1. **Atomic Units**: The mass of electron (m_e), the charge of electron (e), the Bohr radius (a_0) and Planck's constant ($\hbar$) are set to 1 in the scheme of atomic units. Show that the 1 au of time is 2.42×10^{-17} s and that the speed of light is 137.036 a.u.

2. **Prolate Ellipsoidal Coordinates**: Show that the prolate ellipsoidal coordinate system is orthogonal and find the volume element.

3. In ionized Hydrogen molecule (H_2^+) calculation (refer to class notes), show that

$$\nabla^2 \phi_a = \left(\gamma^2 - \frac{2\gamma}{r_a}\right) \phi_a$$

where $\phi_a = (\gamma^3/\pi)^{1/2} e^{-\gamma r_a}$.

4. In ionized Hydrogen molecule (H_2^+) calculation (refer to class notes), evaluate

$$C = \frac{\gamma^3}{\pi} \int d\tau \frac{e^{-2\gamma r_a}}{r_b}$$

and

$$D = \frac{\gamma^3}{\pi} \int d\tau \frac{e^{-\gamma(r_a + r_b)}}{r_b}$$

and

5. Show that following two-spin states are eigenstates of $\mathbf{S^2}$ operator, where $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$:
(a) $\alpha\alpha$ (b) $\beta\beta$ (c) $(\alpha\beta - \beta\alpha)/\sqrt{2}$ (d) $(\alpha\beta + \beta\alpha)/\sqrt{2}$

6. Consider the helium atom in the approximation in which electron-electron interaction is neglected. Write down all possible *2-electron product states* such that one electron is in (1s) state and the other in (2s). From these product states construct eigenstates of electron exchange operator.

7. Prove addition theorem for spherical harmonics. (See section 11.4 of Merzbacher)

8. Show that the first order correction due to electron-electron repulsion to the helium atom is $\frac{5}{8}Z$.

9. Evaluate J_{2s} and K_{2s} for the helium atom.