1. Verify, by direct substitution, that $G_{\pm} = e^{\pm ikr}/r$ are solutions of

$$\left(\nabla^2 + k^2\right) G(\mathbf{r}) = -4\pi\delta(\mathbf{r}).$$

2. Show that

$$k|\vec{r} - \vec{r'}| = kr - k(\hat{r} \cdot \vec{r'}) + \frac{k(\hat{r} \times \vec{r'})^2}{2r} + \cdots.$$

3. Show that the gaussian wave packet moves without appreciable change in the width over time t if $t << 2m/\hbar(\Delta k)^2$.

4. Apply the Born approximation to obtain differential cross section for the following potentials:

   (a) The square well potential

$$\begin{aligned} V(r) &= -V_0 \quad \text{for} \quad r < a & (1) \\ &= 0 \quad \text{for} \quad r > a & (2) \end{aligned}$$

   (b) The Gaussian Potential

$$V(r) = -V_0 \exp\left[-\frac{1}{2}\left(\frac{r}{a}\right)^2\right]$$

   (c) The Exponential Potential

$$V(r) = -V_0 \exp\left(-\frac{r}{a}\right)$$

   Plot the differential cross section in each case.

5. The scattering of fast electrons by a complex atom can be, in many cases, represented fairly accurately by the following form for the potential energy distribution:

$$V = -\frac{Ze^2}{r} + Ze^2 \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r'$$

   For the hydrogen atom in ground state, we may write

$$\rho(r) = |\psi_{1s}|^2$$

   Calculate differential cross section.