Real-time Logics

Expressiveness and Decidability

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Timed Behaviours

- Observable propositions $X_1, X_2, \ldots, X_n$.
- Time frame $(T, <)$ a linear order.
- Behaviour M such that $M(X_i) : T \rightarrow \{\top, \bot\}$.

Some commonly used notions of time and behaviour

- $(\Re, <)$ the standard set of real numbers. Continuous time, Canonical behaviors.
- $(\mathcal{Q}, <)$ the set of rational numbers.
- $(\aleph, <)$ the set of natural numbers. Discrete time.

Behaviours over $(\Re, <)$ are called canonical continous and over $(\aleph, <)$ canonical discrete.

Finite Variability

- M such that in any finite interval $M(X)$ changes finitely often for any X are called Finitely variable behaviours.
  Interval based model $(s_0, I_0), (s_1, I_1), \ldots$.
- $(S, <)$ where S is countably infinite set of sampling points from R which is divergent
  Point based models $(s_0, t_0), (s_1, t_1), \ldots$.

In this talk: Canonical Continuous Models and subclass Finitely Variable models.

Logics of Qualitative Time

Monadic First Order Logic MFO First order logic with equality over linear order $(T, <)$ and monadic predicates (observable propositions) X_i.

Examples

- $\forall x \exists y.(x < y \wedge X_1(y))$
  says that X_1 never stops occuring!
- $\forall x \forall y.(x < y \Rightarrow (\exists z.x < z < y))$

Let $\phi(X_1, \ldots, X_n, y_1, \ldots, y_k)$ be a formula with given free variables. Its model has the form $M = (T, <, P_1, \ldots, P_n, t_1, \ldots, t_k)$. We can define $M \models \phi$ as usual.

Monadic Secondorder Logic (MSO)

Temporal Logics

Example $\Box(P \rightarrow Q\mathcal{U}R)$.

A temporal logic $TL(O_1, \ldots, O_k)$ with modalities $O_1, \ldots, O_k$.

Truth Table For each modality O_i of arity k we have a MFO formula $[O_i(X_1, \ldots, X_k)] \stackrel{\text{def}}{=} \phi(t_0, X_1, \ldots, X_k)$ giving its behaviour.

Examples

- $[\Diamond X] \stackrel{\text{def}}{=} \exists t. t_0 < t \wedge X(t)$
- $[X\mathcal{U}Y] \stackrel{\text{def}}{=} \exists t. t_0 < t \wedge Y(t) \wedge \forall z.((t_0 < z < y) \Rightarrow X(z))$.

A popular temporal logic $TL(\mathcal{U}, \mathcal{S})$ called TL.

Correspondences

Proposition If every modality of a temporal logic TL' has a truth table then for every $\phi(X_1, \ldots, X_n) \in TL'$ we can construct a MFO formula $\hat{\phi}(t_0, X_1, \ldots, X_n)$ such that $M \models \phi$ iff $M \models \hat{\phi}$.

Every such Temporal logic is a fragment of MFO!

Expressive Completeness

Definition A subset L_1 of logic L_2 is expressively complete for L_2 over class of models C provided for all $\phi \in L_2$ there exists $\hat{\phi}_2 \in L_1$ such that $C \models \phi \Leftrightarrow \hat{\phi}$.

Theorems ([Kamp68,GPSS80,GHR94]) Logic $TL(\mathcal{U}, \mathcal{S})$ is expressively complete for MFO over canonical models (discrete or continous).
There exists $TL(\mathcal{U}_s, \mathcal{S}_S)$ which is expressively complete for MFO over the class of all linear order.

Decidability

A logic L is decidable if there exists an algorithm for finding whether $\phi \in L$ is satisfiable (valid).

Results

- MSO over $(\aleph, <)$ is decidable. [Buchi60]
- MFO over $(\Re, <)$ is decidable. [BG85]
- MSO over $(\Re, <)$ is undecidable. MSO with quantification over monadic predicates restricted to countable subsets of reals is decidable. [Shelah75]
- MSO over $(\mathcal{Q}, <)$ is decidabe. [Rabin69]
- MSO over $(\Re, <)$ with finitely variable behaviours is decidable. [Rabin69][Rabinovich98].

Quantitative Realtime Logics

Logic MFO^+

Constructs of MFO, together with $+1$ function.

Example $(\forall t.((t_0 < t < t_0 + 1 \;\Rightarrow\; P(t))) \;\Rightarrow\; Q(t_0 + 1)$

Theorem Logic MFO^+ is undecidable.
Proof Method We can encode accepting runs of a 2-counter machine by a formula.

- Encode each configuration within an interval of time length 1.
- Number of alternations of X_1, X_2 represent value of counters C_1, C_2.
- The configuration can be copied from one unit interval to next.

Guarded Measurements

Logic $QMFO$ [Hirschfeld-Rabinovich]
MFO constructs together with two bounded quantifiers below.

- Let $\phi(t)$ be a QMFO formula with only variable t free.
- Define $(\exists t)^{<t_0+1}_{>t_0}\phi(t) \stackrel{\text{def}}{=} \exists t(t_0 < t < t_0 + 1 \wedge \phi(t))$.
  Dually, $(\forall t)^{<t_0+1}_{>t_0}\phi(t) \stackrel{\text{def}}{=} \forall t(t_0 < t < t_0 + 1 \Rightarrow \phi(t))$.
- Define $(\exists t)^{<t_0}_{>t_0-1}\phi(t) \stackrel{\text{def}}{=} \exists t(t < t_0 < t + 1 \wedge \phi(t))$.
  Dually $(\forall t)^{<t_0}_{>t_0-1}\phi(t) \stackrel{\text{def}}{=} \forall t(t < t_0 < t + 1 \Rightarrow \phi(t))$.

Example $Timer(X, Y) \stackrel{\text{def}}{=} \forall t(Y(t) \Leftrightarrow (\forall t_1)^{<t_0}_{>t_0-1}X(t_1))$.
Y is true at a time iff X has been invariantly true for the previous (open) unit interval.

Temporal Logic QTL

Logic QTL $TL(\mathcal{U}, \mathcal{S}, \overrightarrow{\Diamond}_1, \overleftarrow{\Diamond}_1)$ with two constrained modalities below.

- $\overrightarrow{\Diamond}_1 \ X$ has truth table $(\exists t)_{>t_0}^{<t_0+1} X(t)$
- $\overleftarrow{\Diamond}_1 \ X$ has truth table $(\exists t)_{>t_0-1}^{<t_0} X(t)$
- Define duals $\overrightarrow{\Box}_1 \phi \stackrel{\text{def}}{=} \neg \overrightarrow{\Diamond}_1 \neg \phi$ and $\overleftarrow{\Box}_1 \phi \stackrel{\text{def}}{=} \neg \overleftarrow{\Diamond}_1 \neg \phi$.

Example: $\Box(Y \Rightarrow \overleftarrow{\Box}_1 X)$.

Expressive Completeness

Theorem For any temporal logic $TL(\tau)$ which is expressively complete for MFO, the logic $TL(\tau, \overrightarrow{\Diamond}_1, \overleftarrow{\Diamond}_1)$ is expressively complete for $QMFO$.
Specifically, QTL is expressively complete for $QMFO$.

Consequence: Any temporal modality which has a truth-table in $QMFO$ can be expressed within QTL.

Notation Denote $\overrightarrow{\Diamond}_1 \, X$ by $\overrightarrow{\Diamond}_{(0,1)} \, X$ and $\overleftarrow{\Diamond}_1 \, X$ by $\Diamond_{(-1,0)} X$.

Interval Bounded Modalities

- $(\exists t)_{\geq t_0}^{<t_0+1} X(t) \stackrel{\text{def}}{=} X(t_0) \;\vee\; (\exists t)_{\geq t_0}^{<t_0+1} X(t)$.

  $\Diamond_{[0,1)} X \stackrel{\text{def}}{=} X \vee \Diamond_{(0,1)} X$

- $(\exists t)_{>t_0}^{\leq t_0+1} X(t) \stackrel{\text{def}}{=} (\exists t)_{>t_0}^{<t_0+1} X(t) \;\vee$.

  $First(t_0, X) \wedge (\forall t_1)_{>t_0}^{<t_0+1} (\exists t)_{>t_1}^{<t_1+1} X(t)$

  $\Diamond_{(0,1]} X \stackrel{\text{def}}{=} \Diamond_{(0,1)} X \vee [(\neg X \mathcal{U} X) \wedge \Box_{(0,1)} \Diamond_{(0,1)} X]$

- $\Diamond_{(1,\infty)} X \stackrel{\text{def}}{=} \Box_{(0,1]} \Diamond X$.

- $\Box_{(n,n+1)} X \stackrel{\text{def}}{=} \Box_{(n-1,n)} \Diamond_{(0,1)} \Box_{(0,1)} X$.

A Variety of Modalities

Thus, we can define constrained modality $\Diamond_I$ where I is non-singular

- interval I has integer end-point or infinity as end-points.
- Non-singular, i.e. not a singleton set or empty.
- Can be closed, open, partially closed.

Logic MITL $TL(\mathcal{U}_I, \mathcal{S}_I)$ where I is non-singular interval.

Let $X\,\mathcal{U}_(n, n+m)\;Y \stackrel{\text{def}}{=} (\Box_{(0,n]}(X \wedge X\mathcal{U}Y)) \wedge \Diamond_{(n,n+m)}Y$.

Theorem $MITL$ and QTL have same expressive power.

A Variety of Modalities (Cont)

- Nearest Next [Wilke,Raskin]

$$\rhd_{(n,n+m)}X \stackrel{\text{def}}{=} (\neg X \mathcal{U} X) \wedge \Diamond_{(m,m+n)}X \wedge \neg \Diamond_{(0,n])}X.$$

- Age Constraints [MP93] $Age(X) > n \stackrel{\text{def}}{=} \Diamond_{(-n,0)}X$.

$$Age(X) = n \stackrel{\text{def}}{=} \Box_{(-n,0)}X \wedge \Box_{(-n,-(n-1))}\Diamond_{(-1,0)}\neg X.$$

Summary

- Logic QTL is expressively complete for $QMSO$.
- Modalities from most know decidable timed logics can be defined within QTL and $QMSO$
- Question Is $QMSO$ decidable?

Decidability of QMFO

Overview

- Timer Normal Form $TNF \subset QMFO$
- Transform $QMFO \rightarrow TNF$ (equivalent formula)
- Transform $TNF \rightarrow MFO$ (equisatisfiable formula)
- Decidability of MFO

Timer Normal Form (TNF)

- In QTL, define $TIMER(X,Y) \stackrel{\text{def}}{=} \Box(Y \Leftrightarrow \overleftarrow{\Box}_1 X)$.
  In $QMFO$ define
  $TIMER(X,Y) \stackrel{\text{def}}{=} \forall t(Y(t) \Leftrightarrow (\forall t_1)^{<t_0}_{>t_0-1} X(t_1))$
- $Timer_n(X_1,\ldots,X_n,Y_1,\ldots,Y_n) \stackrel{\text{def}}{=} \bigwedge_i Timer(X_i,Yi)$

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- $Timer_n(X_1,\ldots,X_n,Y_1,\ldots,Y_n) \stackrel{\mathrm{def}}{=} \bigwedge_i Timer(X_i,Yi)$
- Formula is in first order TNF if it has the following form where $\phi \in MFO$ and $\overline{W}$ is list of monadic predicates.

$$\exists \overline{W}.(Timer_n(X_1,\ldots,X_n,Y_1,\ldots,Y_n) \wedge \phi$$

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- If in above definition, if $\phi \in MSO$ we have second order TNF. If $\phi \in TL$ we have TL TNF.

Reducing Future to Past

Aim: To represent $\overrightarrow{\Diamond}_1$ by $\overleftarrow{\Box}_1$ (using $\mathcal{U}, \mathcal{S}$).

Consider witness $\Box(Y \Leftrightarrow \overrightarrow{\Diamond}_1 Y)$. This implies

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- $\psi_1 \stackrel{\text{def}}{=} \Box(Y \Rightarrow Y \mathcal{U} X)$.

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- $\psi_2 \stackrel{\text{def}}{=} \Box(X \Rightarrow \overleftarrow{\Box}_1 Y)$.

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- $\psi_1 \stackrel{\text{def}}{=} \Box(Y \Rightarrow Y \mathcal{U} X)$.
- $\psi_2 \stackrel{\text{def}}{=} \Box(X \Rightarrow \overleftarrow{\Box}_1 Y)$.
- $\psi_3 \stackrel{\text{def}}{=} \Box((\overleftarrow{\Box}_1 \neg X) \Rightarrow \Diamond_{[-1,0)} \neg Y)$
- Let $\psi \stackrel{\text{def}}{=} \psi_1 \wedge \psi_2 \wedge \psi_3$.

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- Let $\psi \stackrel{\text{def}}{=} \psi_1 \wedge \psi_2 \wedge \psi_3$.

Also, $\psi_1 \wedge \psi_3 \Rightarrow (Y \Rightarrow (\overrightarrow{\Diamond}_1 X))$ and $\psi_2 \Rightarrow ((\overrightarrow{\Diamond}_1 X) \Rightarrow Y)$.

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- Let $\psi \stackrel{\text{def}}{=} \psi_1 \wedge \psi_2 \wedge \psi_3$.

Also, $\psi_1 \wedge \psi_3 \Rightarrow (Y \Rightarrow (\overrightarrow{\Diamond}_1 X))$ and $\psi_2 \Rightarrow ((\overrightarrow{\Diamond}_1 X) \Rightarrow Y)$.

- Hence, $\Box(Y \Leftrightarrow \overrightarrow{\Diamond}_1 Y) \Leftrightarrow \psi$.
- $\overrightarrow{\Diamond}_1 Y \Leftrightarrow \exists Y.(Y \wedge \psi)$.

Reducing ψ

$$\Box(Y \Rightarrow Y\mathcal{U}X) \wedge \Box(X \Rightarrow \overleftarrow{\Box}_1 Y) \wedge \Box((\overleftarrow{\Box}_1 \neg X) \Rightarrow \Diamond_{[-1,0)} \neg Y).$$

Reducing ψ

$$\Box(Y \Rightarrow Y\mathcal{U}X) \wedge \Box(X \Rightarrow \overleftarrow{\Box}_1 Y) \wedge \Box((\overleftarrow{\Box}_1 \neg X) \Rightarrow \Diamond_{[-1,0)} \neg Y).$$

- $\Diamond_{[-1,0)} \neg Y \stackrel{\text{def}}{=} (\overleftarrow{\Diamond}_1 \neg Y \ \vee \ (Y \mathcal{S} \neg Y) \wedge \overleftarrow{\Box}_1 \overleftarrow{\Diamond}_1 \neg Y)$ and
- $\overleftarrow{\Diamond}_1 Z \Leftrightarrow \neg \overleftarrow{\Box}_1 \neg Z$.

Reducing ψ

$\Box(Y \Rightarrow Y\mathcal{U}X) \wedge \Box(X \Rightarrow \overleftarrow{\Box}_1 Y) \wedge \Box((\overleftarrow{\Box}_1 \neg X) \Rightarrow \Diamond_{[-1,0)} \neg Y)$.

- $\Diamond_{[-1,0)} \neg Y \stackrel{\mathrm{def}}{=} (\overleftarrow{\Diamond}_1 \neg Y \vee (Y\mathcal{S}\neg Y) \wedge \overleftarrow{\Box}_1 \overleftarrow{\Diamond}_1 \neg Y)$ and
- $\overleftarrow{\Diamond}_1 Z \Leftrightarrow \neg \overleftarrow{\Box}_1 \neg Z$.

Hence, $\psi \Leftrightarrow$

$\Box(Y \Rightarrow Y\mathcal{U}X) \wedge \Box(X \Rightarrow \overleftarrow{\Box}_1 Y) \wedge$
$\Box((\overleftarrow{\Box}_1 \neg X) \Rightarrow (\neg \overleftarrow{\Box}_1 Y \vee (Y\mathcal{S}\neg Y) \wedge \overleftarrow{\Box}_1 \neg \overleftarrow{\Box}_1 Y))$

Eliminating $\overleftarrow{\Box}_1 X$ using Timers

- Consider subformula $\overleftarrow{\Box}_1 X$.
- Introduce $Timer(X, W) \stackrel{\text{def}}{=} \Box(W \Leftrightarrow \overleftarrow{\Box}_1 X)$.
- Hence, $\psi \;\Leftrightarrow\; \exists W.(Timer(X, W) \wedge \psi[\overleftarrow{\Box}_1 X / W]$

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- Hence, $\psi \;\Leftrightarrow\; \exists W.(Timer(X, W) \wedge \psi[\overleftarrow{\Box}_1 X/W]$

We obtain, $\psi \;\Leftrightarrow$

$$\exists T_1, T_2, T_3.\; Timer(\neg X, T_1) \wedge Timer(Y, T_2) \wedge Timer(\neg T_2, T_3) \wedge$$
$$\Box(Y \;\Rightarrow\; Y \mathcal{U} X) \wedge \Box(X \;\Rightarrow\; T_2) \wedge$$
$$\Box(T_1 \;\Rightarrow\; (\neg T_2 \;\vee\; ((Y \mathcal{S} \neg Y) \wedge T_3)))$$

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- Introduce $Timer(X, W) \stackrel{\text{def}}{=} \Box(W \Leftrightarrow \overleftarrow{\Box}_1 X)$.
- Hence, $\psi \;\Leftrightarrow\; \exists W.(Timer(X, W) \land \psi[\overleftarrow{\Box}_1 X / W]$

We obtain, $\psi \;\Leftrightarrow$

$$\exists T_1, T_2, T_3.\; Timer(\neg X, T_1) \land Timer(Y, T_2) \land Timer(\neg T_2, T_3) \land$$
$$\Box(Y \;\Rightarrow\; Y \mathcal{U} X) \land \Box(X \;\Rightarrow\; T_2) \land$$
$$\Box(T_1 \;\Rightarrow\; (\neg T_2 \;\lor\; ((Y \mathcal{S} \neg Y) \land T_3)))$$

Recall that $\overleftarrow{\Diamond}_1 X \;\Leftrightarrow \exists Y.(Y \land \psi))$

Reduction to Timer Normal Form

Theorem For any $\phi(\bar{t}, \overline{Z})$ in $QMFO$, we can associate auxiliary monadic predicates $\overline{X}, \overline{Y}$ and formula $\overline{\phi}(\bar{t}, \overline{Z}, \overline{X}, \overline{Y})$ in MFO such that

$$\phi(\bar{t}, \overline{Z}) \;\Leftrightarrow\; \exists \overline{X}, \overline{Y}. (Timer_n(\overline{X}, \overline{Y}) \wedge \overline{\phi}(\bar{t}, \overline{Z}, \overline{X}, \overline{Y}))$$

- The theorem is true even when $\phi \in QMSO$ and gives a reduction to MSO.
- The theorem is true ven when $\phi \in QTL$ and gives a reduction to $TL(\mathcal{U}, \mathcal{S})$.

Elimination of Metric

Let $\overline{X} = X_1, \ldots, X_n$ and $\overline{Y} = Y_1, \ldots, Y_n$. We transform $Timer(\overline{X}, overlineY)$ into $\overline{Timer}(\overline{X}, \overline{Y})$ in MFO such that satisfiability is preserved (equisatisfiable).

MFO Properties of Timers Formula A_i is conjuction of

- Y_i is true at 0
- Y_i is finitely variable.
- Set of point where Y_i is true is closed. I.e. if Y_i holds for (a, b) it also holds for $[a, b]$.

Formula B_i is conjuction of

- For $t > 0$ if $Y_i(t)$ then X_i is true is small left neighbourhood of t.

Metric Elimination (Cont)

- If X_i continuously true from t onwards then Y_i becomes continuously true from some future point $t' > t$.
- If $Y_i(t)$ and X_i holds in $[t, t')$ then $Y_i(t')$.

Formula $C_{i,j}$ is conjunction of

- If $Y_i(t) \wedge \neg Y_j(t)$ then for some $t' < t$ we have X_i holds invariantly for (t', t) but X_j does not hold invariantly in (t', t).
- If Y_i and Y_j become true at t then for every previous t' we have X_i is true over (t', t) iff X_j is true over (t', t).

Let $\overline{Timer}(\overline{X}, \overline{Y}) \stackrel{\text{def}}{=} \bigwedge_i A_i \wedge B_i \wedge \bigwedge_{i,j} C_{i,j}$.

Timer Elimination Theorem

Theorem [HR03] The predicates $P_1, \ldots, P_n, Q_1, \ldots, Q_n \models \overline{Timer}(\overline{X}, \overline{Y})$ iff there is an order preserving bijection $\rho : \Re rightarrow \Re$ such that $\rho(P_1), \ldots, \rho(P_n), \rho(Q_1), \ldots, \rho(Q_n) \models Timer(\overline{X}, \overline{Y})$

For $\phi \in MFO$, we have $M \models \phi$ iff $\rho(M) \models \phi$.

Decidability of QMFO

Theorem For evert $\phi \in QMFO$ (or $QMSO, QTL$) we can construct $\overline{(}\phi) \in MFO$ (or MSO, TL) which is equisatisfiable.

Corollary

- $QMFO$ is decidable over continous canonical models.
- $QMSO$ is decidable over finitiely variable models. (Alternative proof of $MITL$ decidability.)
- QTL is decidable over continuous canonical models.

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