1. Griffiths: 3.12, 3.14, 3.15
2. Jackson: 2.13, 2.14, 2.17, 2.20, 2.23

Some Answers

Griffiths (3.12)

$$\Phi(x,y) = \frac{8V_0}{\pi}\sum_{j=0}^{\infty}\frac{1}{4j+2}\exp\left(-\frac{(4j+2)\pi x}{a}\right)\sin\left(\frac{(4j+2)\pi y}{a}\right) \tag{1}$$

Griffiths (3.14)

$$\Phi(x,y) = \frac{4V_0}{\pi}\sum_{\mathrm{odd}n}^{\infty}\frac{\sinh(n\pi x/a)\sin(n\pi y/a)}{\sinh(n\pi b/a)} \tag{2}$$

Griffiths (3.15)

$$\Phi(x,y) = \frac{16V_0}{\pi^2}\sum_{\mathrm{odd}\ m}\sum_{\mathrm{odd}\ n}\frac{\sinh\left(\sqrt{m^2+n^2}\pi z/a\right)\sin(m\pi x/a)\sin(n\pi y/a)}{\sinh\left(\sqrt{m^2+n^2}\pi\right)} \tag{3}$$

Jackson (2.13) (Unfortunately I chose wrong geometry. Change $\phi \to \phi - \pi/2$ to get Jackson's answer). For $\rho < b$, the general solution is

$$\Phi(\rho,\phi) = a_0 + \sum_{n=1}^{\infty}\rho^n(a_n\sin(n\phi) + b_n\cos(n\phi)) \tag{4}$$

Boundary conditions:

$$\begin{aligned}\Phi(b,\phi) &= V_1 \qquad 0 \le \phi \le \pi/2 &(5)\\ &= V_2 \qquad \pi/2 \le \phi \le \pi &(6)\end{aligned}$$

Then, $a_0 = (V_1+V_2)/2$, $a_n = 0$ for all even $n > 0$, $a_n = 2(V_1 - V_2)/(\pi n b^n)$ for all odd $n > 0$. $b_n = 0$ for all n. Thus,

$$\begin{aligned}\Phi(\rho,\phi) &= (V_1+V_2)/2 + \frac{2(V_1-V_2)}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{\rho}{b}\right)^{2n+1}\sin((2n+1)\phi) &(7)\\ &= (V_1+V_2)/2 + \frac{2(V_1-V_2)}{\pi}\,\mathrm{Im}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{\rho}{b}e^{i\phi}\right)^{2n+1} &(8)\\ &= (V_1+V_2)/2 + \frac{(V_1-V_2)}{\pi}\,\mathrm{Im}\ \ln\left(\frac{1+\frac{\rho}{b}e^{i\phi}}{1-\frac{\rho}{b}e^{i\phi}}\right) &(9)\\ &= (V_1+V_2)/2 + \frac{(V_1-V_2)}{\pi}\tan^{-1}\left(\frac{2\rho b\sin\phi}{b^2-\rho^2}\right) &(10)\end{aligned}$$

And the charge density is

$$\sigma(\phi) = -\epsilon_0\frac{(V_1-V_2)}{\pi b\cos\phi} \tag{11}$$

Jackson (2.14)

Jackson (2.17) Use following identities to prove the result:

$$\delta(\mathbf{r} - \mathbf{r}') = \frac{1}{\rho}\delta(r - r')\delta(\phi - \phi') \tag{12}$$

$$\delta(\phi - \phi') = \frac{1}{2\pi}\sum_{-\infty}^{\infty} e^{im(\phi-\phi')} \tag{13}$$

To find g_m when $m = 0$, choose

$$g_0(\rho, \rho') = A \qquad \rho < \rho' \tag{14}$$

$$= B \ln \rho \qquad \rho' < \rho \tag{15}$$

But then g_0 does not vanish as $\rho \to \infty$. When can you use this Green's Function?

Jackson (2.20) Use the Green's Function derived above.