[Note: Q10 of t6 and the first four problems will be discussed in the tutorial class.]

1. A linear inhomogeneous dielectric is sandwiched between the plates of a parallel plate capacitor (separation between the plates $= d$) charged to the charge density σ. The permittivity of the dielectric at a distance y from one of the plates, is given by

$$\epsilon = \epsilon_0 \left(1 + K\left(\frac{y}{d}\right)\right)$$

where K is a positive constant. (Neglect edge effects.)

   (a) Find the expressions for E, D, and P. Plot these quantities as a function of y.

   (b) Find the bound charge densities σ_b and ρ_b. Plot ρ_b.

   (c) Find the potential difference between the plates.

2. A current is flowing in a thick wire of radius a. The current is distributed in the wire such that the current density at a distance r from the axis is given by

$$\mathbf{J} = \mathbf{J}_0 \left(1 + \frac{r^2}{a^2}\right).$$

   Find the total current through the wire.

3. Consider a wire, bent in a shape of a parabola, kept in XY plane with focus at origin. The distance from apex to focus is d. The wire carries current I. Find the magnetic field at origin.

4. [G5.44] Use the Biot-Savart law to find the field inside and outside an infinitely long solenoid of radius R, with n turns per unit length, carrying a steady current I. [Write down the surface current density and Eq 5.39. Do z-integration first.]

5. Consider a circular ring, of radius R and carrying current I is placed in the XY plane with its center at origin. Set up the integral to find the magnetic field at a point on the X axis, at a distance $d\,(\gg R)$ from the origin. Now, expand the integrand in the powers of R/d and find the first non-zero term. Express in terms of $m = I(\pi R^2)$.

6. [G5.6]

   (a) A phonograph record carries a uniform density of "static electricity" σ. If it rotates at angular velocity ω, what is the surface current density $\mathbf{K}$ at a distance r from the center?

   (b) A uniformly charged solid sphere, of radius R and total charge Q, is centered about origin and spinning at a constant angular velocity ω about the z axis. Find the current density $\mathbf{J}$ at any point (r, θ, ϕ) within the sphere.

7. [G5.47] Find the magnetic field at a point $z > R$ on the axis of (a) the rotating disk and (b) the rotating sphere, of problem G5.6