

Gentzen's Proof System PK for Propositional Logic

Axiom

$$A \rightarrow A$$

Weakening

$$\text{left} \quad \frac{\Gamma \rightarrow \Delta}{A, \Gamma \rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta, A}$$

Exchange

$$\text{left} \quad \frac{\Gamma_1, A, B, \Gamma_2 \rightarrow \Delta}{\Gamma_1, B, A, \Gamma_2 \rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \rightarrow \Delta_1, A, B, \Delta_2}{\Gamma \rightarrow \Delta_1, B, A, \Delta_2}$$

Contraction

$$\text{left} \quad \frac{A, A, \Gamma \rightarrow \Delta}{A, \Gamma \rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \rightarrow \Delta, A, A}{\Gamma \rightarrow \Delta, A}$$

$\neg$ Introduction

$$\text{left} \quad \frac{\Gamma \rightarrow \Delta, A}{\neg A, \Gamma \rightarrow \Delta}$$

$$\text{right} \quad \frac{A, \Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta, \neg A}$$

$\wedge$ Introduction

$$\text{left} \quad \frac{A, B, \Gamma \rightarrow \Delta}{(A \wedge B), \Gamma \rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \rightarrow \Delta, A \quad \Gamma \rightarrow \Delta, B}{\Gamma \rightarrow \Delta, (A \wedge B)}$$

$\vee$ Introduction

$$\text{left} \quad \frac{A, \Gamma \rightarrow \Delta \quad B, \Gamma \rightarrow \Delta}{(A \vee B), \Gamma \rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \rightarrow \Delta, A, B}{\Gamma \rightarrow \Delta, (A \vee B)}$$

Cut

$$\frac{\Gamma \rightarrow \Delta, A \quad A, \Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta}$$