

CS 302, Theory of Computation  
Even Semester, 2004-2005  
Home Assignment # 1  
Due Date: 25/01/2005

18/01/2005

1. Add the 0-ary connectives 0 (*true*) and 1 (*false*) to the language of propositional logic, with  $v(1) = T$  and  $v(0) = F$  for any truth assignment  $v$ . For any formula  $A$  and atom  $P$ , let  $A_1^P$  be the formula obtained from  $A$  by replacing  $P$  by 1. Similarly for  $A_0^P$ . Let  $A_\star^P = (A_1^P \vee A_0^P)$ . Prove the following results:
  - (a)  $A \models A_\star^P$ , i.e.,  $A_\star^P$  is a logical consequence of  $A$ . (4 points)
  - (b) If  $A \models B$  and  $P$  does not appear in  $B$ , then  $A_\star^P \models B$ . (6 points)
  - (c) (*Semantic Version of Craig Interpolation Lemma*) If  $A \models B$ , then there is some  $C$  (an “interpolant”) all of whose atoms occur both in  $A$  and  $B$  such that both  $A \models C$  and  $C \models B$ . (10 points)