Numerical Calculation of the Stationary Distribution of the Main Multiserver Retrial Queue

A assignment report submitted

for the course

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by

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0.1 Introduction

Many queueing situations have the feature that customers who find all servers busy upon arrival immediately leave the service area and repeat their request after some random time. Between trials a customer is said to be in *orbit*. Then, a flow of repeated attempts for service from the orbit is superimposed on the primary stream of arrivals of first attempts. Retrial queues are presented in as an alternative to classical loss models in telephony which do not take into consideration the real structure of the underlying model. Many other applications include communication protocols, local area networks, and queues in daily life where a customer may leave a long waiting line hoping to come back later when the line may be shorter, so that the area of practical applications of retrial queues is wide. For a comprehensive review of the main results and literature.

0.2 Mathematical description of the main $M/M/c$ retrial queue

We consider a multiserver queueing system in which primary customers arrive according to a Poisson stream of rate λ. The service facility consists of c identical servers and customer service times are independent and exponentially distributed with parameter v. A customer who finds all servers busy upon arrival is obliged to leave the service area and to repeat his demand after an exponential time with parameter μ. As usual, we assume that the interarrival periods, service times and retrial times are mutually independent.

The state of the system can be described by means of a bivariate process $X = \{(C(t), N(t)); t \geq 0\}$, where $C(t)$ is the number of busy servers and $N(t)$ is the number of customers in orbit (sources of repeated demands). Under the above assumptions, the process X is a regular continuous timeMarkov chain with state space $S = \{0, ..., c\} \times N$. Its infinitesimal transition rates $q_{(i,j),(m,n)}$ are as follows. For $0 \leq i \leq c-1$, we have:

$$q_{(i,j),(m,n)} = \begin{cases} \lambda, & if(m,n) = (i+1, j) \\ iv, & if(m,n) = (i-1, j) \\ j\mu, & if(m,n) = (i+1, j-1) \\ -(\lambda + iv + j\mu), & if(m,n) = (i,j) \\ 0, & otherwise, \end{cases}$$

and for $i = c$.

$$q_{(i,j),(m,n)} = \begin{cases} \lambda, & if(m,n) = (c, j+1) \\ cv, & if(m,n) = (c-1, j) \\ -(\lambda + cv), & if(m,n) = (c,j) \\ 0, & otherwise, \end{cases}$$

The ergodicity condition of process X is $\rho = \lambda/cv < 1$. Note that this condition does not depend on the retrial $M/M/c$ queue without retrials. In what follows, we assume that $\lambda < cv$. Then, the limiting probabilities $P_{ij} = \lim_{t\to\infty} p\{c(t) = i, N(t) = j\}$ exist for all $(i,j) \in S$ and are positive. The stationary distribution $\{Pij; (i,j) \in S\}$ cannot be expressed in a tractable analytical form and does not lend to a direct recursive computation. The analysis of numerical methods of calculation of $\{Pij; (i,j) \in S\}$ is the subject matter of this paper.

0.3 Generalized truncated models

In this section we study methods of numerical approximate analysis of the stationary distribution $\{P_{ij}; (i,j) \in S\}$. To this end, we next discuss the use of generalized truncated models which approximate the $M/M/c$ retrial queue by some infinite simpler system. In it is numerically shown that the fact the approximation is based on a infinite system gives better accuracy than those methods based on direct finite truncation.

0.4 Numerical results

In order to understand how much the new generalized model improves the approximation quality, we next compare the approximate values of the blocking probability and the mean number of customers in orbit for the three generalized processes: X^F, X^{NR} and X^{AP}.

To analyze the mean number of customers in orbit, O, we may employ the

$$0 = \frac{(1+\mu)(\lambda - varC(t))}{\mu(c-\lambda)}$$

The above expression reduces the calculation of 0 to the calculation of the variance of the number of busy servers, $VarC(t)$, which is a characteristic of the first component $C(t)$. Since $C(t)$ takes values on the finite set $\{0, ..., c\}$, we may expect to get a better approximation when we use above equation rather than the direct definition $0 = \sum_{i=0}^{c} \sum_{j=0}^{\infty} j \ p_{ij}$.

To simplify notation, we may call the parameters M, N and K as the truncation level of the corresponding generalized model. According to intuitive expectations, the performance characteristics of the truncated models converge to the corresponding characteristics of the main $M/M/c$ retrial queue as the truncation level tends to infinity. Thus, we shall measure the approximation accuracy through the speed of convergence.

0.5 Conclusion

We have discussed the numerical approximation of the main multiserver retrial queue of $M/M/c$ type with exponential repeated attempts. To this end, we have replaced the original intractable system by a simpler calculable one. As opposite to other generalized truncated models , our approach preserves the main feature of a retrial queue, i.e., the non-homogeneity introduced by the existence of a flow of repeated attempts. Our numerical experiments compare the existing approaches and support the use of the generalized method introduced in this paper.

Bibliography

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