Chapter 1

Introduction

1.1 Perspectives

▷ PHYSICS answers HOW not WHY.
For example, Newton's first law states

$$\text{No Force} \implies \text{Uniform Velocity}$$

Not zero velocity! This correctly describes natural phenomena, but this law does not tell us why the Nature has chosen this law.
Simillarly one may ask why is gravitational force is inverse square law. Answer is not known.

▷ Thus, PHYSICS describes the natural world (only observable and measurable) using a mathematical model. For example, the trajectory of a particle in a force field is described by a system of differential equations

$$
\begin{aligned}
m\ddot{\mathbf{x}}(t) &= \mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}, t) \qquad t > 0 \\
\mathbf{x}(0) &= \mathbf{x}_0 \\
\dot{\mathbf{x}}(0) &= \mathbf{v}_0
\end{aligned}
$$

▷ Of course, we would like to have one single model to describe all natural phenomena. Newtonian mechanics, which combined planetory motion and the motion of a ball on the Earth into one single description, could not put light in the same model. Newton did think that the light consisted of tiny particles, however phenomena like interference established wave theory for light.

In addition, the observations (instruments) got better, Newtonian mechanics could not explain the shift of perihelion of Mercury.

Cleary, the search for final description of the world is not over.

▷ Throughout 19th century and in the beginning of 20th century, the microscopic structure of the matter was a puzzling problem. Clearly, the observed electromagnetic emissions by materials could not be explained. The atomic and subatomic particles did not behave as expected in existing models of classical theories.

▷ Thus emerged a new mathematical model called Quantum Mechanics. It would be paramount to understand that this is merely a description of the natural phenomena. At this stage in learning QM, it would not be prudent to ask philosophical questions. What we are going to learn is a model that describes natural world at macroscopic and microscopic length scales and non-relativistic velocities.

▷ Mathematical foundations of Quantum Mechanics are very advanced and abstract. Fortunately, QM can still be practised with sufficient fluency at different levels of mathematical sophistication and possibly, a large number of physicists succesfully do with very little understanding of the mathematical foundations.

1.2 Wave-Particle Duality: History

Particles	Waves
Tiny (localized), indivisible objects with intrinsic properties like mass, charge etc.	Extended divisible objects
Move in definite trajectories	Typically expanding wavefronts
Motion described by position and momentum.	Motion described by wavefront and amplitude.
Particles collide	Waves superimpose
Phenomena like scattering	Diffraction, Interference etc.
Energy is localized	Energy is distributed

▷ Newton BELIEVED that light was made of tiny particles.

▷ However, in 18th century, the wave nature of light was firmly established. It exhibited phenomena like interference and diffraction, just like water waves or elastic waves.

▷ Maxwell's theory sealed the wave nature of light on a firm theoretical platform.

▷ At the same time the community was struggling with the atomic structure of matter. In this context, Plank proposed that the em waves radiated by black bodies must be in *packets* or *quanta*.

▷ Later, Einstein proposed that the all em waves must be *localized* packets of energy. He writes

According to the assumption to be contemplated here, when a light ray is spreading from a point, the energy is not distributed continuously over ever-increasing spaces, but consists of a finite number of energy quanta that are localized in points in space, move without dividing, and can be absorbed or generated only as a whole.

▷ Thus, it was proposed that the light behaves, both as waves(characterized by frequency ν and wavelength λ) and particles (characterized by energy E and momentum $\mathbf{p}$), such that

$$
\begin{aligned}
E &= h\nu \\
\mathbf{p} &= \frac{h}{\lambda}\mathbf{n}
\end{aligned}
$$

where $\mathbf{n}$ is the direction of propagation.

▷ Compton Scattering brilliantly demonstrates the particle nature of light by showing that it participates in collisions with matter particles.

▷ Now, if light has dual nature, do electrons and other matter particles have dual nature too?

▷ Louis de Broglie suggested that the matter has dual nature too. He assigned to particles, *matter waves.*

▷ Davisson-Germer experiment demonstrates the wave nature of electrons through a diffraction experiment.

▷ Thus, Wave-Particle Duality Principle: All matter and waves have dual nature.

▷ Now it may be quiet difficult to imagine or visualize the dual nature of these objects with out classical intuition. It *may* or *may not* be sensible do so.

1.3   Double Slit Experiment with Photons

▷ Consider Young's two slit interference experiment.
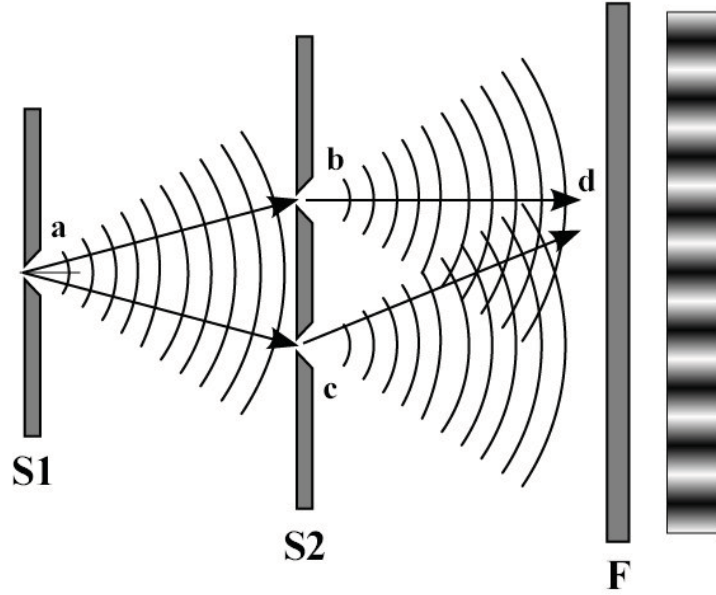
Classically, this is well understood phenomenon exhibited by all waves (Water, Sound, light, etc). In Young's experiment using light, the two slits can be considered as secondary sources. The em waves emitted by two waves are described by respective *wave functions*, say $\psi_1(\mathbf{r}, t)$ and $\psi_2(\mathbf{r}, t)$. Since, the two wave superpose linearly, the net wave function

$$\psi(\mathbf{r}, t) = \psi_1(\mathbf{r}, t) + \psi_2(\mathbf{r}, t).$$

Now the intensity, I, is given by a time average of $|\psi|^2$, thus

$$
\begin{aligned}
I(\mathbf{r}) &= \overline{|\psi|^2} \\
&= \overline{|\psi_1|^2} + \overline{|\psi_2|^2} + \overline{2\mathrm{Re}\psi_1^*\psi_2} \\
&= I_1(\mathbf{r}) + I_2(\mathbf{r}) + I_{\mathrm{int}}(\mathbf{r})
\end{aligned}
$$

Typically, I_{int} is proportional to $\cos^2(\delta/2)$, where δ is the phase difference between ψ_1 and ψ_2.

Now, it is believed that if the intensity of the source is reduced, the wave description is still valid and one should expect an interference pattern with small intensity.

▷ Now, it is possible to detect one photon. Then, if the intensity is reduced to such that there is only one photon in the instrument at a time, and if wave picture is valid, one should expect a interference pattern. Here are the results of experiment by Optics Research Group at Delft University of Technology.

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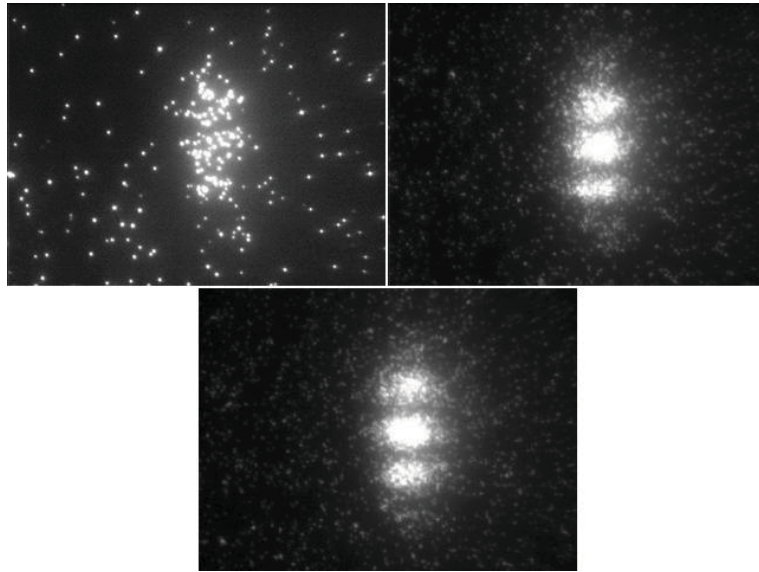
This experiment is performed with about 10^5 photons per second (mean free path $\sim$ 3km).

▷ Points to learn:

    – At the screen, photons are detected as localized particles, not as extended waves.

    – Photons don't interfere with each other, they are well separated in time and space.

    – During transit, photons do not behave like particles, since there is no way of telling how a photon reaches a perticular point on the screen. (Anyway, no interference pattern for particles)

    – Statistical nature of the experiment is unmistakable. Distribution of photons at various points on the screen is given by $I(\mathbf{r})$.

    – The probability that a given photon reaches a perticular point $\mathbf{r}$ on the screen is given by $I(\mathbf{r})$.

▷ Thus, one can conclude:

> The probability that a photon reaches a point in space is given by the square of the classical wave function at that point.

▷ Maxwell's theory including notion of fields is a good description of electromagnetic phenomena only if there are very large number of photons.

1.4 Matter Waves

▷ If the same experiment is performed with electrons instead of light, what is expected?
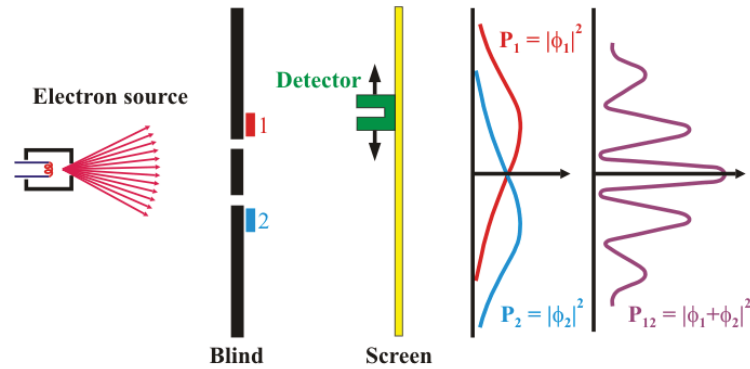
If one of the slits (say slit 1) is closed then the blue pattern is obtained on the screen. This is expected from all particles. If slit 2 is closed and slit 1 opened then we would get red pattern. If both slits are open then we would expect a pattern that is sum of blue and red pattern. However, to one's surprize, the interference pattern is obtained just like the one obtained in case of light. That is even electrons show wave like properties.

▷ What happens when the experiment is performed with one electron at a time. Indeed, in 1989, Tonomura performed double slit experiment with very few electrons.
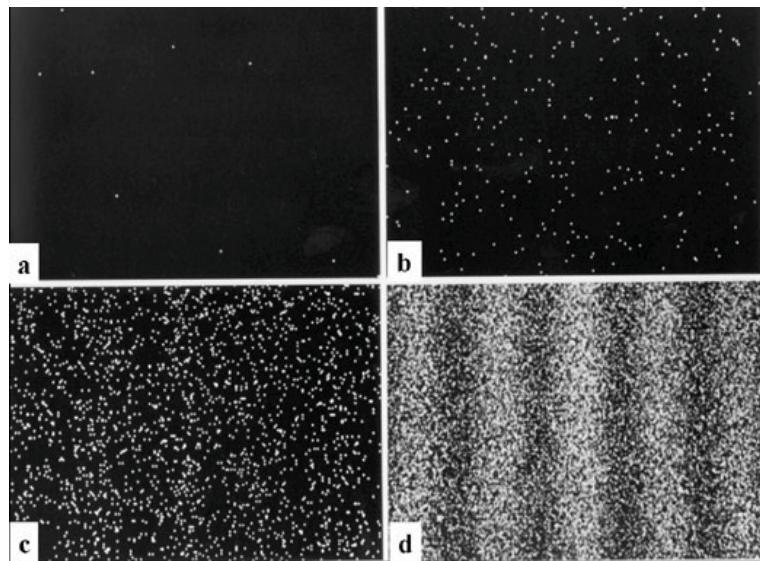
The result is identical to that of light.

▷ This gives one insight into the nature of the de Broglie wave that must be assigned to particles.

  – The particles will be assigned a wave function, square of which will be proportional to the probability of finding the particle at a point in space.

▷ If you can predict only probability distribution of a particle being at a position, which other properties can be predicted with certainty? NONE! The fact that particles donot follow well defined trajectories, means that the velocity and momentum cannot be assigned either.

▷ However, properties like momentum, energy are central in classical mechanics. In quantum mechanics all one can describe is probability distribution of particle having certain momentum.