1. By choosing an appropriate trial function, find the energy of the first excited state of harmonic oscillator.

2. For the Schrödinger equation with a potential $V(x) = g|x|$ $(g > 0)$, use an exponential trial function to estimate the ground state energy. Compare with the estimate from the gaussian trial function.

3. Use the variational principle to estimate the ground state energy of Hydrogen atom using a trial function $\exp(-\gamma r)$.

4. Use the variational principle to estimate the ground state energy for the anharmonic oscillator
$$H = \frac{p^2}{2m} + \lambda x^4.$$
Compare with the exact result
$$E_0 = 1.060\lambda^{1/3}\left(\frac{\hbar^2}{2m}\right)^{2/3}$$
Use a gaussian trial function.

5. Use the variational principle to show that a one-dimensional attractive potential will always have a bound state.

6. Using a gaussian trial function, $e^{-\lambda x^2}$ for a potential well represented by
$$H = \frac{p^2}{2m} - V_0 e^{-\alpha x^2}$$
where V_0 and $\alpha > 0$, find ground state energy.

7. Find the ground state energy of double oscillator described by potential
$$V(x) = \frac{1}{2}m\omega^2(|x| - a)^2$$
(*Hint*: See section 8.5 in Quantum Mechanics by Merzbacher.)

8. **Atomic Units**: The mass of electron (m_e), the charge of electron (e), the Bohr radius (a_0) and Planck's constant $(\hbar)$ are set to 1 in the scheme of atomic units. Show that the 1 au of time is 2.42×10^{-17} s and that the speed of light is 137.036 a.u.

9. **Prolate Ellipsoidal Coordinates**: Show that the prolate ellipsoidal coordinate system is orthogonal and find the volume element.

10. In ionized Hydrogen molecule (H_2^+) calculation (refer to class notes), show that
$$\nabla^2\phi_a = \left(\gamma^2 - \frac{2\gamma}{r_a}\right)\phi_a$$
where $\phi_a = (\gamma^3/\pi)^{1/2}e^{-\gamma r_a}$.

11. In ionized Hydrogen molecule (H_2^+) calculation (refer to class notes), evaluate
$$C = \frac{\gamma^3}{\pi}\int d\tau \frac{e^{-2\gamma r_a}}{r_b}$$
and
$$D = \frac{\gamma^3}{\pi}\int d\tau \frac{e^{-\gamma(r_a + r_b)}}{r_b}$$