MA 402 ASSIGNMENT

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A QUEUEING MODEL FOR METEOR BURST PACKET COMMUNICATION SYSTEMS

INTRODUCTION:

A meteor burst system uses ionization layers formed by naturally occurring meteorite showers in the earth's atmosphere to reflect radio signals thereby letting such layers fulfil the task of a communication satellite. While such showers occur at average intervals of about 10s, the resulting layers provide effective transmission periods of only about 0.5 seconds. Thus, in mathematical terms, we have an intermittently available server.

For point-to-point packet communication from A to B, a continuous probing signal is sent from B to A. Receipt of this signal by A signifies the beginning of an operating period (when the medium is available) and A starts sending packets to B. However, the following complications must be taken into account :

- At the beginning of each operating period, a "preamble" is sent for synchronisation.
- At the end of each operating period, the packets that are en route are lost and need retransmission.
- The channel has a bit error rate and erroneous packets have to be retransmitted

We shall model this system based on a Markov chain belonging to the general class of M/G/1 Markov chains. We shall then compute the distribution and moments of the sojourn time of a message along with several other related quantities. These computations will be substantially simplified due to the special properties of the queuing model. Another advantage of our queuing model is that we can use the empirically observed distributions of operative and inoperative periods (and not any particular distribution) directly as inputs to it.

THE COMMUNICATION SYSTEM MODEL:

Initial Assumptions:

- The time taken to transmit one packet is taken as one time unit.
- Assume that the preamble and the packets lost at the end of a session together constitute M packets.
- The state space of the Markov chain is partitioned into two subsets – operating states and inoperative states. We assume in our model that these periods form an alternating renewal process.
- An operating period is further modelled as consisting of two parts – one in which packets from the message queue are sent and one in which no packets of the message are sent. The second one is the part of the operating period due to the preamble and due to those packets at the end of a transmission period that need retransmission.
- Assume that the wasted portions of the operating period are contiguous and occur at the end of each operating period.

Therefore, the states of the communication medium at successive time points can be described by an irreducible m+M+r state discrete time Markov chain with the following transition matrix

$$A = \begin{bmatrix} T & T^0 & 0 & 0 \\ 0 & 0 & I_{M-1} & 0 \\ 0 & 0 & 0 & \beta \\ \overline{b_M} S^0 \alpha & b_M S^0 & S^0 b & S \end{bmatrix}$$

Where I_{M-1} is the (M-1)X(M-1) identity matrix

T is an mXm substochastic matrix

S is an rXr substochastic matrix

$b = (b_{M-1}, \ldots, b_1)$ is a non-negative vector of probabilities

$\overline{b_M} = 1 - \sum_{j=1}^{M} b_j$

$T^0 = \mathbf{1} - T\mathbf{1}$

$S^0 = \mathbf{1} - S\mathbf{1}$ where **1** is a column vector of appropriate order of 1's

β is an r-component probability vector

α is an m-component probability vector

States 1 to m+M are the available states out of which m+1 to m+M are the states in which the packets transmitted are either part of the preamble or are one of those packets occurring at the end of the available period and requiring retransmission. Thus, in these states the packet queue cannot be depleted.

Therefore, the sojourn time in states m+1 to m+M corresponds to the operating period wasted.

When in states 1 to m, the system transmits packets from the true message queue with bit error rate Є so that probability of correct reception of a message of 'b' bits is given as

$$\bar{e} = (1-\epsilon)^b$$

MODELING ARBITRARY DISTRIBUTIONS:

Now, we shall show that the above model can support any arbitrary distribution for available and unavailable periods.

The durations of operating and inoperative periods can be given as follows:

- Inoperative period : $P(U = k) = \beta S^{k-1} S^0 \quad , \quad k \geq 1$
- Operative period : $P(V = n) = \begin{cases} b_n & if\ n \leq M \\ \overline{b_M} \alpha T^{k-1} T^0 & if\ n = M + k, k \geq 1 \end{cases}$

Evidently, the inoperative period can be described by a phase type distribution PH(β,S) and the operative period by PH(α,S). We know that discrete phase type distributions include as special cases the geometric distribution, the negative binomial distribution, their mixtures and convolutions, as well as any distribution on the non-negative integers with finite support.

Thus, our model allows for very general distributions for the durations of of availability and unavailability of the medium. In fact, we can even use the empirical distributions obtained from observational data directly.

THE QUEUING MODEL:

According to our initial assumption, time is discretized in units equal to the transmission time of a single packet.

We assume that the number of packets arriving in successive intervals [n,n+1) are i.i.d with distribution $\{a_v : v \geq 0\}$ and mean $\bar{a}$.

Define, X_n = number of packets in the system at time n+

J_n = state of the medium at time n-

The process $\{(X_n, J_n) : n \geq 0\}$ is a DTMC with state space $\{(\mathrm{i,j}) : i \geq 0, 1 \leq j \leq m + M + r\}$

Lets define the subset of states $\boldsymbol{i} = \{(i,j): 1 \leq j \leq m + M + r\}$ as level $\boldsymbol{i}$

Partitioning the transition matrix **A** according to levels **0,1,......,** we get

$$P = \begin{pmatrix} B_0 & B_1 & B_2 & B_3 & \cdots \\ A_0 & A_1 & A_2 & A_3 & \cdots \\ 0 & A_0 & A_1 & A_2 & \cdots \\ 0 & 0 & A_0 & A_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{pmatrix}$$ ----------------- **M/G/l** type

Where for v≥0,

$$B_v = a_v A \quad and \quad A_v = A\Delta_v$$

Where

$$\Delta_v = \begin{pmatrix} (\bar{\mathrm{e}}\mathrm{a}_\mathrm{v} + \mathrm{ea}_{\mathrm{v}-1})\mathrm{I}_\mathrm{m} & 0 \\ 0 & \mathrm{a}_{\mathrm{v}-1}\mathrm{I}_{\mathrm{M+r}} \end{pmatrix}$$

denote

$$x(i,j) = \textit{probability that the markov chain } \{(X_n, J_n)\} \textit{is in state } (i,j)$$

$$\boldsymbol{x}_i = \big(x(i,1), \ldots, x(i, m + M + r)\big)$$

$$\boldsymbol{x} = (\boldsymbol{x_0}, \boldsymbol{x_1}, \ldots)$$

$$\boldsymbol{X}(\boldsymbol{z}) = \sum_{i=0}^{\infty} z^i \boldsymbol{x_i}$$

We state the following theorem without proof :

Theorem 1:

The Markov chain given by P is ergodic iff $\pi\beta^* < 1$ where π is the stationary probability vector of the transition matrix and $\beta^* = \sum_{i=1}^{\infty} iA_i$

Clearly, $\pi\beta^* = 1 + \bar{a} - \bar{e}(\boldsymbol{\pi_1}\mathbf{1})$ where $\boldsymbol{\pi_1}$ is the vector comprising of the first m components of $\boldsymbol{\pi}$

This means that $\bar{e}(\boldsymbol{\pi_1}\mathbf{1})$ is the maximum achievable throughput per time slot.

COMPUTATION OF x_0:

For the Markov renewal process(every time **0** is visited) he transition function is given by the generating function

$K(z) = \sum_{i=0}^{\infty} zB_i[G(z)]^i$ where $[G(z)]^i = G_i(z)$ which is the generating function for the first passage time from state **i** to state **0.**

From Markov renewal theory, we can write $\boldsymbol{x_0} = (\kappa\kappa^*)^{-\mathbf{1}}\,\kappa$ where κ is the stationary probability vector with transition matrix K(1) and $\kappa^* = K'(1)\mathbf{1}$

The matrix $G(z) = G_1(z)$ clearly satisfies $G(z) = \sum_{i=0}^{\infty} zA_i[G(z)]^i$

Successive substitutions in this equation, starting with the zero matrix gives a sequence of matrices monotonically increasing to G = G(1). Thus we can compute G, and then from G evaluate the matrix K(1) and the vector κ .

COMPUTATION OF x_i:

The following theorem is stated without proof:

Theorem 2:

$$x_i = \left[x_0\bar{B}_i + \sum_{j=1}^{i-1} x_j\bar{A}_{i+j-1}\right](I - \overline{A_1})^{-1}$$

Where $\bar{B}_i = \sum_{j=i}^{\infty} B_jG^{j-i}$ and $\bar{A}_i = \sum_{j=i}^{\infty} A_jG^{j-i}$

Clearly, the matrices $\bar{B}_i$ *and* $\bar{A}_i \to 0$ as $i \to \infty$. Therefore, we can choose a sufficiently large index i, set $\bar{B}_i = \bar{A}_i = 0$ and compute the others using the backward recursions $\bar{B}_k = B_k + \bar{B}_{k+1}G$ and $\bar{A}_k = A_k + \bar{A}_{k+1}G$ and use these to evaluate the vectors $x_i, i \geq 1$

To confirm the adequacy of the truncation index, we can use $X(1) = \sum_{i=0}^{\infty} x_i = \pi$.

CONCLUSION:

Once these stationary probabilities are computed, we can calculate the joint stationary distributions of the system size and the state of the medium at special times such as the beginning of an operating period, the end of an operating period etc. These provide useful information about how the queue builds up during inoperative periods and gets depleted during operating periods.