Blocking Probability and Channel Assignment in Wireless Network

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***Abstract*— In this summary [1], blocking probability of a call in a multi-hop wireless network with a connection-oriented traffic model and multiple transmission spatially reusable channels is presented. The blocking probability depends upon (a) the channel assignment scheme and (b) the transmission radius of the nodes in network link structure. The blocking probability analysis for a wireless line and grid network is done, and the tradeoff between transmission radius and blocking probability for multi-hop calls in explored. It is shown that for a line network a larger transmission radius substantially reduce the blocking probability of calls, whereas for a grid network with more dense node topology smaller transmission radius is better.**

***Index Terms*— Blocking probability, transmission radius, wireless networks, connection oriented traffic, multi-hop calls, wireless interference, quality of service**

I. Introduction

A multi-hop wireless network is a cooperative network where data streams may be transmitted over multiple wireless hops to reach the destination. The network link structure is dependant upon the transmission radius of the nodes and can be adjusted by varying the transmission power. A spatially reusable multiple channels network without node mobility and with connection-oriented model is considered. The wireless interference and traffic models are explained in the next section. The goal was to investigate the effect of transmission radius of the nodes and the channel assignment scheme on steady state call blocking probability, in such a network. The effect of transmission radius can be understood as follows. A smaller transmission radius of the nodes causes less interference on each hop but the calls have to hop through many nodes to reach the destination. As the same call is served by many nodes along the route, multi-hopping increases the internal load of the network. In contrast, a larger transmission radius reduces the number of hops of a call but increases the interference constraints at each hop. The tradeoff in relation to blocking probability is examined by focusing on two topologies: the line and the grid network for analytical simplicity. Firstly the exact blocking probability analysis for a single channel wireless line network is considered, and then a model is constructed to compute the blocking probabilities in the multiple channel case for the random channel assignment policy. Using the mathematics, it is shown that in a line network a larger transmission radius reduces the blocking probabilities of calls; whereas, for a grid network with an underlying denser node topology it is more desirable to use a smaller transmission radius. The result suggests that for sparse networks the increase in the internal load due to multi-hopping contributes significantly to call blocking whereas for denser networks the increase in the interfering neighboring nodes due to a larger radius is a significant limiting factor. The analysis of blocking probability and dynamic channel assignment has been extensively considered in context of cellular networks. However there are significant differences between a multi-hop wireless network, the focus in this paper, and a cellular network. For example, in a cellular network the communication is with the nearest base-station over a single wireless link; whereas in a multi-hop wireless network, calls hop through various links to reach the destination. This constraint imposes additional complexity as non-conflicting channels must be allocated on the wireless links along the source-destination path. Another contrasting difference between the two networks is that a cellular network has a regular structure that makes the set of interfering cells fixed; whereas in a multi-hop wireless network the set of interfering nodes depends on the node topology and their transmission radii.

II. System Model

A wireless network whose node topology does not change over time and the nodes in the network transmit with equal power using and Omni-directional antenna is considered.

A. Interference model

A disk model of interference is assumed. Let the transmission radius of a node, say T, be defined as the radius

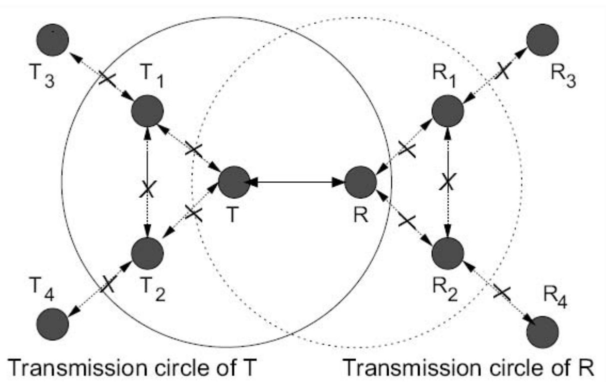

Fig. 1. Interference model for a bi-directional transmission $(T \to R)$

of a circle centered at T such that,(a) outside this circle there is no interference from the signal transmitted by T and (b) within this circle there is complete interference of the signal transmitted by T with other ongoing signal reception. It is also assumed that without any interference from other nodes, the signal transmitted by node T can be perfectly received within its transmission radius. A direct wireless link exists between any two nodes if they lie within each others transmission radius. A node R is said to be a neighbor of node T if R lies within the transmission radius of T. As the nodes have equal transmission radius, T is also a neighbor of R. Let the set of neighbors of T and R be denoted as N_T and N_R respectively. Consider the uni-directional data transfer,$(T \rightarrow R)$, in channel γ. For successful service of this call following criterion need to be satisfied.

1) Nodes T and R must not be involved in any other call transmission/reception in channel γ. This criterion ensures that a node cannot simultaneously serve two calls in channel γ.
2) Neighbors of T ($P \epsilon N_T$, excluding R) must *not receive* from any other node in channel γ. Otherwise the transmission from T will interfere at P
3) Neighbors of R ($Q \epsilon N_R$, excluding T) must *not transmit* to any other node in channel γ. Otherwise the transmission from Q will interfere at R.

For a bi-directional call between nodes T, R to be successful; i.e. data transfer in both directions $(T \rightarrow R)$ and $(R \rightarrow T)$,the above criterion implies that neighbors of node T and node R must neither transmit nor receive in the channel γ. Such nodes which are not involved in transmission/reception in that channel are labeled *inactive* in channel γ, and *active* otherwise. This implies that *for a bidirectional call $T \rightarrow R$ to be successful in γ, neighbors of node T excluding R and neighbors of node R excluding T must be inactive*. Figure 1 illustrates a single hop bi-directional data transfer between nodes T and R in channel γ. Nodes T and R cannot service any other call in channel γ. Neighbors of node $T(T1, T2)$ and neighbors of $R(R1, R2)$ must be inactive while call $T \leftrightarrow R$ is active. In the figure, all data transfers marked $\times$ must not take place for call $T \leftrightarrow R$ to be successful.

B. Traffic Model

In this paper, a connection-oriented model wherein the arriving calls require a dedicated channel on each hop along the path is considered. These channels are held up while the call is in progress and released at the end of the call i.e. in such models the allocated channels are *not* re-assigned very often. The assumption of connection-oriented traffic simply translates into the fact that a channel requests are stochastic with some average rate and there is no queueing of the requests.

III. BLOCKING PROBABILITY ANALYSIS IN A WIRELESS LINE NETWORK

The blocking probability of single hop bi-directional calls in a line network is first analyzed. Also the analysis is first done for single channel network and the solution obtained is then extended to multiple channels. The line network is an important network in practice and serves a good starting point in understanding network tradeoffs.

A. Single channel

A wireless line network with nodes located unit distance apart at positions $x = -m, -m+1, \ldots, m$ is considered with nodes as $X_m, X_{m+1}, \ldots, X_m$. A system is considered to have a *single channel* that can be spatially re-used subject to interference constraints. Each node have a transmission radius of r, where $r \geq 1$ and $r \epsilon Z+$; a positive integer. The calls are taken to be bi-directional with the source and destination nodes r units apart, i.e. between nodes X_k and $X_{k+r} \forall k$. The calls are single hop as each node can communicate directly with a node r units apart. Calls $X_k \leftrightarrow X_{k+r} \forall k$ arrive according to an independent Poisson process of rate λ. The holding period of each call is independent and identically distributed as an Exponential distribution with mean $\frac{1}{\mu}$. If a call cannot be accepted then it is dropped otherwise it occupies the channel while in progress. Such a wireless line network with radius r is denoted as WLN-r for short. A WLN-1 network is depicted in Figure 2.

Theorem 1: The blocking probability of a call in an infinite length $(m \rightarrow \infty)$ WLN-r, $r \epsilon Z^+$, network and $\nu = \frac{\lambda}{\mu}$, $(0 \leq \nu < \infty)$ is,

$$P_B = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \tag{1}$$

where x is unique root in $(0, 1]$ of $\nu x^{2r+1} + x = 1$.

B. Multiple channels

In this section, the extension of analysis of WLN-r to case of multiple channels with random policy of assigning channels to the incoming calls is done. In this policy the new call is assigned a channel randomly from among the free channel on the link. Free channels refer to those channels such that the acceptance of a call in these channels does not violate interference constraints. The random policy is easy to implement in practice. However exact analysis is complicated by the fact that to make a channel allocation decision, the knowledge of the channel already occupied by the ongoing calls, making state space for the system very large and analysis intractable. However since random policy doesn't differentiate between the channels an *approximate* model is constructed based on effective load concept. The results of the blocking probability of a

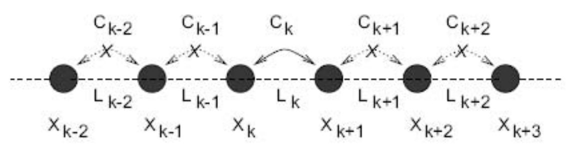

Fig. 2. Constraints representing the simultaneous service of calls in a WLN-1 network.

call for the *single channel* case is used to first construct a single channel tractable markovian model whose parameters are chosen to match the result of (1). This markovian model is then extended with some approximations to incorporate multiple channels. Simulations results verify that the theoretical values from this model closely agree with the numerical results. Consider the link $L_k(X_k \rightarrow X_{k+r})$ of the line network. Initially assuming that there is only a single channel γ in the network on link L_k,with state denoted as S_k. S_k is modeled as a three state process, the free state (F), the busy state (B_u) and the blocked state (B_l) as shown in Figure 3. The link L_k is said to be in the blocked state if the channel is occupied by a call on an interfering link making it unavailable on link L_k. It is in the busy state if there is a call in progress. Let $Y_{F\rightarrow B_l}$ be the random variable that denotes the transition time from state $F \rightarrow B_l$. The distribution of $Y_{F\rightarrow B_l}$ can be computed through its complicated dependence on the various states of the other links. However, a good approximation is to simply assume it to be exponentially distributed with some rate λ'.The random variable $Y_{F\rightarrow B_l}$ can have a general distribution with mean $\frac{1}{\mu'}$. Figure 3 shows the transition rates of S_k. Using the detailed balance equation of the three state Markov process and letting, $\nu' = \frac{\lambda'}{\mu'}$ and $\nu = \frac{\lambda}{\mu}$ we get,

$$\nu' + \nu = 1 + \frac{P_B}{1 - P_B} \tag{2}$$

where P_B is known from (1). Thus, the value of ν' that gives the correct P_B value can be obtained from the above equation. Define an effective load, $\widetilde{\nu} \cong \nu' + \nu$ then, we can interpret the load $\widetilde{\nu}$ as consisting of two components; the external load $\nu$and the load ν' seen by the link that makes the channel blocked. The effect of interference constraints on blocking probability can, thus, be viewed as an additional load $\widetilde{\nu}$. Combining (1) and (2), we get

$$\widetilde{\nu} = \nu' + \nu = \frac{1 + (2r\nu - 1)x^{2r+1}}{x^{2r+1}} \tag{3}$$

The effective load $\widetilde{\nu}$ can be understood as follows. If a link of WLN-r is isolated from the network and load $\widetilde{\nu}$ applied to it, it would have the same blocking probability as experienced within the line network with symmetrical load ν. An isolated link of a single channel WLN-r

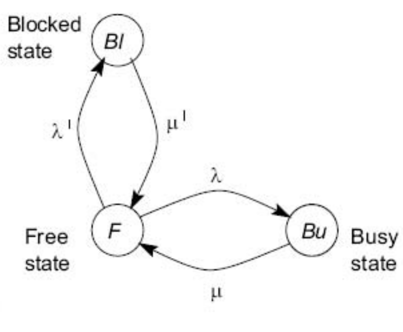

Fig. 3. Three state Markov process model of the channel on a link.

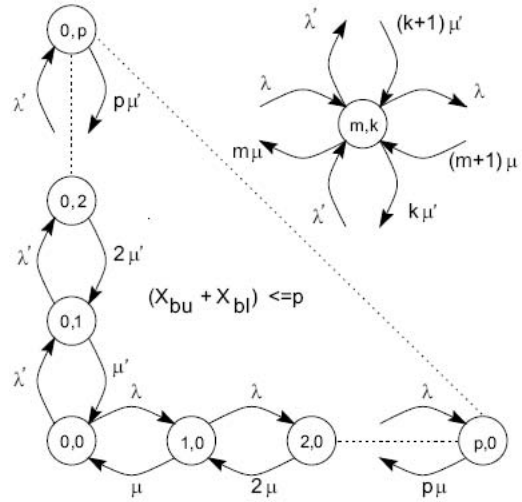

Fig. 4. State transition diagram for the random assignment policy.

is equivalent to a $M/M/1/1$ system. Thus, the above analogy states that in terms of blocking, a single channel WLN-r network with load ν is identical to a $M/M/1/1$ system with load $\widetilde{\nu}$. Generalizing to the multiple channel case, define the state of a link as $X(t) \equiv (X_{bu}(t),\ X_{bl}(t))$ where X_{bu} is the number of busy channels and X_{bl} the number of blocked channels on that link at time t. Let the total number of channels available in the network be p. At any time t, the state $X(t) \equiv (X_{bu}(t)\ X_{bl}(t))$ must satisfy $X_{bu}(t) + X_{bl}(t) \leq p$. Following the single channel process and the fact that the random policy does not differentiate among the channels we approximate the network as a p server system with rate λ that makes the channels busy and rate λ' that makes the channels blocked. The transition rates among the various states of the process $X(t)$ are shown in Figure 4. Let $\pi(i,j)$ denote the steady state probability that X takes value (i,j). The steady state blocking probability,${P_B}^{rand}$, equals $\sum_{i+j=p} \pi(i,j)$. Solving the detailed balance,

$${P_B}^{rand} = \frac{\frac{\widetilde{\nu}^p}{p!}}{1 + \widetilde{\nu} + \frac{\widetilde{\nu}^2}{2!} + \ldots + \frac{\widetilde{\nu}^p}{p!}} \tag{4}$$

where $E(\nu p)$ is the Erlang B formula for load ν and p servers. Thus, (4) is same as the blocking probability of an equivalent $M/M/p/p$ system with load $\widetilde{\nu}$. The results are plotted and it is shown that it is in good agreement with the theory.

IV. EFFECT OF TRANSMISSION RADIUS ON BLOCKING PROBABILITY

It is clear that if the nodes have a smaller transmission radius then the interference constraints on each hop are fewer but the calls hop through many links to reach the destination. This increases the internal load in the system. In contrast, a larger transmission radius reduces the number of hops of a call but increases the interference constraints at each hop. The effect of this tradeoff on blocking probability is non-trivial and leads to different observations under different node topologies. In this section, we study this tradeoff for two simple node topologies; the line and the grid network.

A. Line Network

We begin by considering the following simple but nontrivial example that lends itself to an exact analysis and also clearly highlights the problem. Consider a line network with two channels and with the source-destination nodes of the calls two units apart. The arrival process of each call is an independent Poisson process of rate and the holding time is i.i.d with mean $\frac{1}{\mu}$. Consider the following two schemes. *Scheme A:* The nodes have transmission radius unity, thus are n hops long. The channel is not assigned randomly but on the basis of re-arrangement channel assignment as it uses the channel resources optimally. *Scheme B:* The nodes have transmission radius of $r = n$ and hence all calls are single hop. A sub-optimal channel assignment policy that selects *a* channel randomly from the pool of channels for each new arriving call. If the channel is free(non-blocked and non-busy) then it is allocated otherwise the incoming call is dropped.

It is clear that the channel assignment policy in Scheme B under-utilizes the channels as it rejects a call if the randomly selected channel is not free without considering the state of the other channel. The following theorem shows that even with this inefficient random policy Scheme B with a larger radius has a lower blocking probability as compared to Scheme A. Thus, for any fixed blocking probability threshold β the supportable load ν is higher for Scheme B than Scheme A. The result, thus highlights that in networks with low node density a larger transmission radius can lead to a better network performance. The result, thus, highlights that in networks with low node density a larger transmission radius can lead to better network. The intuition behind this result is that for a line network with a sparse node topology the blocking probability increase due to a larger set of interfering nodes(larger radius) is smaller as compared to an increase due to larger effective link load caused by multi-hopping.

B. Grid Network

Consider an infinite grid network. It is checked that for low ν and moderate number of channels we see it is preferable to use a smaller transmission radius. The intuitive reason is that a grid network has denser node topology than a line network. As a result the number of interfering links increases rapidly with an increase in the transmission radius of the nodes leading to higher blocking probability than using a smaller transmission radius. The simulations are done to justify the conclusion.

V. CONCLUSION

In this summary of the paper, it focused mainly on the effect of transmission radius of the nodes. The blocking probability model and derived formulas yielded useful insights. It is shown that in the line topology using a large transmission radius substantially reduces the blocking probability; while the opposite is true in the more dense grid topology.

REFERENCES

[1] M. Zafer and E. Modiano,'Blocking Probability and Channel Assignment in Wireless Network', *IEEE Trans. Wireless Comm.*, vol. 5, no. 4,April 2006.