1. **A simple harmonic oscillator is perturbed by a constant force. Find the first order correction to the energy eigenvalues.**

   The perturbation potential can be written as $V(x) = Kx$. Since $\hat{x}\,|n\rangle \sim \sqrt{n}\,|n-1\rangle + \sqrt{n+1}\,|n+1\rangle$, the first order correction to the energy eigenvalues is 0.

2. **By choosing an appropriate trial function, find the energy of the first excited state of harmonic oscillator.**

   The Hamiltonian: $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$

   Trial Wave Function: $\psi(x) = xe^{-ax^2}$

   $(\psi, \psi) = \int_{-\infty}^{\infty} x^2 e^{-2ax^2} dx = 2\int_0^{\infty} x^2 e^{-2ax^2}\,dx = \frac{1}{8(\sqrt{a})^3}\lim_{x\to\infty}\left(\sqrt{\pi}\sqrt{2}\right) = \frac{1}{8}\sqrt{\frac{2\pi}{a^3}}$

   $H\psi(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\left(xe^{-ax^2}\right) + \frac{1}{2}m\omega^2 x^3 e^{-ax^2} = \frac{\hbar^2 a}{m}xe^{-ax^2}\left(3 - 2ax^2\right) + \frac{1}{2}m\omega^2 x^3 e^{-ax^2}$

   $(\psi, H\psi) = 2\int_0^{\infty} xe^{-ax^2}\left(\frac{\hbar^2 a}{m}xe^{-ax^2}\left(3 - 2ax^2\right) + \frac{1}{2}m\omega^2 x^3 e^{-ax^2}\right)dx = \frac{3\hbar^2 a}{16m}\sqrt{\frac{2\pi}{a^3}} + \frac{3}{64}\frac{m\omega^2}{a}\sqrt{\frac{2\pi}{a^3}}$

   $E = \frac{3\hbar^2 a}{2m} + \frac{3}{8}\frac{m\omega^2}{a}$

   $\frac{dE}{da} = \frac{3\hbar^2}{2m} - \frac{3}{8}\frac{m\omega^2}{a^2}$

   $\frac{dE}{da} = 0 \Rightarrow a^* = \frac{m\omega}{2\hbar}$

   $E^* = \frac{3\hbar^2 a^*}{2m} + \frac{3}{8}\frac{m\omega^2}{a^*} = \frac{3}{2}\hbar\omega$