1. Exchange operator is defined as

$$(P_{12}f)(x_1, x_2) = f(x_2, x_1)$$

Prove that the exchange operator P_{12} is hermitian

2. Show that following two-spin states are eigenstates of $\mathbf{S^2}$, $\mathbf{S_z}$ and P_{12} operators, where $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$:
(a) $\alpha(1)\alpha(2)$ (b) $\beta(1)\beta(2)$
(c) $(\alpha(1)\beta(2) - \beta(1)\alpha(2))/\sqrt{2}$ (d) $(\alpha(1)\beta(2) + \beta(1)\alpha(2))/\sqrt{2}$

3. Consider two spin-1 particles. Let $\alpha = |1, 1\rangle$, $\beta = |1, 0\rangle$ and $\delta = |1, -1\rangle$. Find the eigenstates of $\mathbf{S^2}$, $\mathbf{S_z}$ and P_{12} operators, where $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$:

4. Two non-interacting identical particles are enclosed in a 1D box of length L. What is the energy of the ground state and the first excited state? Find the degeneracies of these *levels* if particles were

   (a) spin 0 bosons;
   (b) spin 1/2 fermions;
   (c) spin 1 bosons.

5. Two electrons occupy states ϕ_a and ϕ_b. Show that the slater determinant

$$\begin{vmatrix} \phi_a(1)\alpha(1) & \phi_a(2)\alpha(2) \\ \phi_b(1)\beta(1) & \phi_b(2)\beta(2) \end{vmatrix} \tag{1}$$

is not an eigenstate of $\mathbf{S}^2$ operator. Find the eigenstates of $\mathbf{S}^2$ operator by taking linear combinations of slater determinants.

6. Use variational function

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{Z^3}{\pi} \exp\left(-Z(r_1 + r_2)\right) \tag{2}$$

with Z as variational parameter to estimate the ground state energy of the Helium atom.