

# Modeling and verification of cyber-physical systems

## Lecture 1

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# Plan for the lecture 1

- Course administration
- Motivation
  - What are cyber-physical systems?
  - Why care about verification of CPS?
- Modeling Discrete Systems
- Invariance and safety

Algorithms make decisions for us  
everyday

Increasingly, algorithms control our  
physical environment and social  
institutions

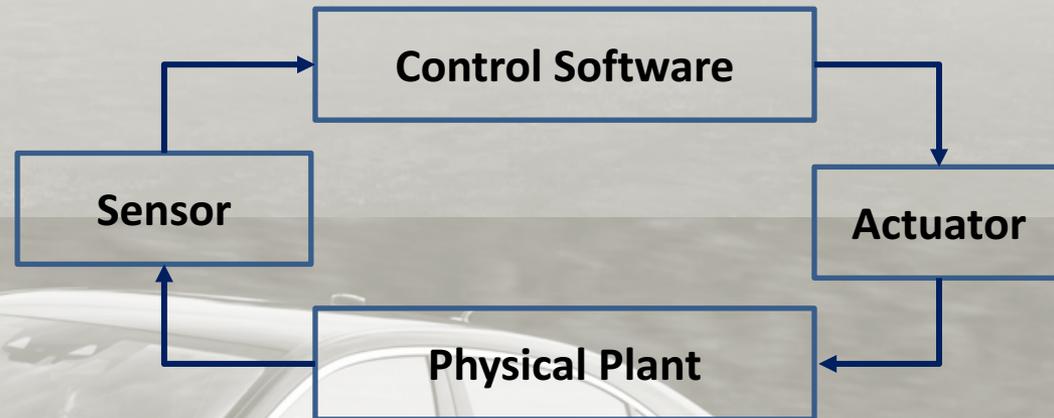


# Cyber-physical Systems

CPS: software monitoring and controlling a dynamical system

We will take a rigorous view of modeling, analyzing, and designing engineering systems

Why?



Autonomous systems

Internet Of Things (IOT)

Cyber-physical System

Industry 4.0

Is the algorithm fair?

# Advanced Driving Assist features (ADAS) to Driverless cars

2004 DARPA Grand challenge I:

no car drives more than 7 miles in  
desert

2005 DARPA challenge II:

Stanford car wins driving 131 miles

2007 DARPA Urban challenge\*

Many successes, CMU wins

Google, Uber, Tesla, GM, Nissan, Toyota, Ford, ...

2016: Public tests at Singapore (nuTonomy),  
Pittsburgh (Uber), Boston

Verification of Periodic Hybrid Systems: Application to An Autonomous Vehicle  
Wongpiromsarn, Mitra, Lamperski and Murray 2009



# Algorithms for predictive policing

# Crime records + Surveillance -> Predictions



- 2008: LAPD starts explorations on forecasting crime using data
- 2013: Better prediction of crime hotspots in Santa Cruz evaluation
- 2016: Used in 50+ police department

Zach Friend. "Predictive Policing: Using Technology to Reduce Crime". Federal Bureau of Investigation. Dec. 2013.

Technology is neither good nor  
bad; *nor is it neutral.*

# Driverless cars will be good for

Productivity: Americans spend 300 hours driving each year

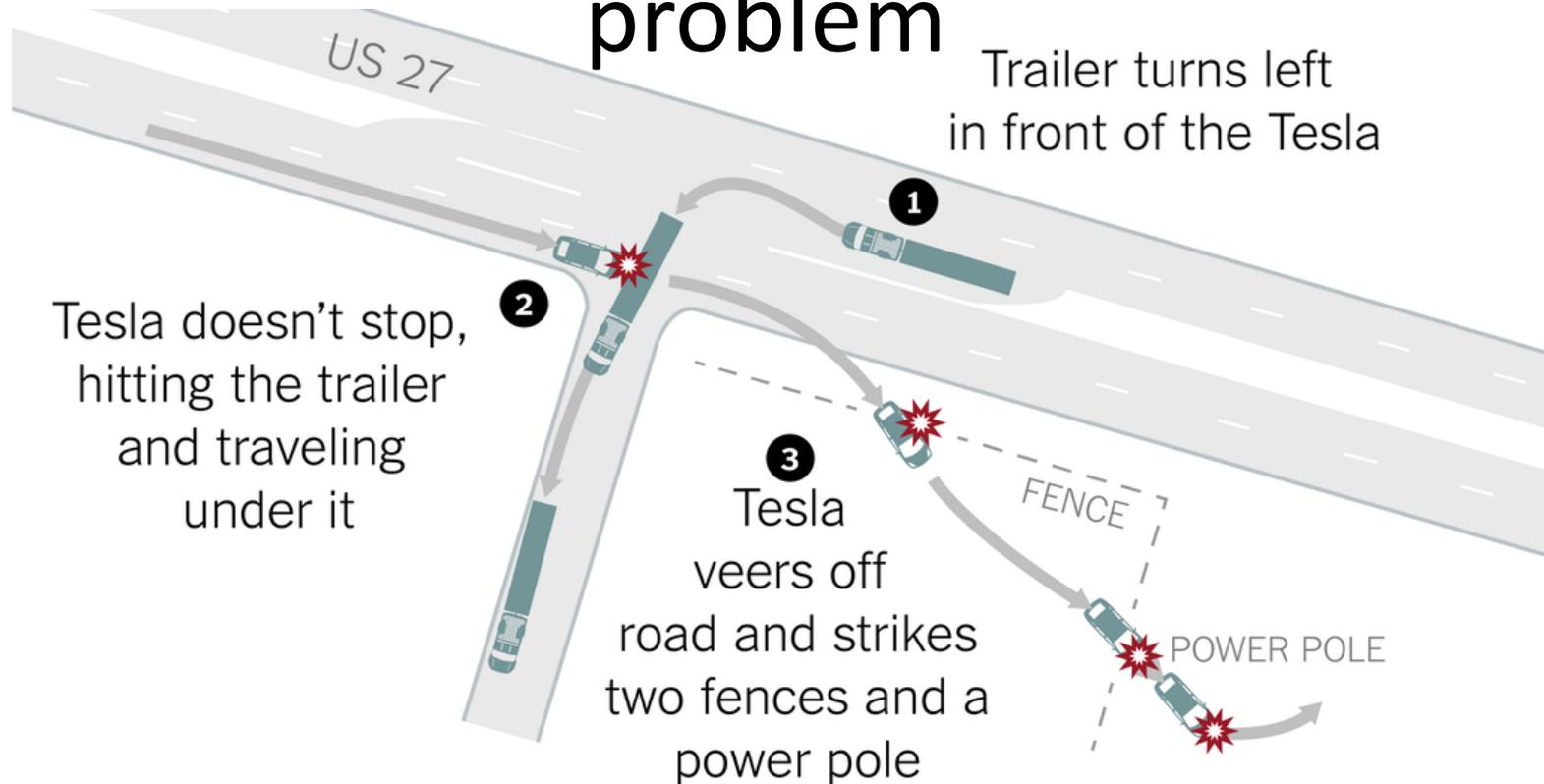
Real estate: 40 % of city surface is parking

Environment: Better fuel efficiency, scheduling, charging

Safety: 32,675 vehicle fatalities in 2014 in the US

10% increase in first half of 2016 !!

# Achieving safety is a very hard problem



NYTimes, July 12 2016

An aerial view of a car assembly plant. The image shows a long, curved conveyor belt system. On the left side, several rows of light blue cars are moving along the belt. In the center, there are more rows of cars, including some white and red ones. On the right side, there are many rows of white cars. The cars are arranged in neat, parallel lines, showing the scale of the manufacturing process. The background is a light-colored concrete or asphalt surface.

# A Record 22 Million Cars Recalled in 2013

Cost in Billions

# 2 Million Device Recalled over the Decade

24% for Design Bugs

# ZA002 Incident

**EVERETT, Wash., Nov. 10, 2010--** During approach to Laredo, Texas, yesterday, airplane **ZA002 lost primary electrical power** as a result of an onboard electrical fire. Backup systems, including the deployment of the **Ram Air Turbine (RAT)**, functioned as expected and allowed the crew to complete a safe landing. The cause of the fire is still under investigation by Boeing.

Initial inspection appears to indicate that a **power control panel in the aft electronics bay will need to be replaced** on ZA002. We are inspecting the power panel and surrounding area near that panel to determine if other repairs will be necessary.

**Boeing: Changes to Power Panel Design** Nov. 24, 2010

Boeing has acknowledged the rumors circulating around that **it was indeed FOD (foreign object damage)** that had caused a short circuit that led to the fire in the P100 power panel.

As a consequence Boeing will undertake a minor redesign of the power panel to reduce the chance of FOD creating and electrical arc or short circuit. Boeing will **also implement software changes to make sure that power distribution is improved**. This appears to be an acknowledgement that the electrical **power redundancies failed as well**.



New in DO178C standard

Formal Methods

Model Based Development

Object Oriented Techniques

# Overflow results in the flight computer crash (2005)

“June 4, 1996 -- Ariane 5 Flight 501. Working code for the Ariane 4 rocket is reused in the Ariane 5, but the Ariane 5's faster engines trigger a bug in an arithmetic routine inside the rocket's flight computer. The error is in the code that converts a 64-bit floating-point number to a 16-bit signed integer. The faster engines cause the 64-bit numbers to be larger in the Ariane 5 than in the Ariane 4, triggering an overflow condition that results in the flight computer crashing.”

*History's Worst Software Bugs, Simson Garfinkel, Wired 2005*



- Unlike “one-shot” function computations, computations of embedded systems are infinite streams
  - Example: Rudder positions computed by an autopilot program:  
L, R, R, R, C, L, ...
- Testing and simulations (at least their naïve applications) can check for only finitely many behaviors
- Not sufficient for **covering** all behaviors of the program and its physical environment

How many miles must an autonomous car drive  
before we call it safe?

200 million miles?

0.07 fatalities per billion passenger miles  
(commercial flight)

Probability of fatal failure per hour of driving  
 $10^{-9}$  [Amnon Shashua, CTO Mobileye]

# How is air-travel so safe?

# Regulations and Audits

What fraction of the cost of developing a new aircraft is in SW?

DO178C



Primary document by which FAA & EASA approves software-based aerospace systems.

DAL establishes the rigor necessary to demonstrate compliance

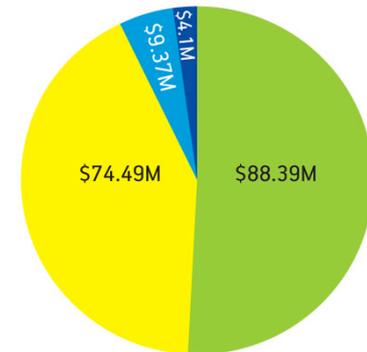
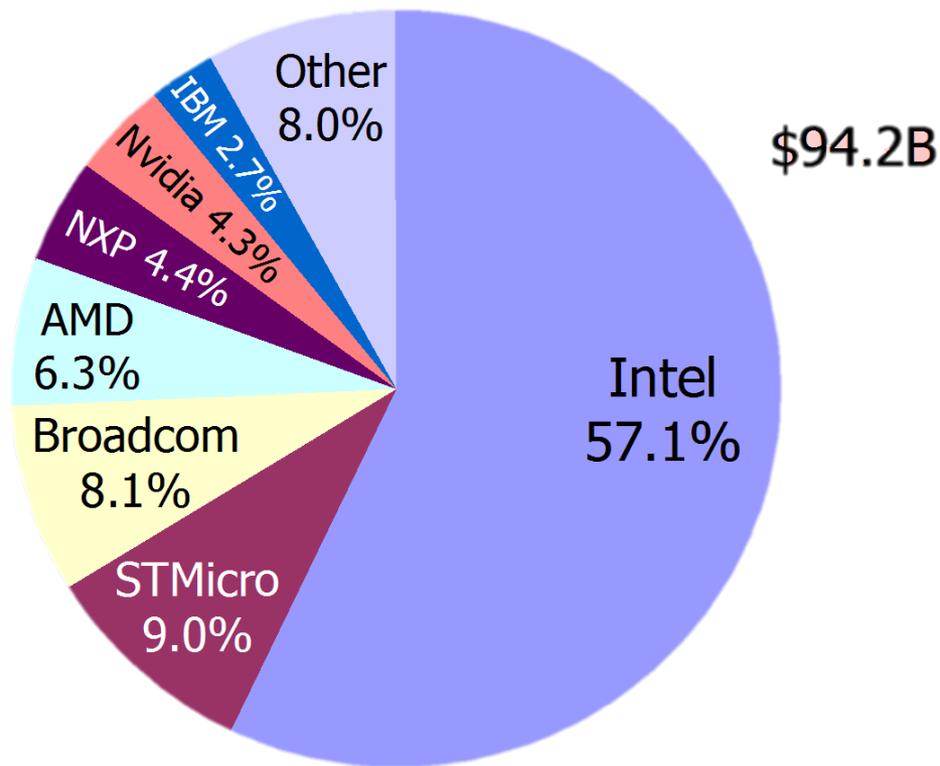
Dev.Assurance Level (DAL)	Hazard Classification	Objectives
A	Catastrophic	71
B	Hazardous	69
C	Major	62
D	Minor	26
E	No Effect	0

**Statement Coverage:** Every statement of the source code must be covered by a

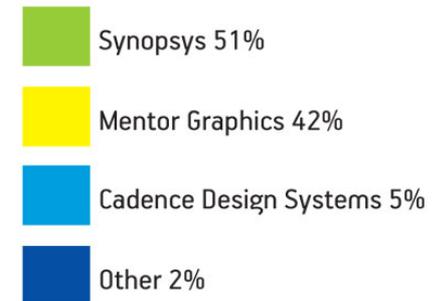
**Condition Coverage:** Every condition within a branch statement must be covered by a test case

Verification of CPS:  
We need standards, processes,  
tools, and trained individuals to  
ensure that cyber-physical systems  
meet the standards

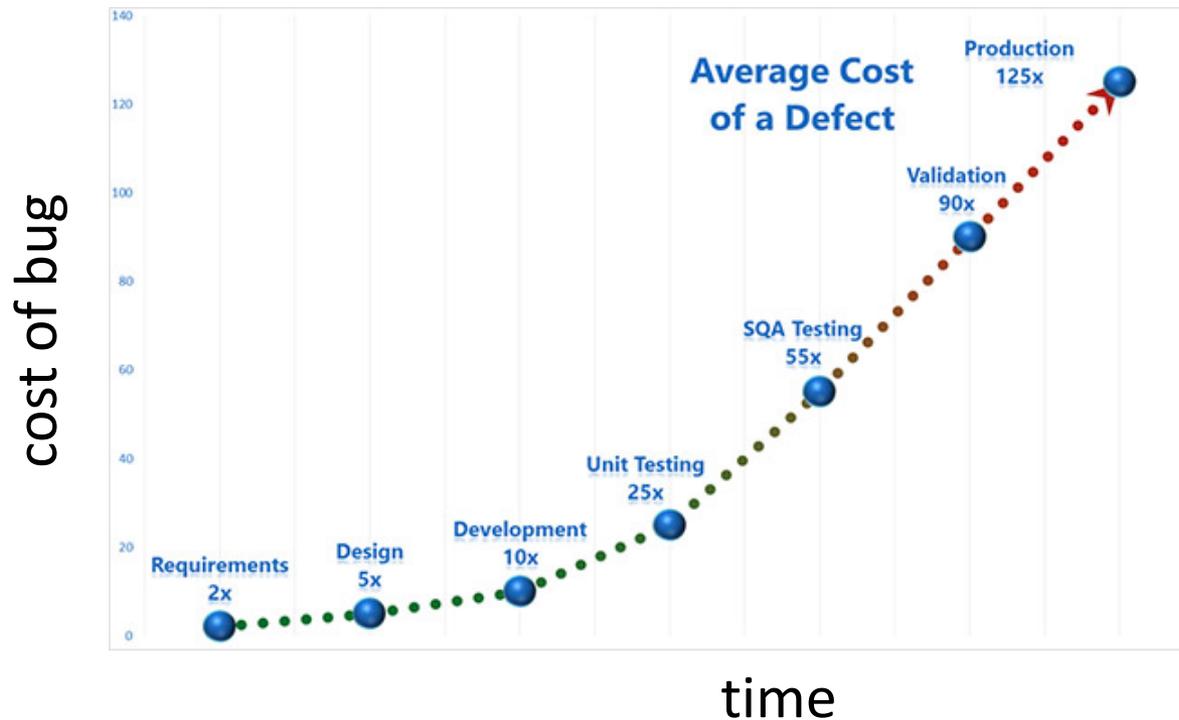
# An earlier instance: microprocessor industry



Electronic design automation industry

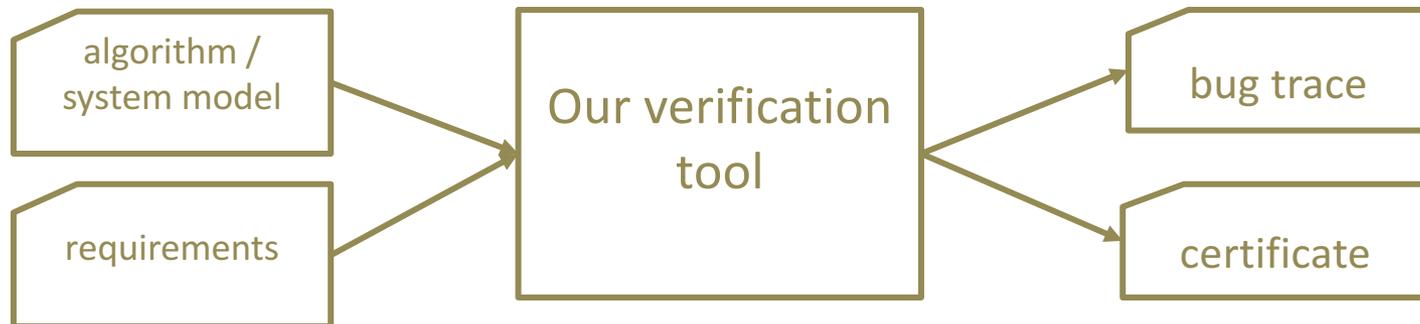


# Defects become more expensive with time



How to Cut Software-Related Medical Device Failures and Recalls, Lisa Weeks

# Audit algorithms with Algorithms and find problems early



Relevant courses: Theory of computation, Program Verification, Formal System Development, Automated Deduction, Control theory, Embedded System Verification

# Example requirements

Safety: “For all nominal behaviors of the car, the separation between the cars must be always  $> 1$  m”

Efficiency: “For all nominal driver inputs, the air-fuel ratio must be in the range  $[1,4]$ ”

Privacy: “Using GPS does not compromise user’s location”

Fairness: “Similar people are treated similarly”

# Example modeling frameworks

Discrete transition  
systems, automata



Dynamical systems

Differential inclusions



Hybrid systems

Markov chains



Probabilistic automata,  
Markov decision processes  
(MDP)



Continuous time,  
continuous state MDPs



Stochastic Hybrid systems

# Young & promising area with rich problems

**Or** why you should stop worrying and love this course

- Intersection of CS and control theory
  - Discrete and continuous math, different analysis techniques
- Formal analysis gaining traction in industry
  - Hardware verification is engineering practice in the industry
  - Many tools used in software, avionics, automotive industries. e.g. SDV (Microsoft), ESTEREL, ASTREE.
  - Mars Code: Gerard Holzman, JPL (see [video](#))
- Big and small commercial enterprises support above customers
  - Synopsis, Mentor Graphics, Cadence, Jasper, Coverity, Galois, SRI,
- Vibrant academic research activities
  - Related Turing Awards: Lamport (2014), Clarke, Sifakis & Emerson (2008), Pnueli (1997), Lampson (1992), Milner (1991), Hoare (1980), Dijkstra (1972) ...
  - Conferences: HSCC, EMSOFT, ICCPS, CAV, TACAS, RTSS...
  - Alums from this course now faculty at U. Conn, UT Arlington, U. Minnesota, Kansas State, Air Force Research Lab, Toyota Research Center, ...

# Learning Objectives

- Techniques for **modeling** systems with dynamics, computation, and communication
- Techniques for **rigorously reasoning** about correctness of systems
- Exposure to using **verification tools** and libraries
  - model checkers, SMT solvers, simulation-driven verifiers and theorem provers
- **Understand** deep and influential ideas in CS and Control theory (from the last 20 years)

Modeling Computation

# DISCRETE SYSTEMS

# Outline

- An Example: Token Ring
- Specification language (syntax)
- Automata (semantics)
- Invariants

# An example: Informal description

A **token-based** mutual exclusion algorithm on a **ring network**

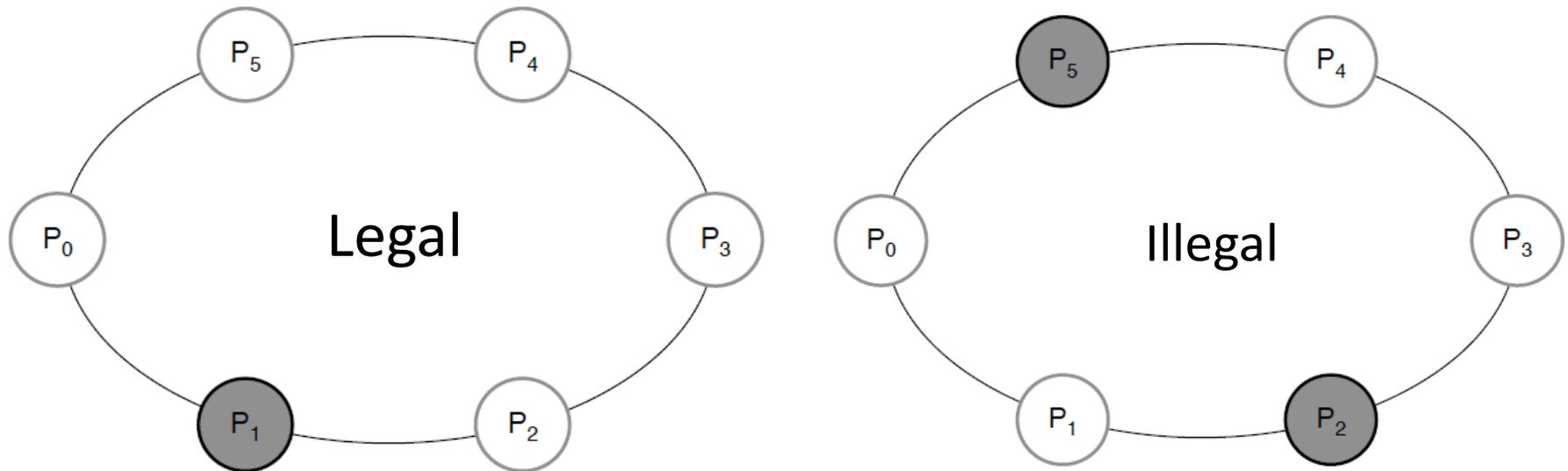
Collection of processes send and receive bits over a ring network so that only one of them has a “token”

Discrete

Each process has variables that take only **discrete values**

Time elapses in **discrete steps** (This is a modeling choice)

# Token ring: Informal problem specification

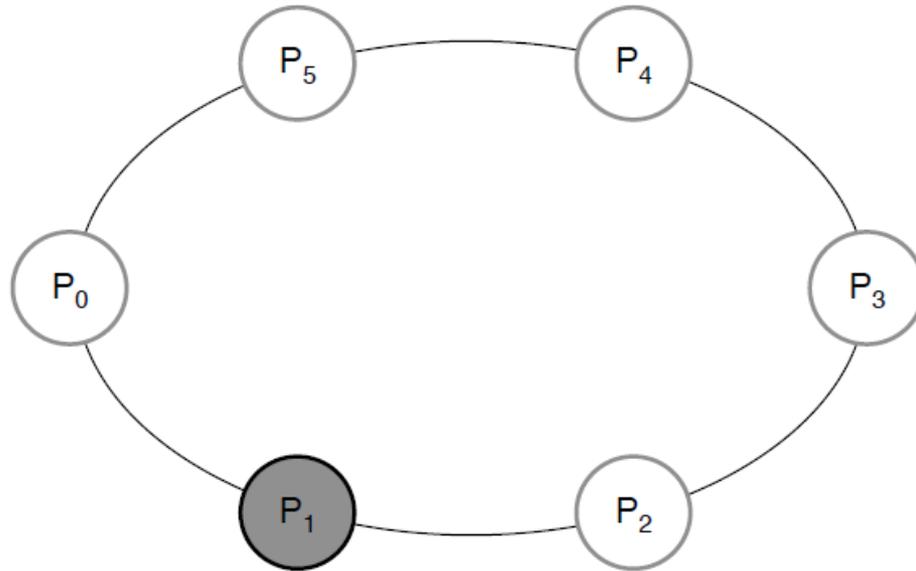


1. There is always at least one token
2. Legal configuration = exactly one “token” in the ring
3. Single token circulates in the ring
4. Even if multiple tokens somehow arise, e.g. with failures, if the algorithm continues to work correctly, then eventually there is a single token

# Properties can be stated as Invariants

- **Invariant** (informal def.): A property of the system that always\* holds
- Examples:
  - “Always at least one process has a token”
  - “Always exactly one process has the token”
  - “Always all processes have values at most  $k-1$ ”
  - “Even if there are multiple tokens, **eventually** there is exactly one token” (not strictly an invariant)

# Dijkstra's Algorithm [Dijkstra 1982]



$n$  processes with indices  $0, 1, \dots, n-1$

state of process  $j$  is  $x[j] \in \{0, 1, 2, k-1\}$ , where  $k > n$

$p_0$      **if**  $x[0] = x[N-1]$  **then**  $x[0] := x[0] + 1 \bmod k$

$p_j$   $j > 0$ , **if**  $x[j] \neq x[j-1]$  **then**  $x[j] := x[j-1]$

( $p_i$  **has TOKEN** if and only if the **conditional** is true)

# A Specification Language: HIOA

**auto** DijkstraTR (n:natural,k:natural)

**type** indices: [0,...,n-1]

**type** values: [0,...,k-1]

**signature**

**internal step**(i:indices)

**variables**

$x$ : [indices  $\rightarrow$  values] **initially**  $\forall i \in \text{indices}, x[i] = 0$

**transitions**

**internal step**(i:indices)

**pre**  $i = 0 \wedge x[i] = x[n-1]$

**eff**  $x[i] := x[i] + 1 \text{ mod } k$ ;

**internal step**(i:indices)

**pre**  $i \neq 0 \wedge x[i] \neq x[i-1]$

**eff**  $x[i] := x[i-1]$ ;

**trajectories**

# Discrete Transition System or Automaton

An **automaton** is a tuple  $\mathcal{A} = \langle X, \Theta, A, \mathcal{D} \rangle$  where

1.  $X$  is a set of names of variables; each variable  $x \in X$  is associated with a type,  $type(x)$ 
  - A **valuation** for  $X$  maps each variable in  $X$  to its type
  - Set of all valuations:  $val(X) = Q$  this is sometimes identified as the **state space** of the automaton
2.  $\Theta \subseteq val(X)$  is the set of **initial** or **start states**
3.  $A$  is a set of names of **actions** or **labels**
4.  $\mathcal{D} \subseteq val(X) \times A \times val(X)$  is the set of **transitions**
  - a transition is a triple  $(u, a, u')$
  - We write  $(u, a, u') \in \mathcal{D}$  in short as  $u \xrightarrow{a} u'$

# HIOA Specs to Automata: variables

**variables**  $s, v$ : Reals;  $a$ : Booleans

$$X = \{s, v, a\}$$

Example valuations also called **states**:

- $u_1 = \langle s \mapsto 0, v \mapsto 5.5, a \mapsto 0 \rangle$
- $u_2 = \langle s \mapsto 10, v \mapsto -2.5, a \mapsto 1 \rangle$

$$\text{val}(X) = \{\langle s \mapsto c_1, v \mapsto c_2, a \mapsto c_3 \rangle \mid c_1, c_2 \in R, c_3 \in \{0,1\}\}$$

**type indices**:  $[0, \dots, n-1]$

**variables**  $x$ : [**indices**->**values**]

- Fix  $n = 6, k = 8$
- $x: [\{0, \dots, 5\} \rightarrow \{0, \dots, 7\}]$
- Example valuations:
  - $u = \langle x \mapsto \langle 0 \mapsto 0, 1 \mapsto 0, 2 \mapsto 0, 3 \mapsto 0, 4 \mapsto 0, 5 \mapsto 0 \rangle \rangle$
  - $v = \langle x \mapsto \langle 0 \mapsto 7, 1 \mapsto 0, 2 \mapsto 0, 3 \mapsto 0, 4 \mapsto 0, 5 \mapsto 0 \rangle \rangle$
  - Notation:  $u.x, u.x[4] = 0$

$$\text{val}(x) = \{\langle x \mapsto \langle i \mapsto c_i \rangle_{\{i=0 \dots 5\}} \mid c_i \in \{0, \dots, 7\}\}$$

# States and predicates

A **predicate** over a set of variables  $X$  is a formula involving the variables in  $X$ . For example:

- $\phi_1: x[1] = 0$
- $\phi_2: \forall i \in \text{indices}, x[i] = 0$

A valuation  $\mathbf{u}$  **satisfies predicate  $\phi$**  if substituting the values of the variables in  $\mathbf{u}$  in  $\phi$  makes it evaluate to **True**. We write  $\mathbf{u} \models \phi$

- $\mathbf{u} \models \phi_1, \mathbf{u} \models \phi_2, \mathbf{v} \models \phi_1$  and  $\mathbf{v} \not\models \phi_2$

$[[\phi]] = \{u \in \text{val}(x) \mid u \models \phi\}$ . Examples

- $[[\phi_1]] = \{\langle x \mapsto \langle 1 \mapsto 0, i \mapsto c_i \rangle_{\{i=0,2,\dots,5\}} \mid c_i \in \{0, \dots, 7\} \rangle\}$
- $[[\phi_2]] = \{\langle x \mapsto \langle 0 \mapsto 0, 1 \mapsto 0, 2 \mapsto 0, 3 \mapsto 0, 4 \mapsto 0, 5 \mapsto 0 \rangle \rangle\}$

# Initial state and invariant assertions

- $\Theta \subseteq val(x)$  initial states
  - Often specified by a predicate
  - $\phi_0 = (\text{Initially } \forall i \in \text{indices}, x[i] = 0)$
  - $\Theta = [[\phi_0]] = \langle x \mapsto \langle i \mapsto 0 \rangle_{i=0, \dots, 5} \rangle$
- Invariant properties
  - “At least one process has the token”.
  - $I_1 = (x[0] = x[5] \vee \exists i \in \{1, \dots, 5\}: x[i] \neq x[i - 1])$
  - $[[I_1]] = \{\langle 0, \dots, 0 \rangle, \langle 1, 0, \dots, 0 \rangle, \dots, \langle k - 1, \dots, k - 1 \rangle\} = val(x)$  (?)
  - “Exactly one process has the token”
  - $I_2 = (x[0] = x[5] \oplus x[1] \neq x[0] \oplus x[2] \neq x[1] \dots)$

# Actions

- **signature** defines the set of Actions
- Examples
  - **internal** *step*(i:indices)
  - $A = \{step[0], \dots, step[5]\}$
  - **internal** brakeOn, brakeOff
  - $A = \{brakeOn, brakeOff\}$

# Transitions

$\mathcal{D} \subseteq \text{val}(X) \times A \times \text{val}(X)$  is the set of **transitions**

**internal**  $\text{step}(i:\text{indices})$

**pre**  $i = 0 \wedge x[i] = x[n-1]$   
**eff**  $x[i] := x[i] + 1 \text{ mod } k;$

$(u, a, u') \in \mathcal{D}$  iff  $u \models \text{Pre}_a$  and  $(u, u') \in \text{Eff}_a$

$(u, \text{step}(i), u') \in \text{Diff}$

**internal**  $\text{step}(i:\text{indices})$

**pre**  $i \neq 0 \wedge x[i] \neq x[i-1]$   
**eff**  $x[i] := x[i-1];$

(a)  $(i = 0 \wedge u.x[0] = u.x[5]$

$\wedge u'.x[0] = u.x[0] + 1 \text{ mod } 6) \mathbf{V}$

(b)  $(i \neq 0 \wedge u.x[i] \neq u.x[i-1]$

$\wedge u'.x[i] = u.x[i-1])$

# Nondeterminism

- For an action  $a \in A$ ,  $\text{Pre}(a)$  is the formula defining its **precondition**, and  $\text{Eff}(a)$  is the relation defining the **effect**.
- States satisfying precondition are said to **enable** the action
- In general  $\text{Eff}(a)$  could be a relation, but for this example it is a function
- **Nondeterminism**
  - Multiple actions may be enabled from the same state
  - There may be multiple post-states from the same action

# Executions, Reachability, & Invariants

An **execution** of  $\mathcal{A}$  is an alternating (possibly infinite) sequence of states and actions

$\alpha = u_0 a_1 u_1 a_2 u_3 \dots$  such that:

–  $u_0 \in \Theta$

–  $\forall i$  in the sequence,  $u_i \xrightarrow{a_{i+1}} u_{i+1}$

A state  $u$  is **reachable** if there exists an execution that ends at  $u$ . The set of reachable states is denoted by  $Reach_{\mathcal{A}}$ .

# Invariants (Formal)

What does it mean for  $I$  to hold “always” for  $\mathcal{A}$ ?

- $I$  holds at all states along any execution  $u_0a_1u_1a_2u_3$
- $I$  holds in all reachable states of  $\mathcal{A}$
- $Reach_{\mathcal{A}} \subseteq [[I]]$

Invariants capture most properties that you will encounter in practice

- safety: “aircraft **always** maintain separation”
- bounded reaction time: “within 15 seconds of press, light must turn to walk”

How to **verify** if  $I$  is an invariant?

- Does there exist reachable state  $u$  such that  $u \not\models I$  ?

# Reachability Problem

- Given a directed graph  $G = (V, E)$ , and two sets of vertices  $S, T \subseteq V$ ,  $T$  **is reachable from**  $S$  if there is a path from  $S$  to  $T$ .
- **Reachability Problem**  $(G, S, T)$  : decide if  $T$  is reachable from  $S$  in  $G$ .

# Algorithm for deciding Reachability $G, S, T$

Set Marked := {}

Queue  $Q := S$

Marked := Marked  $\cup S$

**while**  $Q$  is not empty

$t \leftarrow Q.dequeue()$

**if**  $t \in T$  **return** "yes"

**for each**  $(t, u) \in E$

**if**  $u \notin \text{Marked}$  then

            Marked := Marked  $\cup \{u\}$

$Q := enqueue(Q, u)$

**return** "no"

# Verifying Invariants by solving Reachability

Given  $\mathcal{A} = \langle X, \Theta, A, \mathcal{D} \rangle$  and a candidate invariant  $I$ , how to check that  $I$  is indeed an invariant of  $\mathcal{A}$ ?

Define a graph  $G = \langle V, E \rangle$  where

$$V = \text{val}(X)$$
$$E = \{(u, u') \mid \exists a \in A, u \xrightarrow{a} u'\}$$

**Claim.**  $\llbracket I \rrbracket^c$  is not reachable from  $\Theta$  in  $G$  iff  $I$  is an invariant of  $\mathcal{A}$ .

# Summary

- Well-formed specifications in the HIOA language define automata
- **Invariants**: Properties that hold **at all reachable states**.  $Reach_{\mathcal{A}} \subseteq [[I]]$
- BFS to **verify invariants** automatically for (finite) automata