CYK/2005/PH406/**Mid-Semester Exam**/30 Marks

You may consult your own handwritten notes. Books/Xeroxes/Printed Notes are not allowed. Duration of examination is NOT limited to two hours.

---

1. For the potential shown in the figure, find the approximate ground state energy upto the second order in V_0, assuming that $V_0 << \frac{\hbar^2\pi^2}{2mL^2}$. [5]

2. Consider a particle (mass m) moving in a perturbed 2D infinite well represented by a potential energy function
$$U(x,y) = \begin{cases} \frac{\eta E_0}{L^2} xy & \text{if } x, y \in [0, L] \\ \infty & \text{Otherwise} \end{cases}$$
where η is a small positive real number and $E_0 = \frac{\pi^2\hbar^2}{2mL^2}$. Find the first order correction to the first excited energy level (which is 2-fold degenerate). Also find the correction to the wave functions. [5]

3. To obtain an estimate of the ground state of hydrogen atom, use the trial function $\phi(r) = \frac{1}{\alpha^2+r^2}$ (in atomic units).

   (a) Show that $\langle \phi, \phi \rangle = \frac{\pi^2}{\alpha}$.

   (b) Show that $\frac{1}{2}\langle \phi, p^2\phi \rangle = \frac{\pi^2}{4\alpha^3}$.

   (c) Show that $\left\langle \phi, \frac{1}{r}\phi \right\rangle = \frac{2\pi}{\alpha^2}$.

   (d) Show that the energy estimate is 0.405 au.

   [2+4+2+2]

4. Two electrons, moving in a common harmonic potential are described by a Hamiltonian
$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2\left(x_1^2 + x_2^2\right)$$
in independent particle approximation. Sketch energy spectrum, separately for singlets ($S = 0$) and triplets ($S = 1$), showing atleast five levels. Write down the energy and the degeneracy of each level. [5]

5. Show that the ground state energy of the helium atom, treating the electronic coulomb repulsion as perturbation, is $-Z^2 + \frac{5}{8}Z$. [5]

Useful Information

$\int_0^{\pi/2} \sin^n\theta \, d\theta = \frac{n-1}{n}\int_0^{\pi/2} \sin^{n-2}\theta \, d\theta$

Figure for Problem 1