1. [7 Marks] The wave function of a free particle is given by

$$\psi(x) = \frac{A}{a^2 + x^2} e^{ip_0 x/\hbar}$$

where p_0, A and a are real constants.

   (a) Obtain the momentum space wave function

   (b) What is the most probable value of the momentum.

   (c) Find the average value of the momentum.

   (d) If the wave function above describes 100 independent particles, how many particles would have momentum between p_0 and $p_0 + \frac{a}{\hbar}$

2. [3 Marks] If a particle trapped in a box (of length L) has the wave function

$$\psi(x) = \begin{cases} \frac{1}{\sqrt{L}} & x \in [0, L] \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that the energy measurement yields $9\epsilon_1$ or $16\epsilon_1$? What is the probability that the particle is moving to the right? The mass of the particle is m and $\epsilon_1 = \hbar^2\pi^2/2mL^2$.

The fourier transform of $(1 + x^2)^{-1}$ is $\sqrt{\pi/2}\exp(-|k|)$.