CS 302, Theory of Computation
Even Semester, 2004-2005
Home Assignment # 3
Due Date: Sunday 17/04/2005
20 marks

05/01/2005

1. Suppose $f : \mathbb{N} \to \mathbb{N}$ is a primitive recursive function. Define $F : \mathbb{N} \to \mathbb{N}$ by $F(n) = f^n(n)$, where $f^0(x) = x$ and $f^{n+1}(x) = f(f^n(x))$. Show that F is primitive recursive. (5 marks)

2. Suppose that f is a total recursive bijective (i.e., one-to-one and onto) function from $\mathbb{N}$ to $\mathbb{N}$. Show that its inverse f^{-1} is also total recursive. (5 marks)

3. Without using Rice's theorem, show that the set $T = \{x \mid \{x\}_1 \text{ is a total function}\}$ is not recursive. (10 marks)
   [Hint: Use the special case of the *S-m-n theorem* to show that $K \leq_m T$.]