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1. Calculate the stationary states and energy eigenfunctions of a box with walls at $\pm L/2$.
2. Calculate $\langle X\rangle$, $\langle X^2\rangle$, $\langle P\rangle$, and $\langle P^2\rangle$ for the nth stationary state of the infinite square well. Check validity of the uncertainty principle.
3. Find the probability density function for momentum measurement if the particle is in the n^{th} stationary state of an infinite square well.
4. A particle of mass m in the infinite square well (of width L) starts out in the left half of the well (at $t=0$) and is equally likely to be found at any point in that region.
   (a) What is its initial wavefunction,$\Psi(x,0)$? Assume that it is real.
   (b) What is the probability that a measurement of the energy would yield the value $\pi^2\hbar^2/2mL^2$?
5. A particle in infinite square well has the initial wave function

$$\Psi(x,0)=\begin{cases}Ax, & 0\le x\le L/2,\\ A(L-x), & L/2\le x\le L.\end{cases}$$

   (a) Sketch $\Psi(x,0)$, and determine the constant A.
   (b) Find $\Psi(x,t)$.
   (c) How will you calculate the average energy, that is the expectation value of $\left\langle \hat{H}\right\rangle$. [The wave unction at $t=0$ is not twice differentiable!]
6. A particle in a infinite square well has its intial wave function given by

$$\Psi(x,0)=A\left(\frac{\sin\pi x}{a}\right)^5.$$

   (a) Find A by normalizing the wavefunction.
   (b) Find $\Psi(x,t)$.
   (c) What is the probability that the energy measurement will yield ϵ_3?
7. A particle in the infinite square well has as its initial wave function

$$\Psi(x,0)=\frac{1}{\sqrt{2}}\left(\phi_1(x)+\phi_2(x)\right)$$

   where ϕ_n is the wavefunction of the n^{th}stationary state.
   (a) Write down $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express later as a sinusoidal function of time. Use $\omega=\pi^2\hbar/2mL^2$.
   (b) Find $\langle x\rangle$ as a function of t. It oscillates in time. What is the angular frequency? What is the amplitude?
   (c) Compute $\langle p\rangle$ as a function of t. Check if it obeys Ehrenfest theorem, that is

$$\frac{d}{dt}\langle x\rangle=\frac{1}{m}\langle p\rangle\,.$$

8. Consider a system with a single particle moving in a conservative force field. The potential energy $V(x)$ of particle is bounded below, that is, $V(x)\ge V_{\text{min}}$ for all x. Show that there is no energy eigenstate with eigenvalue less than V_{min}. [Hint: If $E<V_{\text{min}}$, then ψ and its second derivative has same sign. Argue that such functions cannot be square integrable.]