Here are some practice problems in vector calculus and curvilinear coordinates.

1. Find equations for the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point $(2, -1, 5)$.

2. Find the unit outward normal to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at the point $(3, 1, -4)$.

3. Find the divergence and curl of $\hat{\mathbf{r}}/r^2$.

4. Prove:

   (a) $\nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot \mathbf{A} + \phi(\nabla \cdot \mathbf{A})$.

   (b) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$.

5. Show that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. Find the scalar potential. Find the work done in moving an abject in this field from $(1, -2, 1)$ to $(3, 1, 4)$.

6. Given $\phi = 2xyz^2$and a curve $C(t) = (t^2, 2t, t^3)$ from $t = 0$ to $t = 1$. Find $\int_C \phi d\mathbf{r}$.

7. Evaluate $\int_S \mathbf{A} \cdot d\mathbf{S}$ where $\mathbf{A} = 18z\mathbf{i} - 12\mathbf{j} + 3y\mathbf{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant.

8. Prove Green's theorem in a plane. (See any textbook)

9. For a given $R > 0$, define $r_a = \left(x^2 + y^2 + (z - R/2)^2\right)^{1/2}$ and $r_b = \left(x^2 + y^2 + (z + R/2)^2\right)^{1/2}$. The prolate ellipsoidal coordinates are defined as

$$\begin{aligned} \xi &= \frac{1}{R}(r_a + r_b) \\ \eta &= \frac{1}{R}(r_a - r_b) \\ \phi &= \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}$$

   Find inverse transformations, basis vectors e_ξ, e_η, e_ϕ , scale factors h_ξ, h_η, h_ϕ , differential vector $d\mathbf{r}$, length element ds^2 and volume element dv.

10. Let

$$\delta_n(x) = \begin{cases} 0 & x < -\frac{1}{2n} \\ n & -\frac{1}{2n} < x < \frac{1}{2n} \\ 0 & \frac{1}{2n} < x \end{cases}$$

   Show that

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} f(x)\delta_n(x)\, dx = f(0)$$

   assuming that the function f is continuous at $x = 0$.