1. Verify, by direct substitution, that $G_{\pm} = e^{\pm ikr}/r$ are solutions of
$$\left(\nabla^2 + k^2\right) G(\mathbf{r}) = -4\pi\delta(\mathbf{r}).$$

2. Show that
$$k|\vec{r} - \vec{r'}| = kr - k(\hat{r}\cdot\vec{r'}) + \frac{k(\hat{r}\times\vec{r'})^2}{2r} + \cdots.$$

3. Show that the gaussian wave packet moves without appreciable change in the width over time t if $t << 2m/\hbar(\Delta k)^2$.

4. Apply the Born approximation to obtain differential cross section for the following potentials:

   (a) The square well potential
$$\begin{aligned} V(r) &= -V_0 \quad \text{for} \quad r < a &(1)\\ &= 0 \quad \text{for} \quad r > a &(2)\end{aligned}$$

   (b) The Gaussian Potential
$$V(r) = -V_0 \exp\left[-\frac{1}{2}\left(\frac{r}{a}\right)^2\right]$$

   (c) The Exponential Potential
$$V(r) = -V_0 \exp\left(-\frac{r}{a}\right)$$

   Plot the differential cross section in each case.

5. The scattering of fast electrons by a complex atom can be, in many cases, represented fairly accurately by the following form for the potential energy distribution:
$$V = -\frac{Ze^2}{r} + Ze^2\int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r'$$
   For the hydrogen atom in ground state, we may write
$$\rho(r) = |\psi_{1s}|^2$$
   Calculate differential cross section.

6. For square well potential, find the phase shift and partial differential cross section for p-wave. Find low energy limits for phase shift.

7. Expression for scattering amplitude for square well potential was obtained in Born approximation in problem 4(a) of tutorial 5. From low energy analysis, find the scattering amplitude of s- and p-waves.

8. Consider a repulsive potential given by
$$\begin{aligned} V(r) &= V_0 \quad \text{for} \quad 0 < r < a &(3)\\ &= 0 \quad \text{for} \quad r > a. &(4)\end{aligned}$$
   Find phase shifts for s-wave for $E < V_0$. Repeat for $E > V_0$.

9. show that total cross section is related to the scattering amplitude by
$$\sigma = \frac{4\pi}{k}\,\mathrm{Im}\, f_{\mathbf{k}}(0)$$

10. Using the first three partial waves, compute and display on a polar graph the differential cross section for a hard sphere when the de Broglie wavelength of the incident particle equals the circumference of the sphere. Evaluate the total cross section and estimate accuracy of the result.

11. Analysis of the scattering of particles of mass m and energy E from a fixed scattering center with characteristic length a finds the phase shifts are given by

$$\sin \delta_l = \frac{(iak)^l}{\sqrt{(2l+1)\, l!}}$$

Derive a closed expression for the total cross section as a function of incident energy E. At what values of E does S-wave scattering give good estimate of σ?