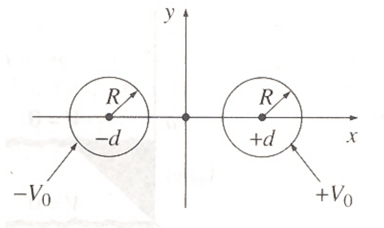
[*Note: Only the first four problems will be discussed in the tutorial class. You must attempt the remaining problems, and if you are stuck, ask your tutor.*]

1. **[G 3.11]** Two long, straight copper pipes, each of radius $R$, are held a distance $2d$ apart. One is at potential $V_0$, the other at $-V_0$ (Fig.). Find the potential everywhere.




2. **[G 3.14]** A rectangular pipe, running parallel to the $z-$axis (from $-\infty$ to $\infty$), has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$, is maintained at a specific potential $V_0(y)$.
   (a) Develop a general formula for the potential within the pipe.
   (b) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).

3. **[G 3.1]** Find the average potential over a spherical surface of radius $R$ due to a point charge $q$ located *inside*. Show that in general,
   $V_{\text{ave}} = V_{\text{center}} + \frac{Q_{\text{enc}}}{4\pi\epsilon_0 R}$,
   where $V_{center}$ is the potential at the center due to all the *external* charges, and $Q_{enc}$ is the total enclosed charge.

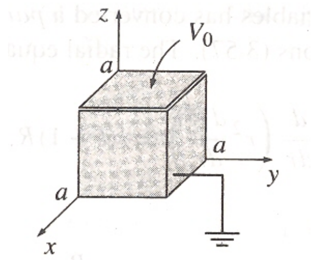
4. **[G 3.3]** Find the general solution to Laplace's equation in spherical coordinates, for the case where $V$ depends only on $r$. Do the same for cylindrical coordinates, assuming $V$ depends only on $s$.

5. **[G 3.7]**
   (a) Using the law of cosines, show that $V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} + \frac{q'}{r'}\right)$ (where $r$ and $r'$ are the distances from $q$ and $q'$ respectively) can be written as follows:
   $V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2+s^2-2rs\cos\theta}} - \frac{q}{\sqrt{R^2+(rs/R)^2-2rs\cos\theta}}\right]$,
   where $r$ and $\theta$ are the usual spherical polar coordinates, with the $z$- axis along the line through $q$. In this form it is obvious that $V = 0$ on the sphere, $r = R$.
   (b) Find the induced surface charge on the sphere, as a function of $\theta$. Integrate this to get the total induced charge.
   (c) Calculate the energy of this configuration.

6. **[G 3.8]** Consider a point charge $q$ situated at a distance $a$ from the center of a grounded conducting sphere of radius $R$. The same basic model will handle the case of a sphere at *any* potential $V_0$ (relative to infinity) with the addition of a second image charge. What charge should you use, and where should you put it? Find the force of attraction between a point charge $q$ and a *neutral* conducting sphere.

7. **[G 3.12]** Two infinite grounded metal plates lie parallel to the $xz$ plane, one at $y = 0$, the other at $y = a$. The left end, at $x = 0$, consists of two metal strips: one, from $y = 0$ to $y = a/2$, is held at a constant potential $V_0$, and the other, from $y = a/2$ to $y = a$, is at potential $-V_0$. Find the potential in the infinite slot.

8. **[G 3.15]** A cubical box (sides of length $a$) consists of five metal plates, which are welded together and grounded. The top is made of a separate sheet of metal, insulated from the others, and held at a constant potential $V_0$. Find the potential inside the box.




### Additional Problem:

1. Consider an insulated spherical conductor with radius $a$ and net charge $q$. Another point charge $q$ is placed outside the conductor at a distance $d$ from the center of the conductor. Show that if $d/a = (1 + \sqrt{5})/2$, the point charge is in equilibrium. [Hint: Show that $d/a$ must be the solution of the quintic equation $x^5 - 2x^3 - 2x^2 + x + 1 = 0$. There are three real and two imaginary solutions. Verify that $x^2 - x - 1$ is a factor of the quintic polynomial.]