MA 402 : Queueing Models for Performance Analysis

(Assignment)

<u>Paper considered</u>: *"The a priori vacation probability in the M/G/1 single vacation models"* by Shmuel N. Labzovski, Abraham Mehrez, Ilia B. Frenkel

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Introduction : -

Single vacation models of the type M/G/1 are those models where the server takes exactly one vacation immediately after service completion and there are no items available in the system. The underlying assumption in this model is that at the moment an item arrives into the system, there exists the probability, denoted as the a priori probability that the server takes a vacation after this item service completion.

Problem Description : -

Vacation model considered has two types of arriving items

1. Regular ones – After service completion, server does not take a vacation and starts to serve next item. Mean process time = b.
2. Special ones – After service completion, a vacation takes place. Mean process time = (b + V).

Where

b is mean process service time

V is mean vacation time

Expected value of waiting time in a Bernoulli schedule M/G/1 model with two sub-types of arriving items (1-ρ),ρ is given by

$$E(W) = \frac{\lambda}{2}\frac{(1-\rho)b_1 + \rho b_2}{1-\lambda[(1-\rho)b_1 + \rho b_2]},$$

Where b_1 and b_2 are the mean service times of sub-types, λ is the item arrival rate

Thus, expected value of waiting time for the vacation model considered is given by

$$E(W) = \frac{\lambda}{2}\frac{b^{(2)} + pV^{(2)} + 2pbV}{1-\lambda(b+pV)}.$$

It can be noticed that expected value of item waiting time is function of λ, b and V. Hence, the expression for the a priori probability p can be obtained as

$$p = \frac{(1-\lambda b)f(\lambda, b, V) - b^{(2)}}{V^{(2)} + 2bV + \lambda V f(\lambda, b, V)}.$$

Examples of a priori vacation probability deviation

1) **Exhaustive service model –** server takes exactly one vacation immediately at the end of each service completion, if there are no items available in the system.

   Expected value of waiting time is

$$E(W) = \frac{\lambda}{2}\left[\frac{b^{(2)}}{1-\lambda b} + \frac{V^{(2)}}{V^*(\lambda) + \lambda V}\right]$$

   Thus,

$$f(\lambda, b, V) = \frac{b^{(2)}}{1-\lambda b} + \frac{V^{(2)}}{V^*(\lambda) + \lambda V}.$$

and

$$p = \frac{2(1-\lambda b)(\lambda V^{(2)}/(1+2\lambda^2 V))}{V^{(2)} + bV + \lambda V[(b^{(2)}/(1-\lambda b)) + 2\lambda V^{(2)}/(1+2\lambda^2 V)]}.$$

2) **Gated service model –** when server returns from a vacation it serves continuously only those items that are waiting at that time, ignoring until the next vacation all the items arriving during the service period.
   The expected value of waiting time is given by

$$E(W) = \frac{\lambda}{2}\left[\frac{b^{(2)}}{1-\lambda b} + \frac{V^{(2)}}{V^*(\lambda) + \lambda V}\right] + \frac{(V^*(\lambda) + \lambda V)b}{1-\lambda b}.$$

From the above expression $f(\lambda, b, V)$ and thus a priori probability p can be computed.

3) **Model with fixed vacation policy –** vacation periods initiation via fixed number of served items.
   Expected value of waiting time in queue is

$$E(W) = \frac{\lambda}{2(1-\lambda b)}\left[\frac{b^{(2)} - b^2 + V^2}{2n^{0.78}} + b^{(2)} + \frac{2bV}{n}\right],$$

   Where n is number of items.
   Thus, $f(\lambda, b, V)$ turns of to be

$$f(\lambda, b, V) = \frac{1}{1-\lambda b}\left[\frac{b^{(2)} - b^2 + V^2}{2n^{0.78}} + b^{(2)} + \frac{2bV}{n}\right].$$

   Using $f(\lambda, b, V)$ the a priori vacation probability can be calculated.

Conclusions :-

A single vacation model of the type M/G/1 with Bernoulli schedule is considered with different vacation disciplines. A way of computing a priori vacation probability is presented for the vacation disciplines with the knowledge it they depends on a function of item arrival rate, mean process service time and mean vacation time which in turn is nothing but twice of expected value of waiting time in the queue.

SCOPE : - a priori probabilities for different types of vacation models can be researched.