1. **[G 4.4]** A point charge q is situated at a large distance r from a neutral atom of polarizability α. Find the force of attraction between them.

2. Consider a localized (of small dimension) charge distribution ρ with zero net charge and dipole moment $\mathbf{p}$, placed in an external field $\mathbf{E}_{\text{ext}}$. Let 0 be some suitable origin.

   (a) Show that the force on the charge distribution is given by

   $$\mathbf{F} = (\mathbf{p} \cdot \nabla)\,\mathbf{E}_{\text{ext}}(0) + \cdots$$

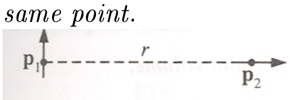
   (b) Show that the torque on the charge distribution is given by

   $$\mathbf{N} = \mathbf{p} \times \mathbf{E}_{\text{ext}}(0) + \cdots$$

   (c) Show that the energy of the charge distribution is given by

   $$U = -\mathbf{p} \cdot \mathbf{E}_{\text{ext}}$$

3. **[G 4.5, G 4.29]** In Fig., $\mathbf{p_1}$and $\mathbf{p_2}$ are (perfect) dipoles a distance r apart. What is the torque on p_1 due to p_2? What is the torque on $p_2$due to p_1?
   For the same configuration, calculate the *force* on $\mathbf{p_2}$ due to $\mathbf{p_1}$, and the force on $\mathbf{p_1}$ due to $\mathbf{p_2}$. Are the answers consistent with Newton's third law?
   Also, find the total torque on $\mathbf{p_2}$ *with respect to the center of* $\mathbf{p_1}$, and compare it with the torque on $\mathbf{p_1}$ *about that same point.*



4. **[G 4.13]** A very long cylinder, of radius a, carries a uniform polarization $\mathbf{P}$ perpendicular to its axis. Find the electric field inside the cylinder. Show that the field outside the cylinder can be expressed in the form

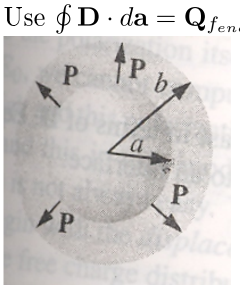
   $$\mathbf{E}(\mathbf{r}) = \frac{a^2}{2\epsilon_0 s^2} \left[2(\mathbf{P} \cdot \hat{\mathbf{s}})\,\hat{\mathbf{s}} - \mathbf{P}\right].$$

5. **[G 4.15]** A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization

   $$\mathbf{P}(\mathbf{r}) = \frac{k}{r}\hat{\mathbf{r}},$$

   where k is a constant and r is the distance from the center (Fig.). (There is no *free* charge in the problem.) Find the electric field in all three regions by two different methods:

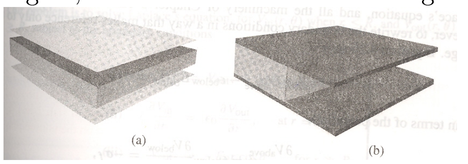
   (a) Locate all the bound charge, and use Gauss's law to calculate the field it produces.
   (b) Use $\oint \mathbf{D} \cdot d\mathbf{a} = \mathbf{Q}_{fenc}$, to find $\mathbf{D}$, and then get $\mathbf{E}$ from $\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P}$.



6. **[G 4.11]** A short cylinder, of radius a and length L, carries a "frozen-in" uniform polarization $\mathbf{P}$, parallel to its axis. Find the bound charge, and sketch the electric field (i) for $L \gg a$, (ii) $L \ll a$ and (iii) $L \approx a$.

7. **[G 4.31]** A dielectric cube of side a, centered at the origin, carries a "frozen-in" polarization $\mathbf{P} = k\mathbf{r}$, where k is a constant. Find all the bound charges, and check that they add up to zero.

8. **[G 4.19]** Suppose you have enough linear dielectric material, of dielectric constant ϵ_r, to *half*-fill a parallel-plate capacitor (Fig.). By what fraction is the capacitance increased when you distribute the material as in (a) of given Fig.? How about (b) of the same? For a given potential difference V between the plates, find **E**, **D**, and **P**, in each region, and the free and bound charge on all surfaces, for both cases.



9. **[G 4.32]** A point charge q is imbedded at the center of a sphere of linear dielectric material (with susceptibility χ_e and radius **R**). Find the electric field, the polarization, and the bound charge densities, ρ_b and σ_b. What is the total bound charge on the surface? Where is the compensating negative bound charge located?

10. **[G 4.36]** A conducting sphere at potential V_0 is half embedded in linear dielectric material of susceptibility χ_e, which occupies the region $z < 0$ shown in the first Fig. *Claim:* the potential everywhere is exactly the same as it would have been in the absence of the dielectric! Check the claim as follows:

    (a) Write down the formula for the proposed potential $V(r)$, in terms of V_0, **R**, and r. Use it to determine the field, the polarization, the bound charge, and the free charge distribution on the sphere.

    (b) Show that the total charge configuration would indeed produce the potential $V(r)$.

    (c) Appeal to the uniqueness theorem to complete the argument.

    (d) Could you solve the configurations in the second Fig. with the same potential? If not, explain *why*.