Erlang Capacity of a Power Controlled CDMA system

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1 Introduction

A limited amount of bandwidth is allocated for wireless services. A wireless system is required to accommodate as many users as possible by effectively sharing the limited bandwidth. Therefore, in the field of communications, the term multiple access could be defined as a means of allowing multiple users to simultaneously share the finite bandwidth with least possible degradation in the performance of the system. There are several techniques how multiple accessing can be achieved. There are various basic schemes like Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) , Code Division Multiple Access (CDMA), Space Division Multiple Access (SDMA), Random Access (Packet Radio).

Code Division Multiple Access(CDMA) evolved from spread spectrum technology. Spread Spectrum systems were developed to combat unauthorized access and prevent jamming of signals while operating at least external interference, low spectral density and providing multiple access capability.In CDMA, all the users occupy the same bandwidth, however they are all assigned separate codes, which differentiates them from each other . CDMA systems utilize a spread spectrum technique in which a spreading signal, which is uncorrelated to the signal and has a large bandwidth, is used to spread the narrow band message signal. Direct Sequence Spread Spectrum (DS-SS) is most commonly used for CDMA[1]. In DS-SS, the message signal is multiplied by a Pseudo Random

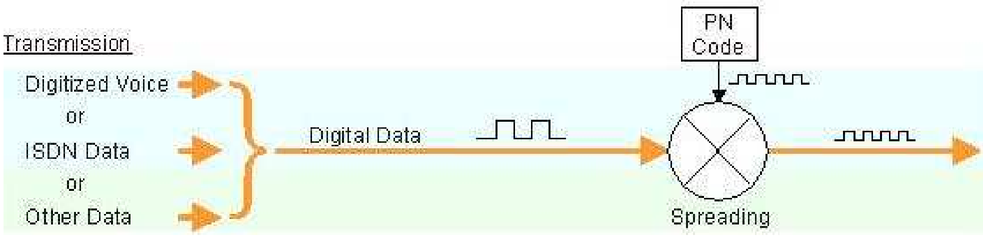

Figure 1: Basic transmitter for DS-CDMA

Noise Code (PN code), which has noise-like properties. Each user has his own codeword which is orthogonal to the codes of other users. In order to detect the user, the receiver is required to know the codeword used by the transmitter. At the base station, there is a separate receiver for each user, and all receivers are presented with the same incoming signal, which is the sum of the signals of all users and the noise(AWGN). Each receiver correlates the incoming signal with a synchronized copy of the desired PN code to generate a decision statistic which is used to estimate the transmitted data stream. Thus, conventional correlation detectors operate by enhancing the desired user, and treat the MAI which is inherent in CDMA as additive noise. This would work well if the MAI were truly uncorrelated with the desired signal.

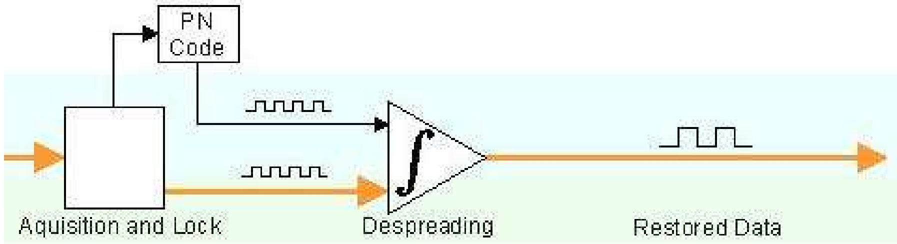

Figure 2: Basic receiver system for DS-CDMA

2 Power Control

Power control is a basic system requirement for CDMA. Originally, power control was proposed for a CDMA system to overcome the near-far problem so that the received power for all the (homogeneous) connections are the same in order to achieve the same QoS. In CDMA systems with heterogeneous traffic, power control could also be used to achieve a target power for each connection in order to limit the MAI interference to other users and to achieve its required SINR.

In 3G W-CDMA and cdma2000 systems, the reverse link power control includes the *open loop power control* and the *closed loop power control*[2]. The open loop power control is based on the principle that a mobile closer to the BS needs to transmit less power as compared to a mobile that is far away from the BS. The mobile adjusts its transmitted power based on the received power level from the BS. If the received power is high, the mobile reduces its transmit power. Otherwise, the mobile increases its transmit power.The reversed link closed loop power control consists of two parts : the reverse *inner loop power control* and the reverse *outer loop power control.*The inner loop power control keeps the mobile as close to its target signal-to-interference ratio(SIR)as possible and the outer loop power control adjusts the target SIR for a given mobile. For a single cell CDMA system, each user's transmitted power is controlled by the BS. New users can be accepted as long as there are receiver-processors to service them, independent of time and frequency allocations. It is assumed that a sufficient number of such processors is provided in the common base station such that the probability of a new arrival finding them all busy is negligible.

3 Capacity of Conventional Techniques

The CDMA system's capacity can be compared with the traditional multiple access techniques such as TDMA (Time Division Multiple access) or FDMA (Frequency Division Multiple access) . FDMA is one of the earliest multiple-access techniques for cellular systems in which the bandwidth is divided into a number of channels and distributed among the users. TDMA is a complimentary access technique to FDMA where the entire bandwidth is available to the user but only for a finite period of time. Here the overhead is an important issue but the interference level is highly minimized.

Thus these conventional multiple access systems (FDMA and TDMA) traffic channels are allocated to users as long as there are channels available, after which all incoming traffic is blocked until a channel becomes free at the end of a call. Thus their blocking probability is obtained from the Erlang analysis of the M/M/S/S queue, where the first M refers to a Poisson arrival rate of λ calls/s; the second M refers to exponential service time with mean $1/\mu$ s/call. The first S refers to the number of servers (channels); and the second refers to the maximum number of users supported before blockage occurs.The Erlang-B formula gives the blocking probability as follows:

$$P_{blocking} = \frac{(\lambda/\mu)^S/S!}{\sum_{k=0}^{S}(\lambda/\mu)^k/k!}$$

Where λ/μ is the offered average traffic and $(\lambda/\mu)(1-P_{blocking})$ as the active number of users . The offered traffic Erlang capacity per sector for a blocking probability 2 percent is respectively, equal to

$$(\lambda/\mu)_{AMPS} = 12.34\ Erlang$$

4 CDMA reverse link Erlang Capacity approximation :

The capacity of CDMA systems are limited by the number of interfering signals both of the cell under consideration and the neighboring cells which raise the noise floor of the system and puts restriction on the number of users that can be serviced maintaining the desired Qos for all the users.

For the analysis of the system three main assumptions considered are as follows :

- The number of active calls is a Poisson random variable with mean λ/μ
- Each user is switched ON with probability ρ
- Each user's required energy-to-interference Eb/Io ratio is varied according to propagation conditions to achieve the desired frame error rate .

So the non blocking condition can be given by :

$$Z \equiv \sum_{i=1}^{k} \nu_i \epsilon_i + \sum_{j}^{othercells} \sum_{i=1}^{k} \nu_i{}^{(j)} \epsilon_i{}^{(j)} \leq (W/R)(1-\eta)$$

W = spread-spectrum bandwidth,R = data rate,Eb = bit energy,No = thermal (or background) noise density, Io = maximum total acceptable interference density (inter- ference power normalized by W), and f = ratio of other cell interference (at base station for given sector)-to-own sector interference η = No/Io

The probability of blocking becomes

$$P_{blocking} = Pr[Z > (W/R)(1-\eta)]$$

Thus, this probability of blocking is kept sufficiently low so that good quality sevice could be ensured to the users. Here in CDMA system since the entry of the users are independent of the parameters such as time slots or channel bandwidth the probability of k active users in the system is modeled as the M/M/ ∞ [3] and can be given by

$$P_k = \frac{(\lambda/\mu)^k}{k!} e^{-\lambda/\mu}$$

The power control in CDMA is not perfect because of the channel characteristics being dynamic and suffers much in case of fast fading channel where it changes quickly. This inaccuracy in power control can be approximated by the log normal distribution.So the ϵ (Eb/Io ratio of a single user) can be approximated using log normal approximation and its moments can be given using a gaussian random variable x with mean m and variance σ as :

$$E(\epsilon) = E(e^{\beta x}) = exp[(\beta\sigma)^2/2]exp(\beta m)$$

$$E(\epsilon^2) = E(e^{2\beta x}) = exp[2(\beta\sigma)^2]exp(2\beta m)$$

The blocking probability can be approximated by the Central limit theorem which reduces the complexity of computation and provides a satisfactory result.

$$P_{blocking} \approx Q\left(\frac{A - E(Z')}{\sqrt{VarZ'}}\right)$$

Where

$$A \equiv \frac{(W/R)(1-\eta)}{exp(\beta m)}$$

$$E(Z') = (\lambda/\mu)\rho\ exp[(\beta\sigma)^2/2]$$

and

$$Var(Z') = (\lambda/\mu)\rho\ exp[2(\beta\sigma)^2]$$

The interference experienced by the users in a cell also depends on the neighboring cell users that act as interfering signals and raise the noise floor. The other interfering user signal is also under the effect of power control and hence they can also be considered to be log normally distributed. The total number of other cell users can be much larger but their effective power is equivalent to kf user (where f represents the fraction of the interference of the other cell to the own sector interference). The modified Erlang capacity considering the interference from neighboring cells can be given by

$$\frac{\lambda}{\mu} = \frac{(1-\eta)(W/R)F(B,\sigma)}{\rho(1+f)(E_b/I_o)_{median}}\ Erlangs/sector$$

where

$$F(B,\sigma) = exp[-(\beta\sigma)^2/2].\left\{1 + (B/2)exp[3(\beta\sigma)^2/2].\left(1 - \sqrt{1 + 4exp[-3((\beta\sigma)^2/2]/B}\right)\right\}$$

and

$$B = \frac{[Q^{-1}(P_{blocking}]}{A}$$

Here we can see that the power control inaccuracy leads to capacity reduction of the system. Now comparing the Erlang Capacity of the CDMA system with that of the AMPS system for conventional multiple access technique we conclude that the capacity of the system increases tremendously.

5 Transmitted Power consideration :

Instead of assuming the total received interference-to-noise ratio Io/No to be fixed and evaluating the blocking probability as a function of average user loading, blocking probability is made fixed and for each user loading λ/μ the minimum value of Io/No is determined.

As reflected by the graph above when traffic is light, much lower interference-to-noise ratios can be imposed . Its important to note that the per-user power can be reduced for lightly loaded cells. This also justifies the previous assumption that when the interference-to-noise ratio exceeds a given level (about 10 dB), the interference per additional user grows very rapidly, yielding diminishing returns and potentially leading to instability.

6 Initial Access

CDMA system unlike other multiple access techniques is interference limited. The arriving users are not power controlled until their power level reaches a threshold level. The initial power level could be assumed to be uniformly distributed from 0 to a maximum value Em. Thus, the initial energy can be given as γEm where γ is uniformly distributed random variable.

The user increases its power exponentially till it reaches a level where its request could be acknowledged. Thus energy at any time t can be given by

$$E(t) = \gamma Em e^{\delta t}$$

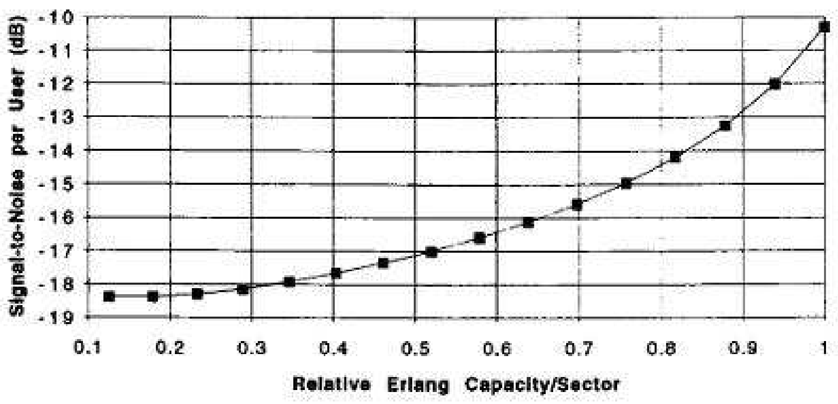

Figure 3: Graph of signal to noise per user as a function of relative sector capacity

Where $1/\delta$ refers to the mean initial request time which is exponentially distributed. This is an important assumption since it helps us to use the fact that when the arrival is poisson and the service time is exponentially distributed then output distribution is also poisson process. Thus the new capacity of the system incorporating initial access could be given by using the previously used formula with basic modifications:

Where

$$E(Z'') = E(Z') + (\lambda/\delta)E(\gamma Em/I_o)/e^{2\beta m}$$

$$Var(Z'') = Var(Z') + (\lambda/\delta)E(\gamma^2 Em/I_o)/e^{2\beta m}$$

$$E(Z'') = \left\{(\lambda/\mu)\rho\ exp[(\beta\sigma)^2/2] + (\lambda/\delta)(1/2)\theta\ exp[(\beta\sigma)^2/2]\right\}[1+f]$$

$$= \rho(\lambda/\mu)\left(1 + \frac{\theta\mu}{2\rho\delta}\right) exp[(\beta\sigma)^2/2][1+f]$$

$$Var(Z'') = \left\{(\lambda/\mu)\rho\ exp[2(\beta\sigma)^2] + (\lambda/\delta)(1/3)\theta^2\ exp[2(\beta\sigma)^2]\right\}[1+f]$$

$$= \rho(\lambda/\mu)\left(1 + \frac{\theta^2\mu}{3\rho\delta}\right) exp[2(\beta\sigma)^2][1+f]$$

Where $\theta E_b/No = E_m/No$ This shows that the initial access decreases the Erlang traffic due to the inclusion of the factor

$$(1 + \frac{\theta^2\mu}{3\rho\delta})$$

but this decrease is considerably less.

7 Conclusion :

Erlang capacity was defined in terms of 1 percent blocking probability for the spread-spectrum DS-CDMA system. The considerations involved Initial access requests sent by the users and the interference offered by the users considering both, the cell under consideration as well as the neighboring cell users.

The main conclusion derived is the tremendous increase in the capacity of the system in comparison with the traditional AMPS systems. It is also observed that the mobile can work at much lower power when the traffic is light thus leading to enhanced battery life. Thus the CDMA system promises to be the technology for the B3G (beyond 3G) system which can support high data rates for various applications.

References

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