1. A rotator whose orientation is specified by the angular coordinates θ and ϕ performs a *hindered rotation* described by the Hamiltonian

$$H = A\mathbf{L}^2 + B\hbar^2 \cos 2\phi$$

with $A >> B$. Calculate the S, P and D energy levels of this system in the first order perturbation theory, and work out unperturbed energy eigenfunctions.

2. Find the shift in the ground state energy of a 3D harmonic oscillator due to relativistic correction to the kinetic energy.

3. If the general form of a spin-orbit coupling for a particle of mass m and spin $\mathbf{S}$ moving in a central potential $V(r)$ is

$$H_{SO} = \frac{1}{2m^2c^2}\mathbf{S}\cdot\mathbf{L}\frac{1}{r}\frac{dV(r)}{dr},$$

what is the effect of the coupling on the spectrum of 3D harmonic oscillator?

4. By choosing an appropriate trial function, find the energy of the first excited state of harmonic oscillator.

5. For the Schrödinger equation with a potential $V(x) = g|x|$ $(g > 0)$, use an exponential trial function to estimate the ground state energy. Compare with the estimate from the gaussian trial function.

6. Use the variational principle to estimate the ground state energy of Hydrogen atom using a trial function $\exp(-\gamma r)$.

7. Use the variational principle to estimate the ground state energy for the anharmonic oscillator

$$H = \frac{p^2}{2m} + \lambda x^4.$$

Compare with the exact result

$$E_0 = 1.060\lambda^{1/3}\left(\frac{\hbar^2}{2m}\right)^{2/3}$$

Use a gaussian trial function.

8. Use the variational principle to show that a one-dimensional attractive potential will always have a bound state.

9. Using a gaussian trial function, $e^{-\lambda x^2}$ for a potential well represented by

$$H = \frac{p^2}{2m} - V_0 e^{-\alpha x^2}$$

where V_0 and $\alpha > 0$.

10. Find the ground state energy of double oscillator described by potential

$$V(x) = \frac{1}{2}m\omega^2(|x| - a)^2$$

(*Hint*: See section 8.5 in Quantum Mechanics by Merzbacher.)