1. Let $A(k) = \frac{A_0}{k^2+\alpha^2}$, where A_0 and α are positive constants. The wave packet is given by
$$\psi(x) = \int_{-\infty}^{\infty} A(k)\, e^{ikx} dk = A_0 \pi e^{-\alpha|x|}.$$
[The integral is usually solved by method of contour integrals, which students will study in the third semester.]
   (a) Plot $A(k)$ and $\psi(x)$.
   (b) If Δk and Δx are defined as widths at which corresponding functions ($A(k)$ and $\psi(x)$) become $1/e$ times their maximum value, show that $\Delta k \Delta x > 1$.

2. For every observable, which correspond to an operator $\widehat{A}$, the uncertainty is defined as
$$\Delta A = \sqrt{\left\langle \widehat{A}^2 \right\rangle - \left\langle \widehat{A} \right\rangle^2}.$$
For harmonic oscillator the average energy is given by $E = \left\langle \frac{P^2}{2m} + \frac{1}{2} m\omega^2 x^2 \right\rangle$.
   (a) For harmonic oscillator, argue that $\langle \widehat{p} \rangle = \langle \widehat{x} \rangle = 0$.
   (b) Calculate the average energy in terms of Δx, assuming that $\Delta x \Delta p = \hbar/2$ (this is the best we can do in QM).
   (c) Find the lowest possible energy by minimising it wrt Δx.

3. For the ground state of a particle in a box, find $\Delta x \Delta p$ using the definitions given in problem 2.

4. Show that [Assuming that ψ and its derivatives vanish at $\pm\infty$]
$$\langle \widehat{p} \rangle = m \frac{d \langle \widehat{x} \rangle}{dt}.$$

5. Show that if $\widehat{A}$ is a linear operator, $\widehat{A}^n$ is also a linear operator. (n is positive integer.)

6. Show that $\widehat{D} = \frac{d}{dx}$ is not a hermitian operator.

7. Show that if two hermitian operators commute, then their product is also hermitian.

8. Show that wave functions of a particle in a box are eigenfunctions of $\widehat{p}^2$ operator, but not of $\widehat{p}$ operator.

9. In a particle in a box problem, define a parity operator $\widehat{T}$ as
$$\left(\widehat{T} f\right)(x) = f(L/2 - x).$$
Show that the energy eigenstates are eigenfunctons of the parity operator. What are the eigenvalues of the parity operator?

10. Sketch a possible solution to the Schrödinger equation for each of the potential energies shown in the figure. In each case, show several cycles of the wave function. (See figure 5.24 on page 172 of Modern Physics by Krane)