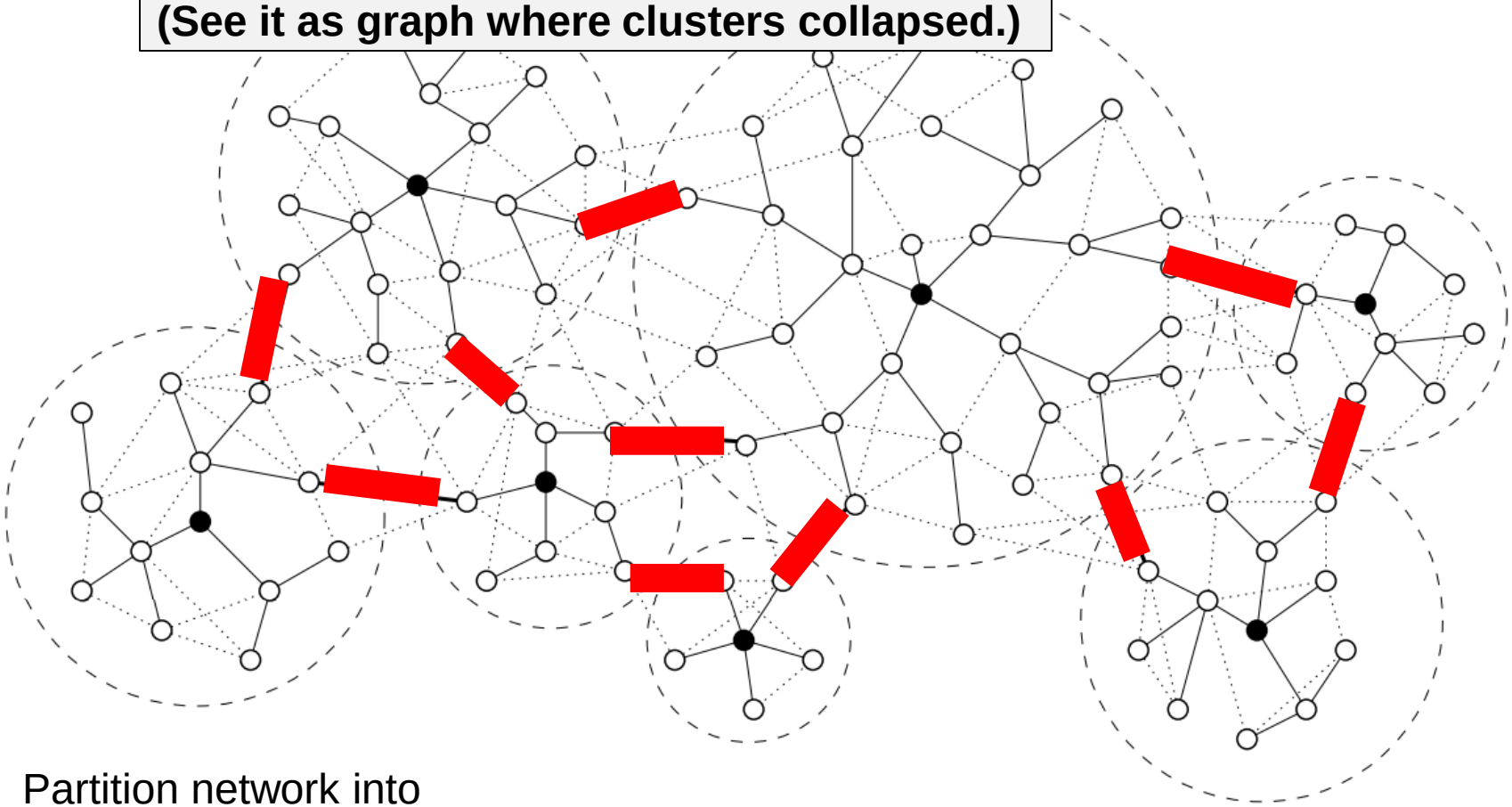
The Hybrid Synchronizer γ

Idea:

Between clusters, local synchronizer α!
Small number of edges.
(See it as graph where clusters collapsed.)



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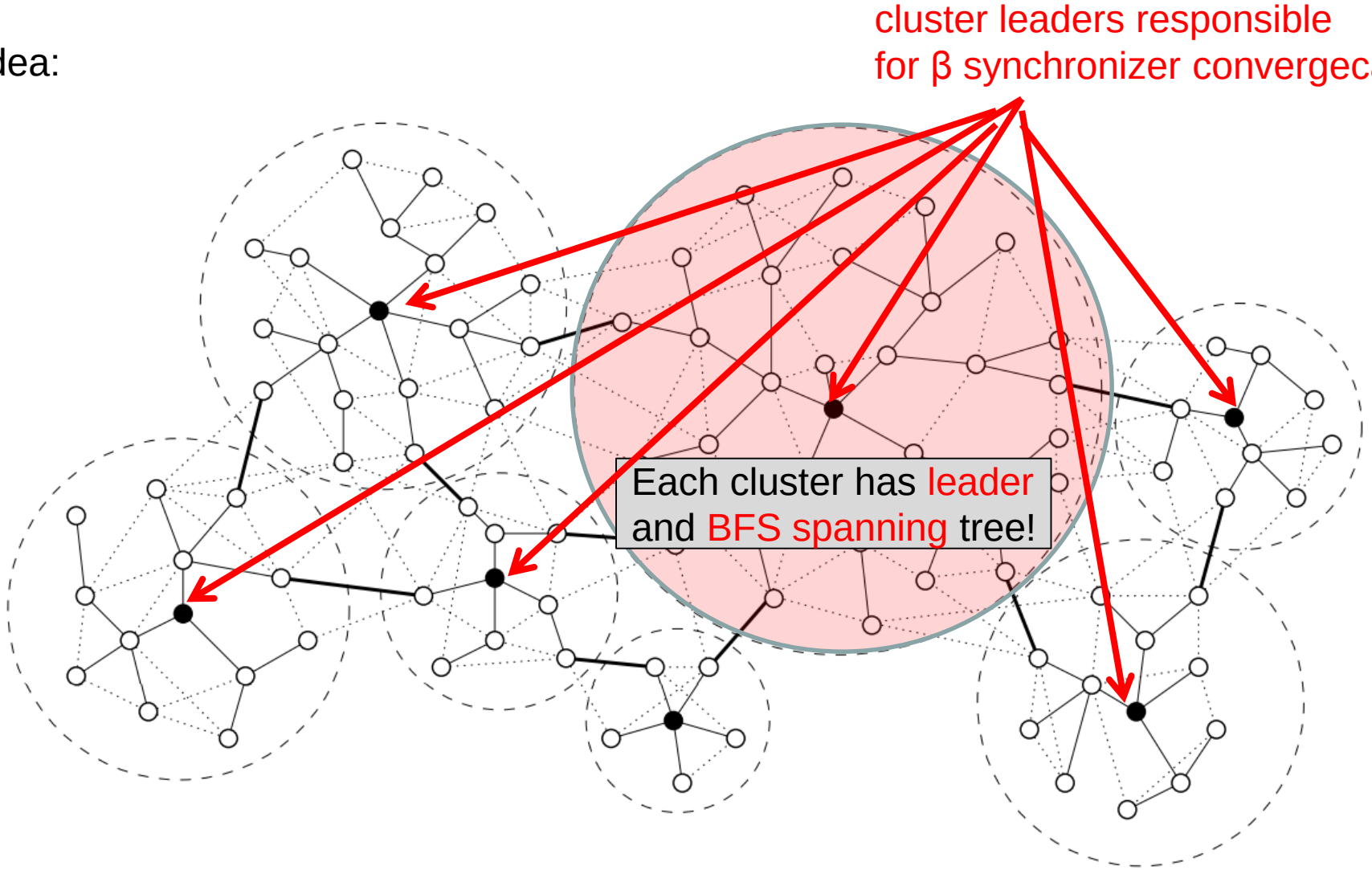
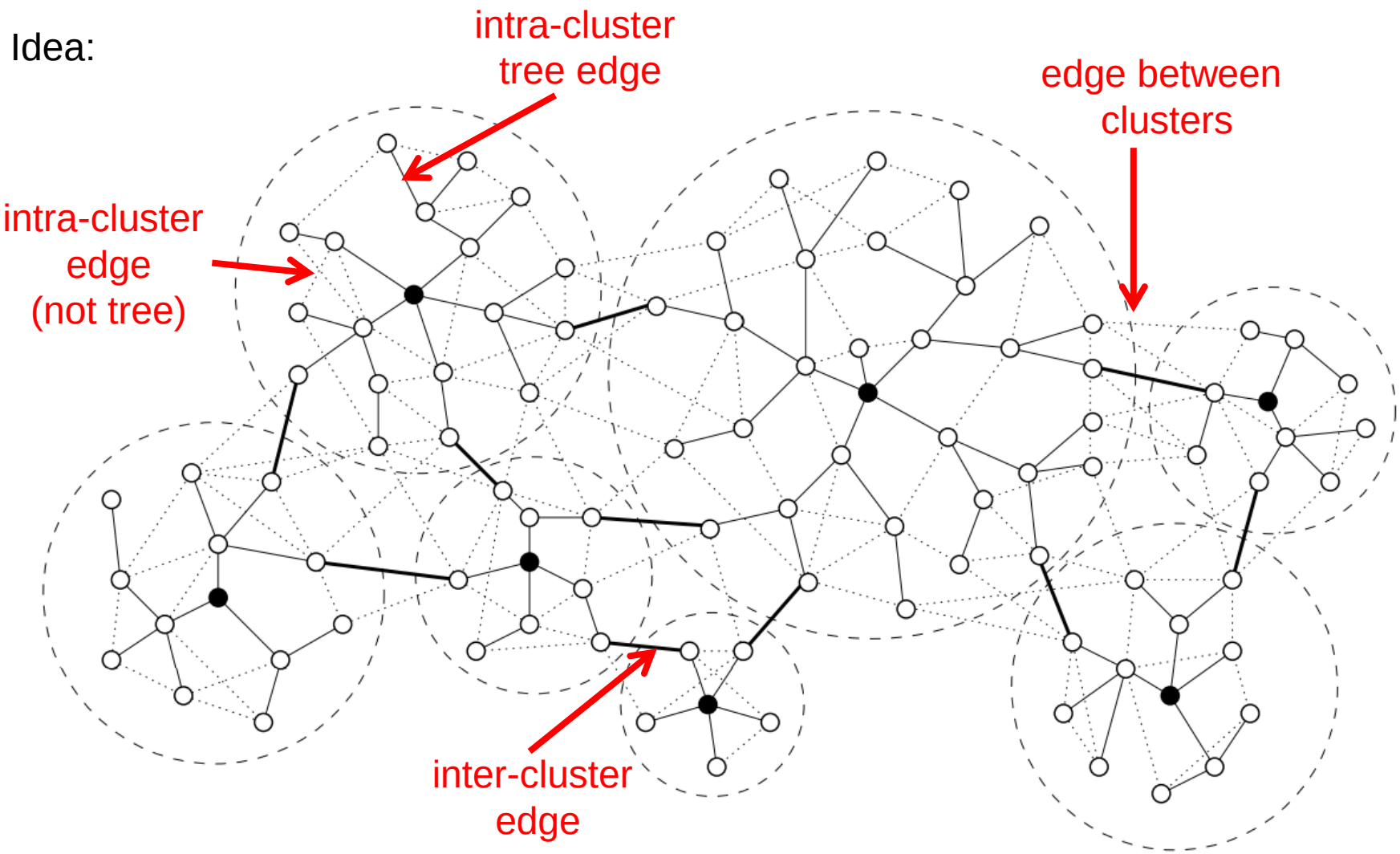


Edge Types in the Hybrid Synchronizer γ

Idea:



The Hybrid Synchronizer γ

Idea: First make cluster safe, then make inter-cluster safe.

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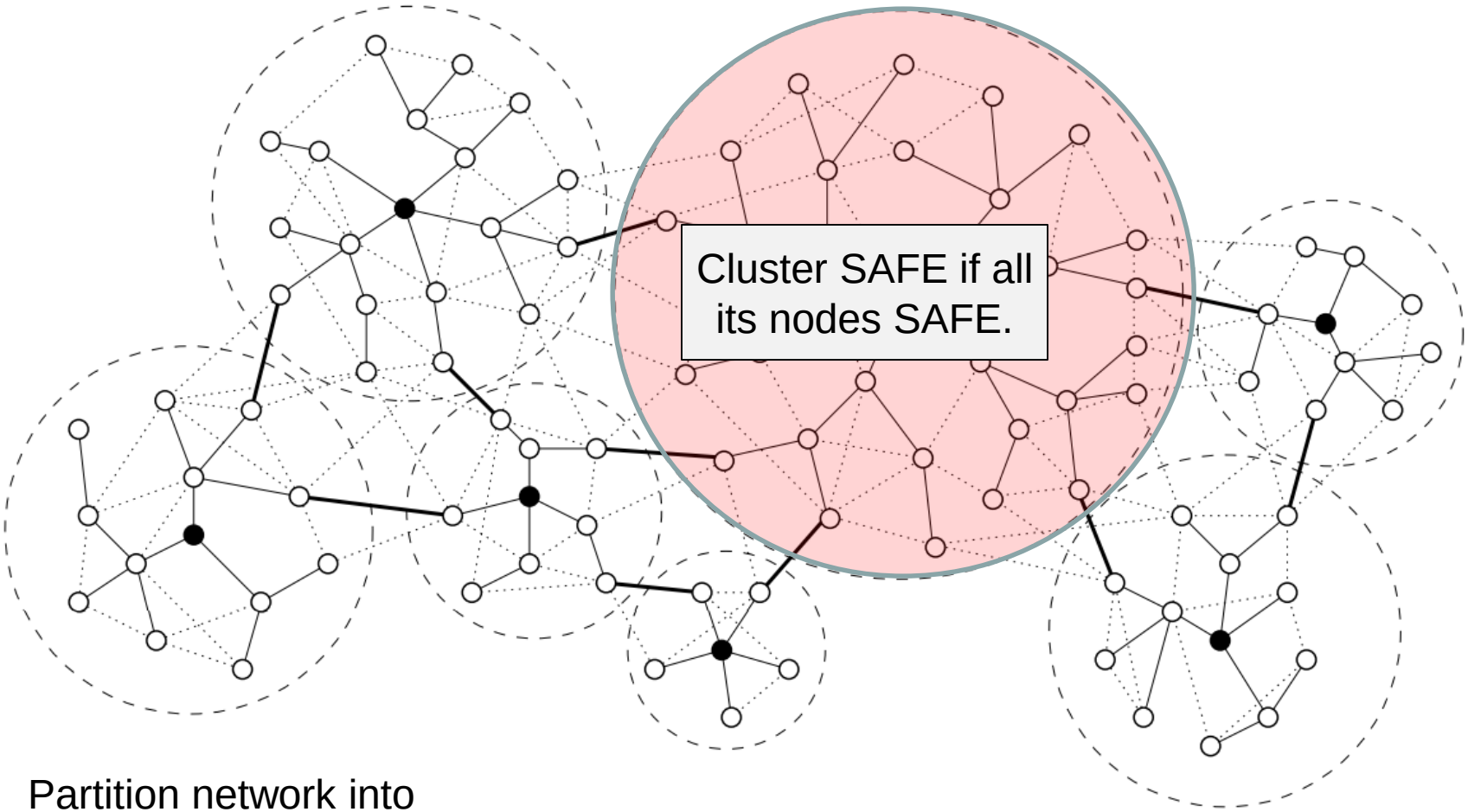

Partition network into small-diameter clusters.

The Hybrid Synchronizer γ

Idea: First make cluster safe, then make inter-cluster safe.

Idea:

1. Phase 1: Apply Synchronizer **β** inside each cluster;
   when done inform leaders in neighbor clusters
2. Phase 2: Generate next pulse when neighbor clusters are SAFE (Synchronizer **α**)

Synchronizer γ

For node v:

wait until v is SAFE
wait until v receives SAFE from all children in intra-cluster tree
send SAFE to parent in tree
wait for CLUSTERSAFE message from parent
send CLUSTERSAFE to children
wait until NEIGHBORSAFE received from all incident
inter-cluster edges and children in intra-cluster
send NEIGHBORSAFE to parent
wait for PULSE and forward

Synchronizer γ

Let mc be number of inter-cluster edges and let k be the maximum cluster radius (max dist leaf to leader in BFS). Then:

$$T(\gamma) = O(k)$$
$$M(\gamma) = O(n+m_c)$$

Synchronizer γ

Let mc be number of inter-cluster edges and let k be the maximum cluster radius.
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Dominant is the global **β** synchronizer: time at most the cluster radius k! O(1) time for **α** synchronizer.

Local synchronizer **α** for inter cluster at most m_c many messages. Intra-cluster along spanning tree at most n.

Synchronizer γ

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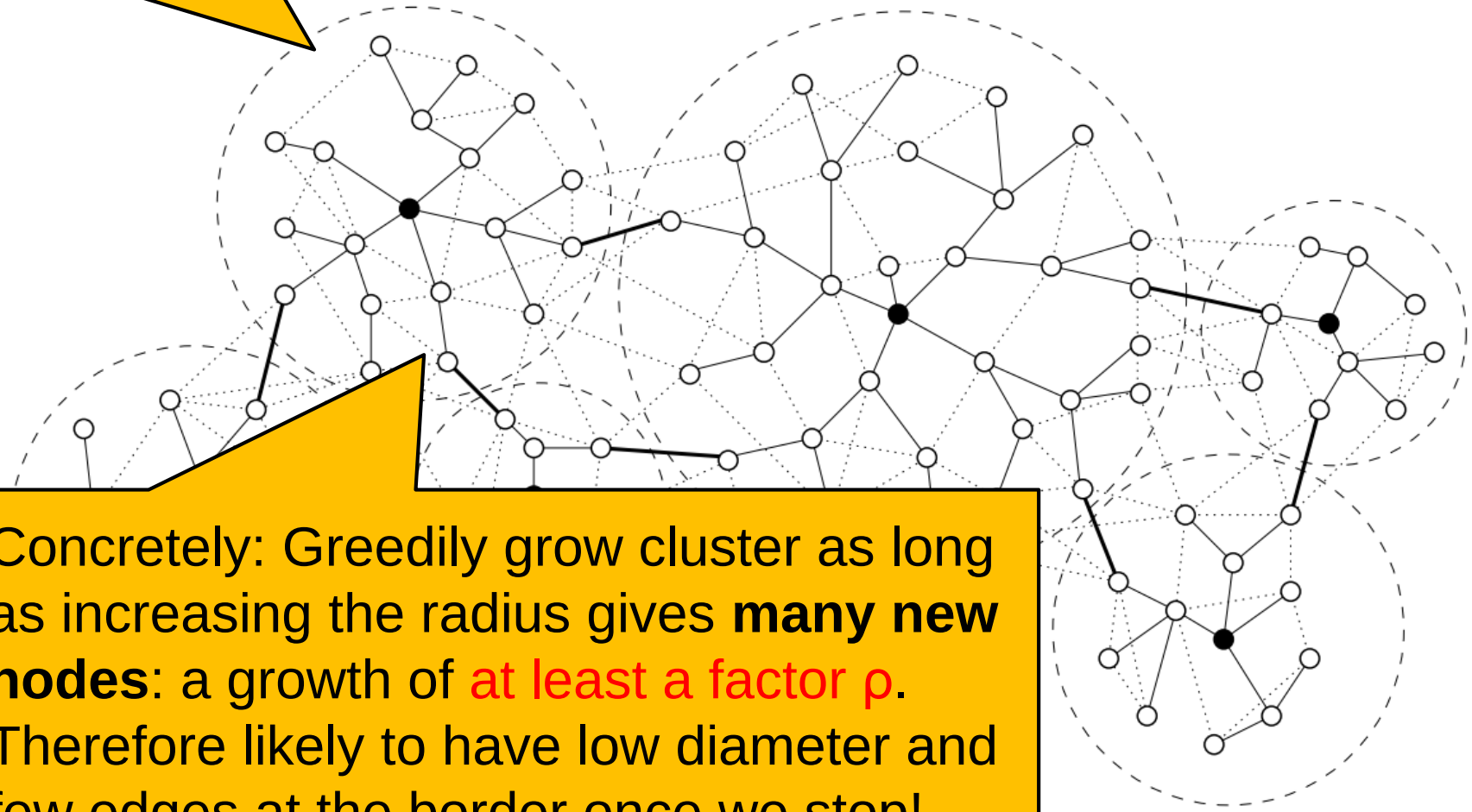
How to cluster the network so that m_c and k are minimal?

Idea: grow clusters one by one!



Concretely: Greedily grow cluster as long as increasing the radius gives **many new nodes**: a growth of at least a factor ρ. Therefore likely to have low diameter and few edges at the border once we stop!

Network Partition Algorithm

Idea:
1. Construct one cluster after another; start cluster at random non-covered node
2. Grow as long as “growth significant” (factor ρ)

Define: B(v,r) = Ball of radius r around v

Cluster Construction

```
while unprocessed nodes:
        select arbitrary unprocessed node v
        r:=0
        while |B(v,r+1)| > ρ * |B(v,r)| do
                r := r+1
        end while
        makeCluster(B(v,r))
end while
```

Partition Properties

The resulting network partition:

(1) consists of clusters of radius at most $\log_\rho n$

(2) at most $(\rho - 1)*n$ intercluster edges

Radius grows only if cluster size increases by factor ρ.
As there are at most n nodes, this can happen at most $\log_\rho n$ times.

Partition Properties

The resulting network partition:

(1) consists of clusters of radius at most $\log_\rho n$

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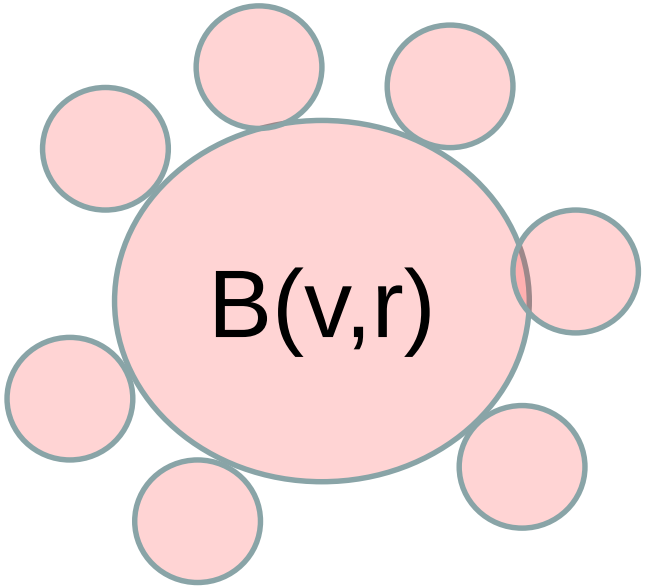
We know that for ball B:
$|B(v,r+1)| \leq \rho * |B(v,r)|$



Define $|C| := |B(v,r)|$. The size of the "border of the cluster" is at most
$| B(v,r+1) \setminus B(v,r) | \leq \rho * |C| - |C|$.

Summing over all clusters (n nodes in total, in worst case each one is a cluster): at most $\sum (\rho - 1)* |C| = (\rho - 1)* \sum |C| = (\rho - 1)* n$ inter-cluster edges

Quality of Partition

Partition Properties

The resulting network partition:

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Asymptotically optimal tradeoff!

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The resulting network partition:

(1) consists of clusters of radius at most $\log_\rho n$

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Asymptotically optimal tradeoff!

Example: $\rho = 2$

log(n) time synchronization overhead,
but only O(n) inter-cluster edges (messages)

Example: $\rho = n^{1/k}$

k time overhead, $O(n^{1+1/k})$ inter-cluster edges

End of Lecture