decision problems for ω-automata have been addressed by Landweber [261] and later by Emerson and Lei [143] and Sistla, Vardi, and Wolper [373]. For a survey of automata on infinite words, transformations between the several classes of ω-automata, complementation operators and other algorithms on ω-automata, we refer to the articles by Choueka [81], Kaminsky [229], Staiger [376], and Thomas [390, 391]. An excellent overview of the main concepts of and recent results on ω-automata is provided by the tutorial proceedings [174].

Automata and linear-time properties. The use of Büchi automata for the representation and verification of linear-time properties goes back to Vardi and Wolper [411, 412] who studied the connection of Büchi automata with linear temporal logic. Approaches with similar automata models have been developed independently by Lichtenstein, Pnueli, and Zuck [274] and Kurshan [250]. The verification of (regular) safety properties has been described by Kupferman and Vardi [249]. The notion of persistence property has been introduced by Manna and Pnueli [282] who provided a hierarchy of temporal properties. The nested depth-first algorithm (see Algorithm 8) originates from Courcoubetis et al. [102] and its implementation in the model checker SPIN has been reported by Holzmann, Peled, and Yannakakis [212]. The Murφ verifier developed by Dill [132] focuses on verifying safety properties. Variants of the nested depth-first search have been proposed by several authors, see, e.g., [106, 368, 161, 163]. Approaches that treat generalized Büchi conditions (i.e., conjunctions of Büchi conditions) are discussed in [102, 388, 184, 107]. Further implementation details of the nested depth-first search approach can be found in the book by Holzman [209].

4.7 Exercises

Exercise 4.1. Let $AP = \{ a, b, c \}$. Consider the following LT properties:

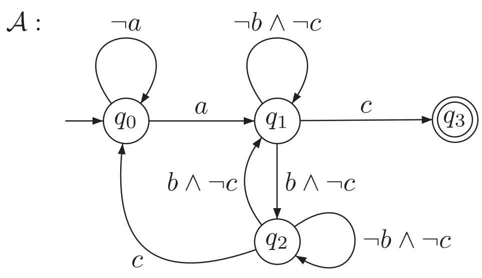
(a) If a becomes valid, afterward b stays valid ad infinitum or until c holds.

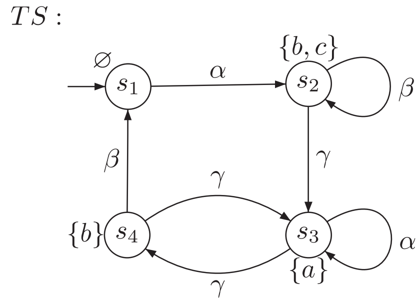
(b) Between two neighboring occurrences of a, b always holds.

(c) Between two neighboring occurrences of a, b occurs more often than c.

(d) $a \wedge \neg b$ and $b \wedge \neg a$ are valid in alternation or until c becomes valid.

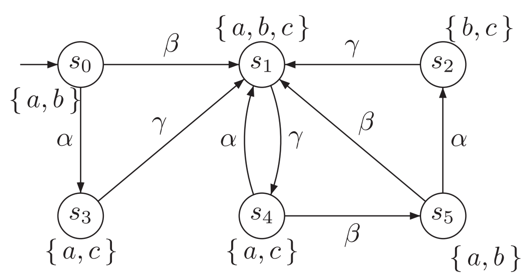
For each property P_i ($1 \leqslant i \leqslant 4$), decide if it is a regular safety property (justify your answers) and if so, define the NFA $\mathcal{A}_i$ with $\mathcal{L}(\mathcal{A}_i) = \mathit{BadPref}(P_i)$. (*Hint: You may use propositional formulae over the set AP as transition labels.*)





Construct the product $TS \otimes \mathcal{A}$ of the transition system and the NFA.

EXERCISE 4.6. Consider the following transition system TS



and the regular safety property

$$P_{safe} = \begin{array}{l}\text{"always if } a \text{ is valid and } b \wedge \neg c \text{ was valid somewhere before,} \\ \text{then } a \text{ and } b \text{ do not hold thereafter at least until } c \text{ holds"}\end{array}$$

As an example, it holds:

$$\begin{array}{ll} \{b\}\varnothing\{a,b\}\{a,b,c\} & \in \mathit{pref}(P_{safe}) \\ \{a,b\}\{a,b\}\varnothing\{b,c\} & \in \mathit{pref}(P_{safe}) \\ \{b\}\{a,c\}\{a\}\{a,b,c\} & \in \mathit{BadPref}(P_{safe}) \\ \{b\}\{a,c\}\{a,c\}\{a\} & \in \mathit{BadPref}(P_{safe}) \end{array}$$

Questions:

(a) Define an NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = \mathit{MinBadPref}(P_{safe})$.

(b) Decide whether $TS \models P_{safe}$ using the $TS \otimes \mathcal{A}$ construction.
Provide a counterexample if $TS \not\models P_{safe}$.

EXERCISE 4.7. Prove or disprove the following equivalences for ω-regular expressions:

(a) $(\mathsf{E}_1 + \mathsf{E}_2).\mathsf{F}^\omega \quad \equiv \quad \mathsf{E}_1.\mathsf{F}^\omega + \mathsf{E}_2.\mathsf{F}^\omega$

(b) $\mathsf{E}.(\mathsf{F}_1 + \mathsf{F}_2)^\omega \quad \equiv \quad \mathsf{E}.\mathsf{F}_1^\omega + \mathsf{E}.\mathsf{F}_2^\omega$

(c) $\mathsf{E}.(\mathsf{F}.\mathsf{F}^*)^\omega \quad \equiv \quad \mathsf{E}.\mathsf{F}^\omega$

(d) $(\mathsf{E}^*.\mathsf{F})^\omega \quad \equiv \quad \mathsf{E}^*.\mathsf{F}^\omega$

where $\mathsf{E}, \mathsf{E}_1, \mathsf{E}_2, \mathsf{F}, \mathsf{F}_1, \mathsf{F}_2$ are arbitrary regular expressions with $\varepsilon \notin \mathcal{L}(\mathsf{F}) \cup \mathcal{L}(\mathsf{F}_1) \cup \mathcal{L}(\mathsf{F}_2)$.

EXERCISE 4.8. Generalized ω-regular expressions are built from the symbols $\underline{\varnothing}$ (to denote the empty language), $\underline{\varepsilon}$ (to denote the language $\{\varepsilon\}$ consisting of the empty word), the symbols $\underline{A}$ for $A \in \Sigma$ (for the singleton sets $\{A\}$) and the language operators "+" (union), "." (concatenation), "$*$" (Kleene star, finite repetition), and "ω"(infinite repetition). The semantics of a generalized ω-regular expression G is a language $\mathcal{L}_g(\mathsf{G}) \subseteq \Sigma^* \cup \Sigma^\omega$, which is defined by

- $\mathcal{L}_g(\underline{\varnothing}) = \varnothing, \quad \mathcal{L}_g(\underline{\varepsilon}) = \{\varepsilon\}, \quad \mathcal{L}_g(\underline{A}) = \{A\}$,
- $\mathcal{L}_g(\mathsf{G}_1 + \mathsf{G}_2) = \mathcal{L}_g(\mathsf{G}_1) \cup \mathcal{L}_g(\mathsf{G}_2)$ and $\mathcal{L}_g(\mathsf{G}_1.\mathsf{G}_2) = \mathcal{L}_g(\mathsf{G}_1).\mathcal{L}_g(\mathsf{G}_2)$,
- $\mathcal{L}_g(\mathsf{G}^*) = \mathcal{L}_g(\mathsf{G})^*$, and $\mathcal{L}_g(\mathsf{G}^\omega) = \mathcal{L}_g(\mathsf{G})^\omega$.

Two generalized ω-regular expressions G and G' are called equivalent iff $\mathcal{L}_g(\mathsf{G}) = \mathcal{L}_g(\mathsf{G}')$.

Show that for each generalized ω-regular expression G there exists an equivalent generalized ω-regular expression G' of the form

$$\mathsf{G}' = \mathsf{E} + \mathsf{E}_1.\mathsf{F}_1^\omega + \ldots \mathsf{E}_n.\mathsf{F}_n^\omega$$

where $\mathsf{E}, \mathsf{E}_1, \ldots, \mathsf{E}_n, \mathsf{F}_1, \ldots, \mathsf{F}_n$ are regular expressions and $\varepsilon \notin \mathcal{L}(\mathsf{F}_i)$, $i = 1, \ldots, n$.

EXERCISE 4.9. Let $\Sigma = \{ A, B \}$. Construct an NBA $\mathcal{A}$ that accepts the set of infinite words σ over Σ such that A occurs infinitely many times in σ and between any two successive A's an odd number of B's occur.

EXERCISE 4.10. Let $\Sigma = \{ A, B, C \}$ be an alphabet.

(a) Construct an NBA $\mathcal{A}$ that accepts exactly the infinite words σ over Σ such that A occurs infinitely many times in σ and between any two successive A's an odd number of B's or an odd number of C's occur. Moreover, between any two successive A's either only B's or only C's are allowed. That is, the accepted words should have the form

$$wAv_1Av_2Av_3\ldots$$

where $w \in \{ B, C \}^*$, $v_i \in \{ B^{2k+1} \mid k \geqslant 0 \} \cup \{ C^{2k+1} \mid k \geqslant 0 \}$ for all $i > 0$. Give also an ω-regular expression for this language.

(b) Repeat the previous exercise such that any accepting word contains only finitely many C's.

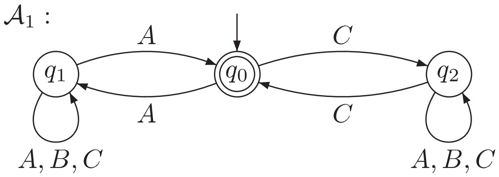
(c) Change your automaton from part (a) such that between any two successive A's an odd number of symbols from the set $\{ B, C \}$ may occur.

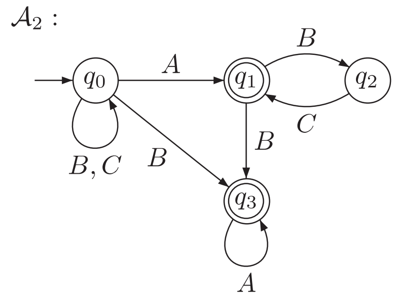
(d) Same exercise as in (c), except that now an odd number of B's *and* an odd number of C's must occur between any two successive A symbols.

EXERCISE 4.11. Depict an NBA for the language described by the ω-regular expression

$$(AB + C)^*((AA + B)C)^\omega + (A^*C)^\omega.$$

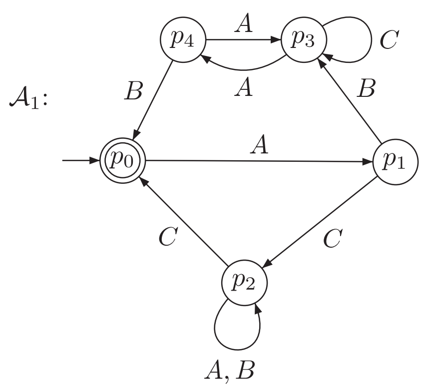
Exercise 4.12. Consider the following NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ over the alphabet $\{A, B, C\}$:

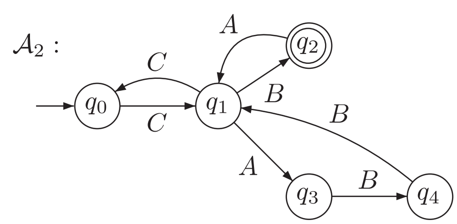




Find ω-regular expressions for the languages accepted by $\mathcal{A}_1$ and $\mathcal{A}_2$.

Exercise 4.13. Consider the NFA $\mathcal{A}_1$ and $\mathcal{A}_2$:





Construct an NBA for the language $\mathcal{L}(\mathcal{A}_1).\mathcal{L}(\mathcal{A}_2)^\omega$.

Exercise 4.14. Let $AP = \{a, b\}$. Give an NBA for the LT property consisting of the infinite words $A_0 A_1 A_2 \ldots (2^{AP})^\omega$ such that

$$\overset{\infty}{\exists} j \geqslant 0. (a \in A_j \wedge b \in A_j) \quad \text{and} \quad \exists j \geqslant 0. (a \in A_j \wedge b \notin A_j).$$

Provide an ω-regular expression for $\mathcal{L}_\omega(\mathcal{A})$.

Exercise 4.15. Let $AP = \{a, b, c\}$. Depict an NBA for the LT property consisting of the infinite words $A_0 A_1 A_2 \ldots (2^{AP})^\omega$ such that

$$\forall j \geqslant 0. A_{2j} \models (a \vee (b \wedge c))$$