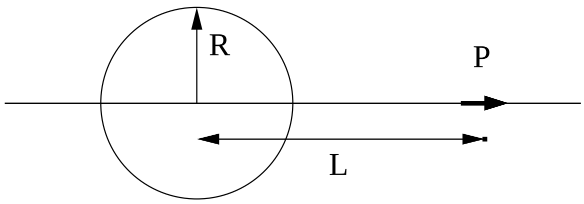
1. Surface of a sphere, of radius R, carries a charge density $\sigma(\theta, \phi) = P\cos\theta$ where P is a constant.
   (a) Using **direct integration**, find the potential at a distance d along the z-axis. [6 Marks]
   (b) Using the result of (a), argue that the potential ϕ at any point (r, θ, ϕ) is given by
   $$\phi(r,\theta,\phi) = \begin{cases} \frac{P}{3\epsilon_0} r\cos\theta & r < R \\ \frac{PR^3}{3\epsilon_0 r^2}\cos\theta & r > R \end{cases}$$
   [2 Marks]
   (c) Using the result of part (b) find the electric field $\mathbf{E}(r, \theta, \phi)$, both inside and outside the sphere. [3 Marks]
   (d) Using the result of part (b) find the total electrostatic energy, W, of this charge distribution. [Hint: You may want to use $W = \frac{1}{2}\int \sigma\phi d\tau$.] [4 Marks]

2. A dipole of strength P is kept at a distance L from the center of a grounded, conducting sphere of radius R, as shown in the figure. To find the image charge configuration, follow these steps:



   (a) The dipole can be approximated by a couple of charges q and $-q$ kept at distances $L+l$ and $L-l$, respectively, such that $2ql = P$. Find the image charges q_1 and q_2 (of q and $-q$, respectively) and their distances d_1 and d_2.
   (b) Find the total image charge $Q' = q_1 + q_2$ and the dipole moment of the image charges $P' = q_1 d_1 + q_2 d_2$.
   (c) Now, find $\lim Q'$ and $\lim P'$ as $q \to \infty$ and $l \to 0$ such that $2ql = P$. Thus, show that the image charges of a dipole consists of a monopole and a dipole. [5 Marks]

3. A square pipe, running parallel to the z-axis (from $-\infty$ to ∞), has three grounded metal sides, at $y = 0$, $y = a$ and $x = 0$. The fourth side, at $x = a$, is maintained at a constant potential V_0.
   (a) Find the potential inside the pipe.
   (b) Find the net charge per unit length (along z-axis) on the side *opposite* to V_0. [10 Marks]

Useful Formulae

$$\int \frac{tdt}{\sqrt{1-at}} = -\frac{2(2+at)\sqrt{1-at}}{3a^2}$$

$$\sum_{m=1}^{\infty} \frac{1}{m\sinh(m\pi)} = \frac{\ln(2)}{8}$$