Assume that the primed frame S' moves with respect to unprimed frame S with speed v along common x-x' axis etc.

1. Show that $x_0^2 - (x_1^2 + x_2^2 + x_3^2)$ is invariant under Lorentz transformation (Boost).

2. Show that, with $u'^2 = {u'_x}^2 + {u'_y}^2 + {u'_z}^2$ and $u^2 = u_x^2 + u_y^2 + u_z^2$, we can write

$$c^2 - u^2 = \frac{c^2(c^2 - u'^2)(c^2 - v^2)}{(c^2 + u'_x v)^2}$$

   From this result show that if $u' < c$ and $v < c$, then u must be less than c.

3. Show that the $[u_0, u_1, u_2, u_3]$ where $u_i = \frac{1}{\sqrt{1-u^2/c^2}} dx_i/dt$ is a four vector. (Hint: Use the result of the previous problem)

4. Show that the operator $\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$ is invariant under Lorentz transformation.

5. Suppose that α_x, α_y, α_z and β are *mutually anti-commuting* square matrices of order n, satisfying

$$\alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 = 1.$$

   (a) α and β are traceless.
   (b) Eigenvalues of α and β are ± 1.
   (c) n must be even.

6. Show that every unimodular, hermitian, traceless matrix of order 2 can be written as a linear combination of Pauli matrices. Hence, show that there is no unimodular, hermitian, traceless matrix of order 2 which anticummutes with Pauli matrices.

7. Show that

$$(\sigma \cdot \mathbf{A})(\sigma \cdot \mathbf{A}) = \mathbf{A} \cdot \mathbf{B} + i\sigma \cdot (\mathbf{A} \times \mathbf{B}) \quad (1)$$
$$(\alpha \cdot \mathbf{A})(\alpha \cdot \mathbf{A}) = \mathbf{A} \cdot \mathbf{B} + i\mathbf{\Sigma} \cdot (\mathbf{A} \times \mathbf{B}) \quad (2)$$

   where $\mathbf{A}$ and $\mathbf{B}$ are vectors.

8. Show that the Klein-Gordon equation has plane wave solutions

$$\psi(\mathbf{r}, t) = \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

   where

$$(\hbar\omega)^2 = (\hbar ck)^2 + m^2c^4$$

9. Find the first-order correction to the time independent Schrödinger equation from Klein-Gordon equation.

10. Show that the four free spin-1/2 particle eigenstates corresponding a certain energy are orthogonal to each other.