1. Two fair and identical dice are thrown.
   (a) Write down the sample space.
   (b) What is the probability that the sum ≤ 4?
   (c) What is the probability that the sum is divisible by three?
   (d) If one forms a two digit number from the outcome, what is the probability that the two digit number is greater than 33?

2. Consider a semicirle $S = \{(x, y) \mid x^2 + y^2 = 1,\ y \geq 0\}$. A particle moves back and forth on S with uniform speed. An experiment is performed to find the x coordinate of the particle at some random time. Denote the outcome by X. What is the probability density function $\rho(x)$ for X. Plot $\rho(x)$. Find $\langle x \rangle$, $\langle x^2 \rangle$. Find average value of y coordinate.

3. It can be shown that for an ideal (classical) gas, the probability density function of the distance that molecule travels between collisions be x, is $e^{-x/\lambda}$, where λ is a constant. Show that the average distance between collisions (called the *mean free path*) is λ. Find the probability that the free path is greater than 2λ.

4. Find the fourier series of
$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi/2 \\ 0 & \pi/2 < x < \pi. \end{cases}$$

5. Find the fourier transform of
$$f(x) = \begin{cases} \cos x & |x| < \pi/2 \\ 0 & |x| > \pi/2 \end{cases}$$

6. The family of functions $\delta_L(k)$ is defined by
$$\delta_L(k) = \frac{1}{2\pi} \int_{-L}^{L} e^{ikx} dx.$$
Evaluate the integral and show that $\delta_L(k)$ behaves like a delta function as $L \to \infty$.

7. The gaussian probability density function is given by
$$g(x) = A \exp\left[-\frac{(x-a)^2}{2\sigma^2}\right]$$
where a and σ are positive constants.
   (a) Normalize g such that $\int_{-\infty}^{\infty} g(x)dx = 1$.
   (b) Find $\langle x \rangle$, $\langle x^2 \rangle$ and standard deviation of x.
   (c) Sketch the function.

8. The wave function of a particle at $t = 0$ is given by
$$\psi(x, 0) = \begin{cases} A\left(a^2 - x^2\right), & |x| \leq a \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find normalization constant A.

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$ and standard deviation σ_x of x.

(c) Find $\langle p \rangle$, $\langle p^2 \rangle$ and standard deviation σ_p of p.

(d) Is your answer consistent with uncertainty principle?

9. A free particle has the initial wave function

$$\psi(x,0) = B \exp\left[i\frac{p_0 x}{\hbar}\right] \exp\left[-\frac{|x|}{2\Delta_x}\right].$$

Normalize $\psi(x,0)$. Find momentum space wave function $A(p)$. Suggest a reasonable definition of uncertainty in p and denote it byΔ_p. Show that $\Delta_x \Delta_p \gtrsim \hbar$.

10. Suppose that the momentum space wave function of a gaussian wave packet is given by

$$A(p) = \left(\frac{1}{\sqrt{\pi}\Delta_p}\right)^{\frac{1}{2}} \exp\left[-\frac{(p-p_0)^2}{2\Delta_p^2}\right].$$

Find $\psi(x,t)$ for $t > 0$. What is the width (defined as the standard deviation of x) of the wave packet at time t?