1. A modified infinite well potential is given by

$$V(x) = \begin{cases} \epsilon x & \text{if} \quad 0 \le x \le b \\ \infty & \text{otherwise} \end{cases} \tag{1}$$

Obtain approximate energy eigenvalues to the first order in ϵ. Also find the second order correction to the ground state energy.

2. The bottom of an infinite well is changed to have the shape

$$V(x) = \epsilon \sin \frac{\pi x}{b} \qquad 0 \le x \le b \tag{2}$$

Calculate the energy shifts for all the states to first order in ϵ.

3. A simple harmonic oscillator is perturbed by a constant force. Find the energy eigenvalues. Calculate second order perturbation correction to the energy values and compare with the exact answer.

4. A simple harmonic oscillator is perturbed by a potential $V(x) = \lambda x^4$. Show that the perturbation can be written as

$$V(x) = \lambda \left(\frac{\hbar}{2m\omega}\right)^2 \left(a^4 + a^2(4\hat{n} - 2) + (6\hat{n}^2 + 6\hat{n} + 3) + {a^\dagger}^2(4\hat{n} + 6) + {a^\dagger}^4\right) \tag{3}$$

where $\hat{n} = a^\dagger a$. Obtain the first order correction to the energies. Discuss validity of this approximation for states with large $\hat{n}$ eigenvalues.

5. The Hamiltonian of a rigid rotator in magnetic field is of the form $A\mathbf{L}^2 + BL_z + CL_y$, if quadratic terms are neglected. Obtain exact eigenvalues and eigenfunctions of this Hamiltonian. Assuming $B \gg C$, use second order perturbation theory to obtain approximate values of energy and compare with the exact answers.

6. A charged particle is constrained to move on a spherical shell in a weak uniform electric field. Obtain the energy spectrum to the second order in the field strength. (Is it ok to use non-degenerate perturbation theory?)

7. In hydrogenic atoms, assume that the nucleus is uniformly charged sphere of radius R. Calculate the energy shift for $n = 1$ and $n = 2$ states.

8. Find out the energy shifts due to linear Stark effect in $n = 3$ state of hydrogen atom.

9. Consider a Hamiltonian of the form

$$\mathbf{H} = \begin{pmatrix} E_0 & 0 \\ 0 & -E_0 \end{pmatrix} + \lambda \begin{pmatrix} \alpha & U \\ U^* & \beta \end{pmatrix} \tag{4}$$

Find the energy shift to first and second order in λ. Compare your results with exact eigenvalues.

10. The perturbing hamiltonian for Hydrogen atom in constant magnetic field is given by

$$V = -\frac{e}{2mc}\mathbf{B} \cdot (\mathbf{L} + \mathbf{S}) \tag{5}$$

Show that, in this case a level of given total angular momentum quantum number j splits in $(2j + 1)$ levels according to the formula $E^{(1)}_{jm} = -g_j \mu B m$ where g_j is Lande's factor.