CS 302, Theory of Computation
Even Semester, 2004-2005
Home Assignment # 1 (Alternative)
Due Date: 15/02/2005

07/02/2005

1. Assume that the set of atoms (and hence formulas) in propositional logic is countable, i.e. there is an enumeration $A_1, A_2, \ldots$ of all formulas. A set Φ of formulas is called *consistent* if every finite subset of Φ is satisfiable. Φ is *maximally consistent* if it is consistent and if no proper superset of Φ is consistent. Define by recursion the following sequence of sets of formulas, where Φ is a consistent set:

$$\Delta_0 = \Phi, \quad \Delta_{n+1} = \begin{cases} \Delta_n \cup \{A_{n+1}\} & \text{if this set is consistent} \\ \Delta_n \cup \{\neg A_{n+1}\} & \text{otherwise} \end{cases}$$

Prove the following results without using the compactness theorem.

   (a) Each Δ_n is consistent. (4 points)

   (b) $\Delta = \bigcup_n \Delta_n$ is consistent. (6 points)

   (c) (Lindenbaum's Theorem) Show that every consistent set Φ can be extended to a maximally consistent set. (10 points)