1. **Calculate the probability of excitation to the 2p state of a hydrogen atom, originally in its ground state, due to a homogeneous electric field with time dependence**

$$E = \frac{E_0}{\pi} \frac{\tau}{t^2 + \tau^2}.$$

Discuss the limits of large and small value of τ and their significance.

Suppose that the electric field points in the z direction. Then $V(t) = eE_0 z \frac{1}{t^2+\tau^2}$. The transition amplitude is given by

$$\begin{aligned} c_{2p} &= (i\hbar)^{-1} \int_0^\infty \langle 2p |V| 1s\rangle e^{i\omega t'} dt' \\ &= (i\hbar)^{-1} eE_0 \langle 2p |z| 1s\rangle \int_{-\infty}^{\infty} \frac{1}{t'^2 + \tau^2} e^{i\omega t'} dt' \\ &= \frac{qE_0\pi}{i\hbar\tau} \langle 2p |z| 1s\rangle e^{-\omega\tau} \end{aligned}$$

(Using method of residues, $\int_{-\infty}^{\infty} \frac{1}{t^2+\tau^2} e^{i\omega t} dt = 2\pi i \lim'_{z\to i\tau} \frac{1}{z+i\tau} e^{i\omega z} = \frac{\pi}{\tau} e^{-\omega\tau}$)

$|1s\rangle = \frac{2}{\sqrt{4\pi}} a^{-3/2} \exp(-r/a)$

$|2p\rangle = \frac{1}{\sqrt{4\pi}} (2a)^{-3/2} \left(\frac{r}{a}\right) \exp(-r/2a) \cos\theta$

Then

$$\langle 2p |z| 1s\rangle = \frac{128}{243} \sqrt{2} a$$

2. **Two level System:** Solve

$$\begin{aligned} i\hbar \dot{C}_a(t) &= V_{aa} C_a(t) + V_{ab} e^{i\omega_{ab} t} C_b(t) \quad (1) \\ i\hbar \dot{C}_b(t) &= V_{ba} e^{-i\omega_{ab} t} C_a(t) + V_{bb} C_b(t) \quad (2) \end{aligned}$$

3. **Consider a particle of charge q and mass m, in SHM along x-axis. A homogeneous electric field $E(t) = E_0 \exp(-t/\tau)$ directed along x-axis is switched on at $t = 0$. If the particle was in the ground state before $t = 0$, find the probability that it will be found in an excited state as $t \to \infty$.**

Probability that the particle will jump to state $|m\rangle$ is given by $|c_m|^2$, where

$$\begin{aligned} c_m &= (i\hbar)^{-1} \int_0^\infty \langle m |V| 0\rangle e^{i\omega_{m0} t'} dt' \\ &= -(i\hbar)^{-1} qE_0 \langle m |x| 0\rangle \int_0^\infty e^{-t'/\tau} e^{i\omega_{m0} t'} dt' \\ &= -\frac{qE_0\tau}{\hbar} \frac{1}{i + \tau\omega_{m0}} \langle m |x| 0\rangle \\ &= -\frac{qE_0\tau}{\hbar} \frac{1}{i + \tau\omega_{m0}} \sqrt{\frac{\hbar}{2m\omega}} \delta_{m,1} \end{aligned}$$

Since $x = \sqrt{\frac{\hbar}{2m\omega}} (a + a^*)$, it is easy to see that $\langle m |x| 0\rangle = \sqrt{\frac{\hbar}{2m\omega}} \delta_{m,1}$. Hence

$$P_{1\leftarrow 0} = \frac{1}{2} \frac{q^2 E_0^2}{\hbar m\omega} \frac{\tau^2}{(1 + \tau^2\omega^2)}$$

- Large τ implies that a near constant field is applied, hence the final state is mixture of ground state and first excited state (same result as the one you would get from time independent perturbation theory

4. **Consider a particle of charge q and mass m, in SHM along x-axis. A homogeneous electric field $E(t) = E_0 \exp\left(-(t/\tau)^2\right)$ directed along x-axis is applied. If the particle was in the ground state at $t = -\infty$, find the probability that it will be found in an excited state as $t \to \infty$.**

   This problem is same as the previous one, except for the time dependence of electric field and initial conditions. Probability that the particle will jump to state $|m\rangle$ is given by $|c_m|^2$, where

$$\begin{aligned} c_m &= (i\hbar)^{-1} \int_0^{\infty} \langle m | V | 0 \rangle \, e^{i\omega_{m0} t'} dt' \\ &= -(i\hbar)^{-1} qE_0 \langle m | x | 0 \rangle \int_0^{\infty} e^{-(t'/\tau)^2} e^{i\omega_{m0} t'} dt' \\ &= -\frac{qE_0\tau}{\hbar} \sqrt{\pi} \langle m | x | 0 \rangle \, e^{-\omega_{m0}^2 \tau^2/4} \\ &= -\frac{qE_0\tau}{\hbar} \sqrt{\frac{\pi\hbar}{2m\omega}} e^{-\omega_{m0}^2 \tau^2/4} \delta_{m,1} \end{aligned}$$

   Hence

$$P_{1\leftarrow 0} = \frac{1}{2} \frac{\pi q^2 E_0^2}{\hbar m \omega} \tau^2 e^{-\omega^2 \tau^2/2}$$

5. **Show that the number of modes per unit volume per unit frequency range for electromagnetic radiation confined to a cubical box is given by $\omega^2/\pi^2 c^3$.**

   In a cubical box the emwave must have a solution given by

$$\sin k_x x \sin k_y y \sin k_z z \exp(i\omega t)$$

   And then, using boundary conditions, we can show that $k_x = \pi n_x / L$ etc. and $k_x^2 + k_y^2 + k_z^2 = \omega^2/c^2$. For given ω, total number of solutions is given by number of k-points on the surface of sphere of radius ω/c in the first octant of k-space. The volume of a shell of width dk is given by $4\pi k^2 dk$. There is one k-point in volume $\left(\frac{\pi}{L}\right)^3$. Hence number of solutions is given by $4\pi k^2 dk / \left(\frac{\pi}{L}\right)^3 / 8 = \frac{1}{2\pi^2} k^2 dk L^3$. Since $\omega = ck$ and that there are two polarizations for each solutions, number of solutions per unit volume is $\frac{\omega^2}{\pi^2 c^3} d\omega$.

6. **Calculate how may photons per second are radiated from a monochromatic source, 1 watt in power, for the following wavelengths (a) 10 m (radio wave) (b) 10 cm (microwave) (c) 5890 A (optical waves) (d) 1 A (x-rays). At a distance of 10 m from the source, calculate the number of photons passing through unit area, normal to the direction of propagation, per unit time and the density of photons, in each case.**

   (a) If $\lambda = 10$ m, $\nu = \frac{c}{\lambda} = 3 \times 10^7$Hz. The energy per photon is $h\nu = 6.6261 \times 10^{-34} \times 3 \times 10^7 = 1.9878 \times 10^{-26}$J. Since the source radiates 1 J per second, number of photons emitted per second is 5.0307×10^{25}. At 10 m from the source these photons cross a surface area 400π. Number of photons crossing surface per unit area $5.0307 \times 10^{25} / (400\pi) = 4 \times 10^{22}$. The density is given by $4 \times 10^{22} / (3 \times 10^8) = 1.3333 \times 10^{14}$

7. Show that, with the gauge condition $\nabla \cdot \mathbf{A} = 0$, $\mathbf{p}$ commutes with $\mathbf{A}$ and hence $\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p} = 2\mathbf{A} \cdot \mathbf{p}$.

8. Generalise Einstein's results in case the two levels E_a and E_b are degenerate with degeneracies g_a and g_b respectively.

9. State and prove the Thomas-Reiche-Kuhn sum rule for oscillator strengths.

10. Calculate the Einstein's coefficient A for the 2p - 1s transition in a hydrogenic atom, and find the half life of the 2p level.