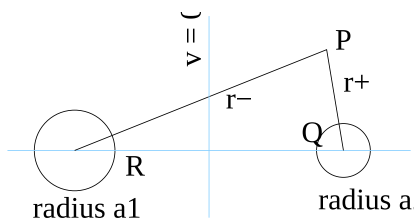
1. Jackson: Problems 1.6, 1.7, 1.8, 1.10, 1.12, 1.13
2. Jackson: Problems 2.2, 2.3, 2.4, 2.7, 2.11

Some Answers

Jackson (1.6) Capacitance in each case: (a) $\epsilon_0 A/d$ (b) $4\pi\epsilon_0 ba/(b-a)$ (c) $2\pi\epsilon_0/\ln(b/a)$ (per unit length)

Jackson (1.7) Here we assume that the conducting wires are far apart, hence the charge is uniformly distributed over the surface of each wire.



Hence the potential is given by

$$V(\mathbf{P}) = \frac{\lambda}{2\pi\epsilon_0}\log\left(\frac{r_+}{r_-}\right) \tag{1}$$

Thus potential of the two wires are $V_1(R) = \frac{\lambda}{2\pi\epsilon_0}\log\left(\frac{a_1}{d-a_1}\right)$ and $V_2(Q) = \frac{\lambda}{2\pi\epsilon_0}\log\left(\frac{d-a_2}{a_2}\right)$. capacitance is

$$\frac{\lambda}{|v_1 - v_2|} = \frac{2\pi\epsilon_0}{|\ln(a_1 a_2/(d-a_1)(d-a_2))|} \approx \frac{\pi\epsilon_0}{|\ln(\sqrt{a_1 a_2}/d)|} \tag{2}$$

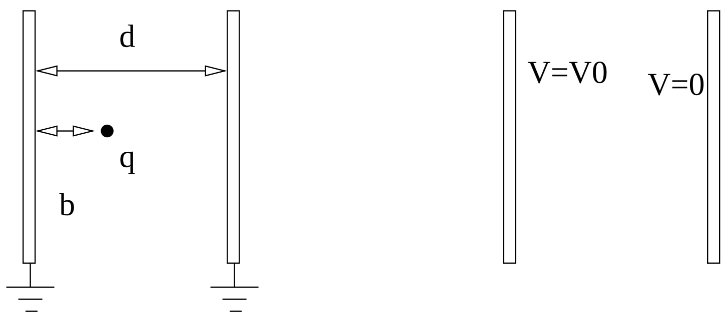
Jackson (1.8) In all cases the answer is $\frac{1}{2}QV$.

Jackson (1.10) By integral formula (Eq 1.36 of Jackson) for potentials (Given that $\rho(\mathbf{x}') = 0$ for $\mathbf{x}' \in V$)

$$\phi(\mathbf{x}) = \frac{1}{4\pi}\oint\left(\frac{\partial\phi}{\partial n'}\right)\frac{dS'}{|\mathbf{x}-\mathbf{x}'|} - \frac{1}{4\pi}\oint\phi(\mathbf{x}')\frac{\partial}{\partial n'}\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)dS' \tag{3}$$

Since $|\mathbf{x}-\mathbf{x}'| = R$ and $\frac{\partial}{\partial n'}\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right) = -1/R^2$ for $\mathbf{x}' \in S$, the first integral vanishes by Gauss law and second integral is simply the average value of the potential on the surface of the sphere.

Jackson (1.12) Hint: Use Green's second identity (Eq 1.35 of Jackson). Instead of ψ choose ϕ'. Remember that $\rho = -\epsilon_0\nabla^2\phi$ and $\sigma = +\epsilon_0\nabla\phi\cdot\hat{\mathbf{n}}'$ and simillarly for ϕ'.



Situation a　　　Situation b

Jackson (1.13) See Figure In situation a, we know $\rho(\mathbf{x}) = q\delta(\mathbf{x} - b\hat{\mathbf{i}})$ for $\mathbf{x} \in V$ and $\phi(\mathbf{x}) = 0$ for $\mathbf{x} \in S$. We dont know σ on S and ϕ in V.
In situation b, consider a parallel plate capacitor. Here, we know $\rho'(\mathbf{x}) = 0$ for $\mathbf{x} \in V$, $\phi'(\mathbf{x}) = 0$ for $\mathbf{x} \in S_1$ and $\phi'(\mathbf{x}) = V_0$ for $\mathbf{x} \in S_2$. We also know that $\phi'(\mathbf{x}) = V_0(1 - x/d)$, where $x = \mathbf{x} \cdot \mathbf{i}$. Using the result of the previous problem, we get

$$\int_{S_2} \sigma V_0 ds = -\int_V \delta(\mathbf{x} - b\mathbf{i})\phi'(\mathbf{x}) dv = -q(1 - b/d)$$

Jackson (2.2)

1. Given that the a charge $q$is kept inside a hollow conducting sphere (radius a) at a distance y from the center (along z axis, say), the image charge of magnitude $-qa/y$ must be kept outside at a distance a^2/y. Potential is given by

$$\phi(\mathbf{x}) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{|\mathbf{x} - y\mathbf{k}|} - \frac{(a/y)}{|\mathbf{x} - (a^2/y)\mathbf{k}|}\right]$$

2. Induced surface charge density

$$\sigma(\theta) = \frac{q}{4\pi a}\frac{(a^2 - y^2)}{(a^2 + y^2 - 2ay\cos\theta)^{3/2}}$$

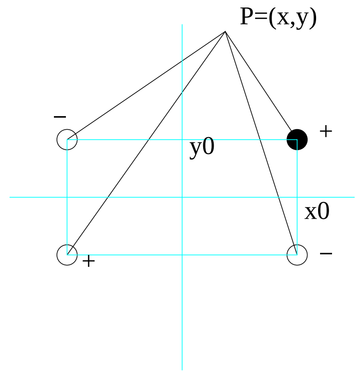
3. Force on q

$$\mathbf{F}_q = \frac{q^2 ay}{4\pi\epsilon_0(a^2 - y^2)^2}\mathbf{k}$$

4. All you need to do is to add a constant V_0 to the result of part (a). The induced charge densities are same!

Jackson (2.3)

(a) Place image charges (wires, in this context) as shown in the figure.



The potential at point P is given by

$$\phi(P) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{r_3 r_4}{r_1 r_2}\right)$$

where r_1 and r_2 are distances from $+\lambda$ wires and r_3 and r_4 from $-\lambda$ wires. Equipotential surfaces are shown in the figure.

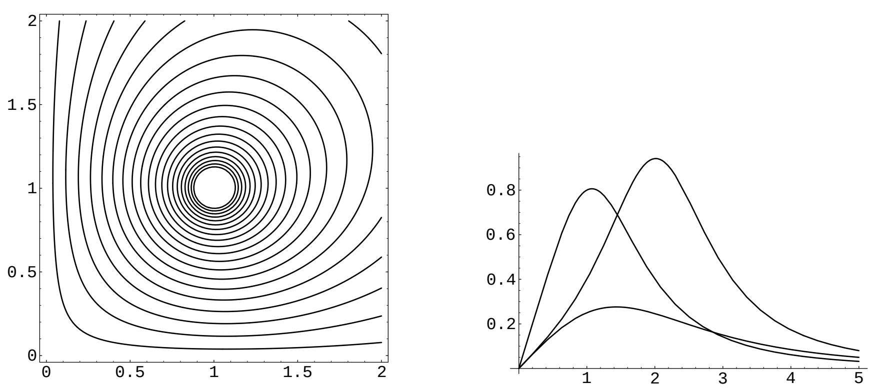


Figure 1: Jackson 2.3 (a) Equipotential surfaces.(b) Charge density along x axis

(b) The required plots are shown in the figure.

$$\sigma(x > 0, y = 0) = \frac{\lambda y_0}{\pi}\left[\frac{1}{(x+x_0)^2 + y_0^2} - \frac{1}{(x-x_0)^2 + y_0^2}\right]$$

(c) Integrate σ.

Jackson (2.4) (a) The force on charge q is given by

$$\mathbf{F}_q = \frac{q^2 \hat{\mathbf{k}}}{4\pi\epsilon_0}\left[\frac{d+R}{d^2} - \frac{dR}{(d^2 - R^2)^2}\right] \tag{4}$$

Equating magnitude of the force to zero, we get three real solutions for $d/R = -0.61803, 0.7549, 1.61803$. Only one solution is outside the sphere.

Jackson (2.7) The required Green's Function is

$$G(\mathbf{x}, \mathbf{x}') = \frac{1}{|\mathbf{x} - \mathbf{x}'|} - \frac{1}{|\mathbf{x} - \mathbf{x}' + 2(\mathbf{x}' \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}|} \tag{5}$$

In cylindrical coordinates,

$$\frac{\partial G}{\partial n'} = -\frac{2z}{(r'^2 + r^2 + z^2 - 2rr' \cos(\phi - \phi'))^{3/2}} \tag{6}$$

Then, the potential at any point ($z > 0$) is given by

$$\phi(\mathbf{x}) = \frac{zV}{2\pi} \int_0^{2\pi} d\phi' \int_0^R r' dr' \frac{1}{(r'^2 + r^2 + z^2 - 2rr' \cos(\phi - \phi'))^{3/2}} \tag{7}$$