Gentzen's Proof System LK for First Order Logic

In addition to the rules for PK, LK has the following rules for introducing the quantifiers.

$\forall$ Introduction

left $\dfrac{A(t), \Gamma \rightarrow \Delta}{\forall x A(x), \Gamma \rightarrow \Delta}$ right $\dfrac{\Gamma \rightarrow \Delta, A(b)}{\Gamma \rightarrow \Delta, \forall x A(x)}$

Restriction: In the $\forall$ **right** rule the free variable b must not appear in the conclusion (bottom sequent).

$\exists$ Introduction

left $\dfrac{A(b), \Gamma \rightarrow \Delta}{\exists x A(x), \Gamma \rightarrow \Delta}$ right $\dfrac{\Gamma \rightarrow \Delta, A(t)}{\Gamma \rightarrow \Delta, \exists x A(x)}$

Restriction: In the $\exists$ **left** rule the free variable b must not appear in the conclusion (bottom sequent).