1. **[G 1.27]** Prove that the curl of a gradient is always zero. Check it for the function $f(x, y, z) = x^2y^3z^4$.

2. Find the length of one turn of a helical wire(with radius R and pitch p).

3. Find the work done by the force field $\mathbf{F}(x, y) = x\hat{\mathbf{x}} + (y + 2)\hat{\mathbf{y}}$ in moving an object along an arch of the cycloid $\mathbf{r}(t) = (t - \sin t)\,\hat{\mathbf{x}} + (1 - \cos t)\,\hat{\mathbf{y}}$, $0 \leq t \leq 2\pi$.

4. Evaluate $\int\int \mathbf{A} \cdot \hat{\mathbf{n}}\, ds$, where $\mathbf{A} = 18z\,\hat{\mathbf{x}} - 12\,\hat{\mathbf{y}} + 3y\,\hat{\mathbf{z}}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant.

5. **[G 1.30]** Calculate the volume integral of the function $T = z^2$ over the tetrahedron with corners at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

6. **[G 1.31]** Check the fundamental theorem for gradients, using $T = x^2 + 4xy + 2yz^3$, the points $\mathbf{a} = (0, 0, 0)$, $\mathbf{b} = (1, 1, 1)$, and the three paths in Fig.:

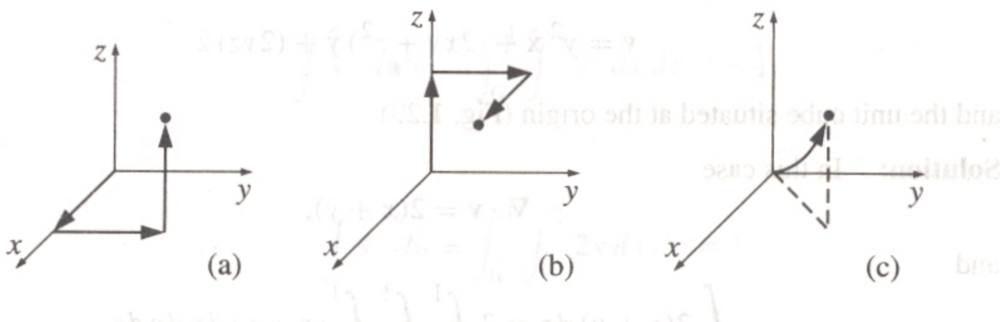


Figure 1: Problem 7

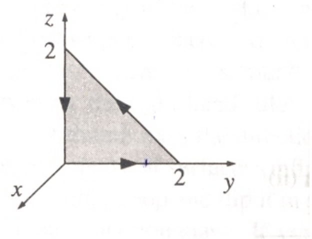
   (a) $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 0) \to (1, 1, 1)$;
   (b) $(0, 0, 0) \to (0, 0, 1) \to (0, 1, 1) \to (1, 1, 1)$;
   (c) the parabolic path $z = x^2$; $y = x$.

7. **[G 1.33]** Test Stokes' theorem for the function $\mathbf{v} = (xy)\hat{\mathbf{x}} + (2yz)\hat{\mathbf{y}} + (3zx)\hat{\mathbf{z}}$, using the triangular shaded area of Fig. .

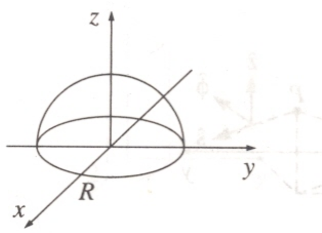
8. **[G 1.39]** Compute the divergence of the function

$$\mathbf{v} = (r\cos\theta)\hat{\mathbf{r}} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}.$$

   Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (Fig.).



(a) Problem 7



(b) Problem 8

9. **[G 1.41]** Derive the relations for unit vectors of cylindrical coordinate system:

$$\begin{aligned}\hat{\mathbf{s}} &= \cos\phi\hat{\mathbf{x}} + \sin\phi\hat{\mathbf{y}},\\ \hat{\phi} &= -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}},\\ \hat{\mathbf{z}} &= \hat{\mathbf{z}}.\end{aligned}$$

Invert the formulas to get $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ in terms of $\hat{\mathbf{s}}$, $\hat{\phi}$, $\hat{\mathbf{z}}$ (and ϕ).

10. **[G 1.44]** Evaluate the following integrals:

(a) $\int_{-2}^{2}(2x+3)\delta(3x)dx$.

(b) $\int_{0}^{2}(x^3+3x+2)\delta(1-x)dx$.

(c) $\int_{-1}^{1}9x^2\delta(3x+1)dx$.

(d) $\int_{-\infty}^{a}\delta(x-b)dx$.