Physics I

Lecture 8

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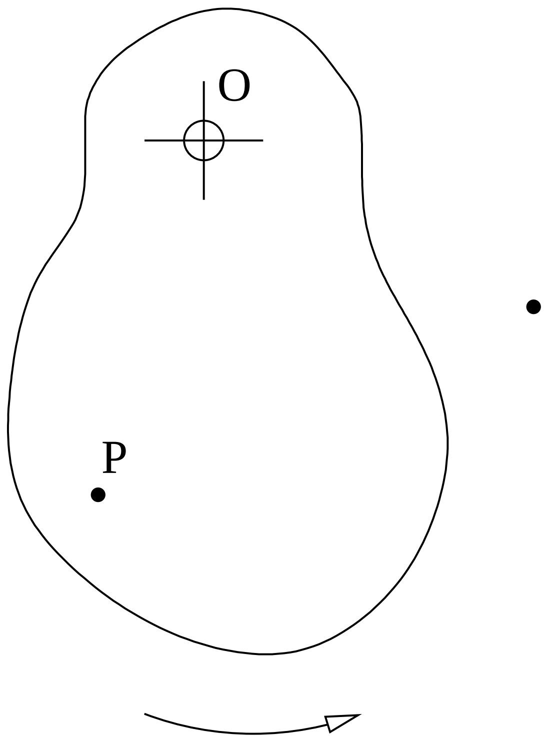
Pure Rotation in 2D

- A planar rigid body in a plane
- O is a fixed point of the plane
- P is arbitrary point of the rigid body
- Pure Rotation about O if

$$d(O, P) = \text{constant}$$

for all points P, at all times



Examples

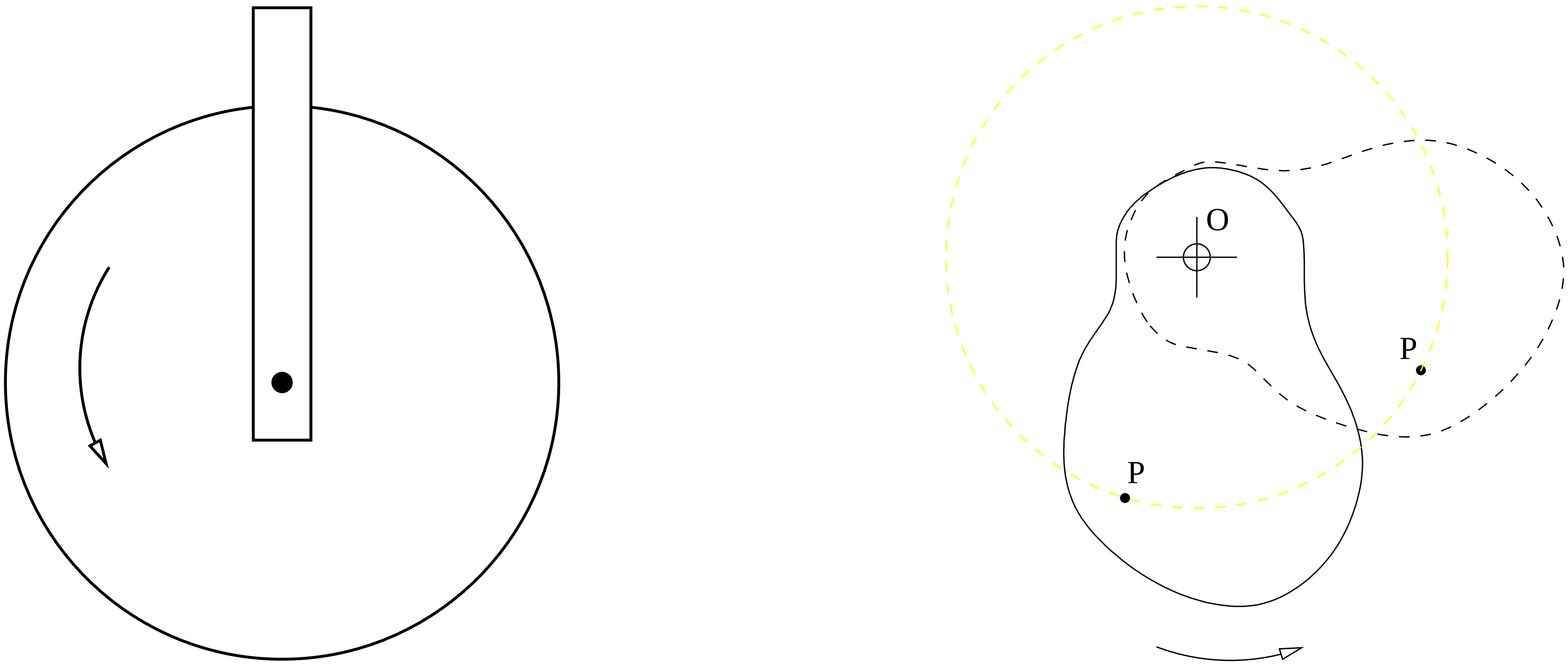


Figure 1: Pure Rotation.

Each point of the rigid body performs a circular motion about O.

Examples

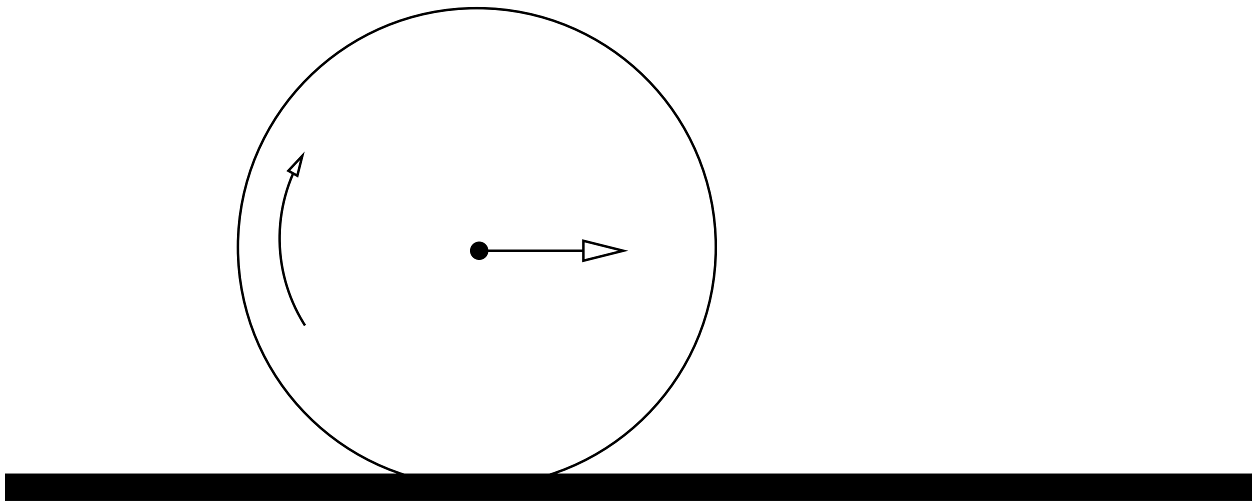


Figure 2: Not Pure Rotation

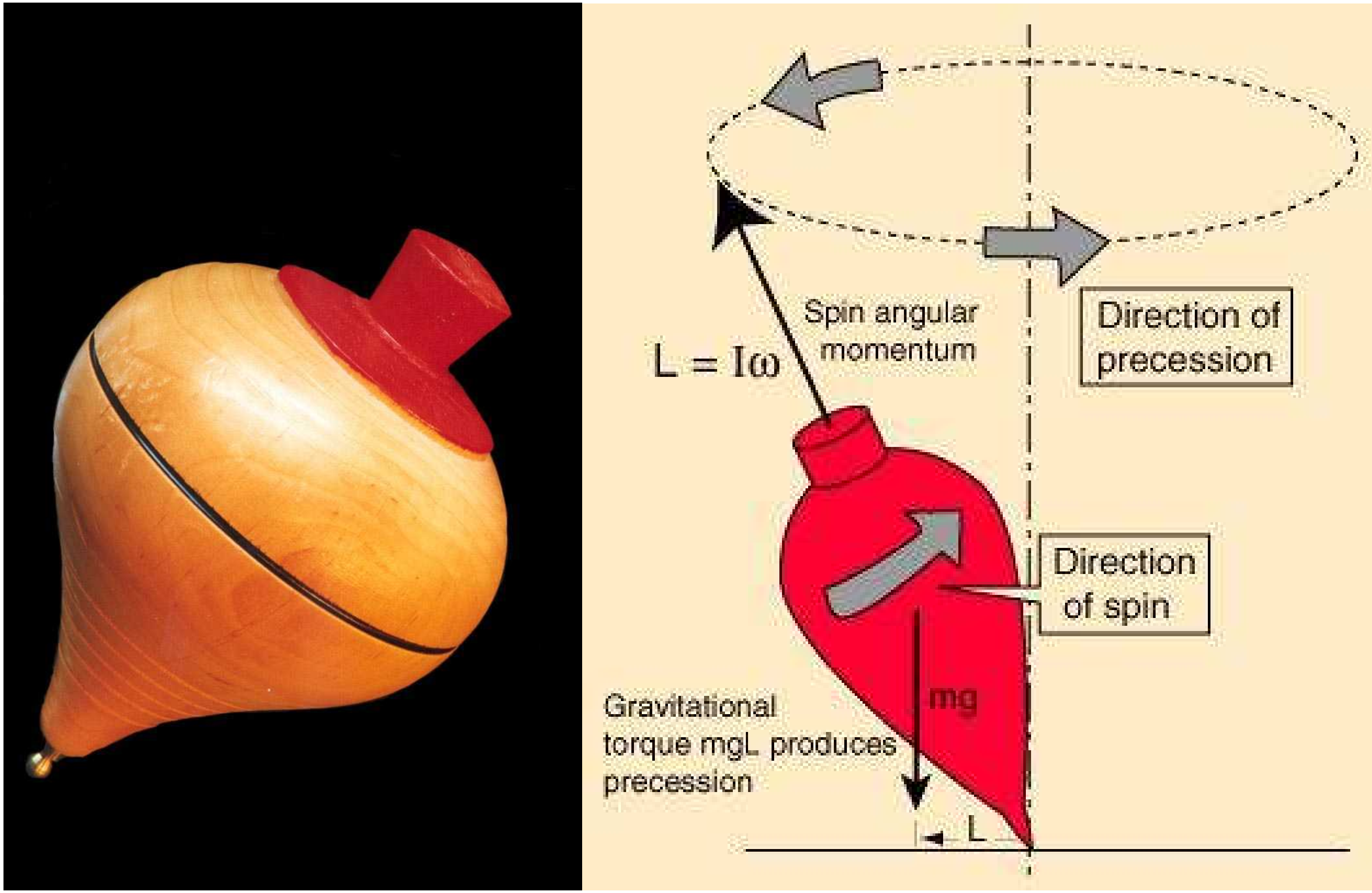
Pure Rotation in 3D

- A rigid body in Space
- O is a fixed point of the space
- P is arbitrary point of the rigid body
- Pure Rotation about O if

$$d(O, P) = \text{constant}$$

for all points P, at all times

Example



All points of the top are restricted to a spherical surface.

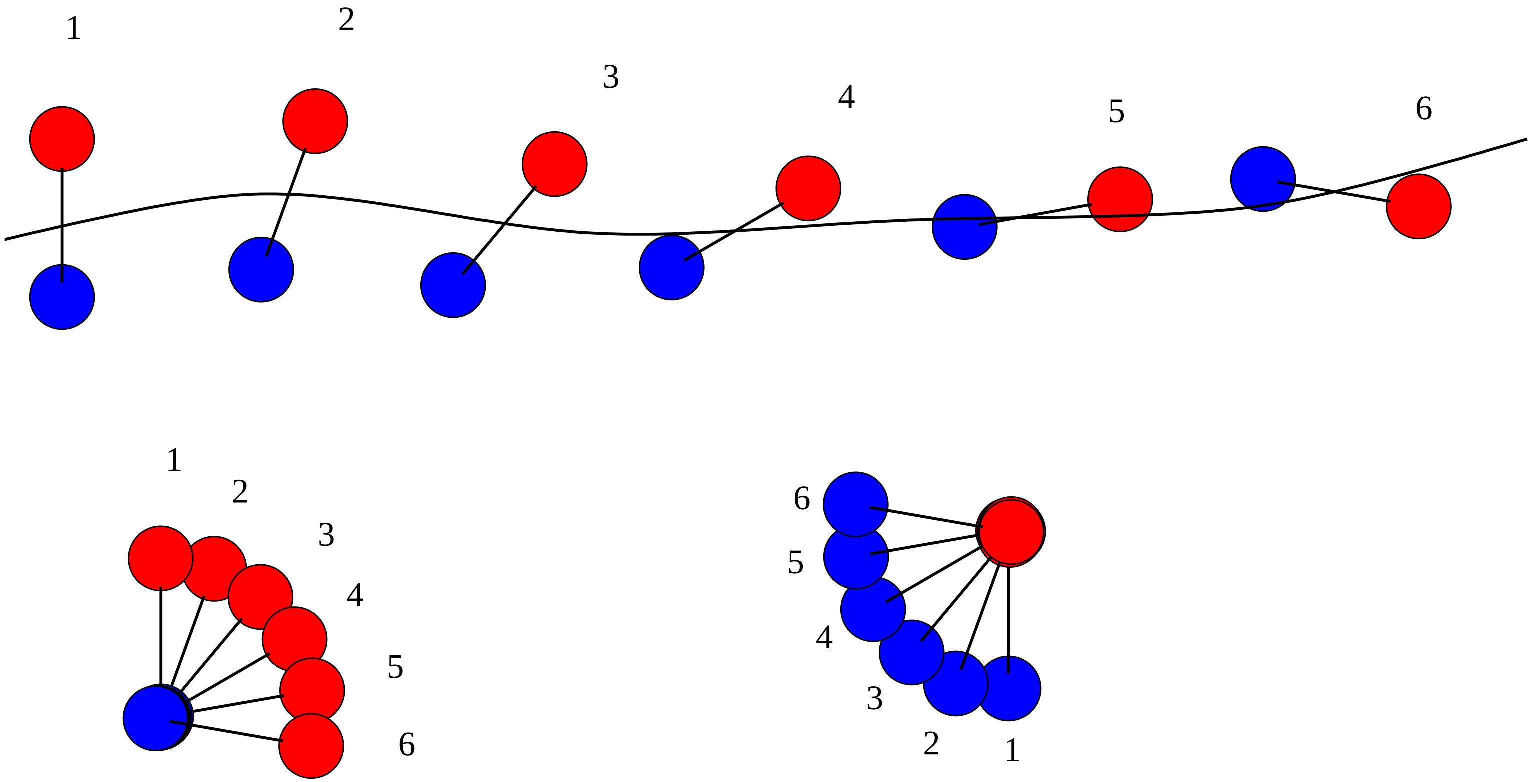
Fixed Axis Rotation

- Special case of pure rotation in 3D.
- Distance of the points of rigid body from a fixed LINE in space is constant.
- The fixed line is called the axis of rotation.

Example

All points of the tyre are in circular motion.

Rotation and Translation in 2D



For observer sitting on the blue ball, dumbbell motion is pure rotation.

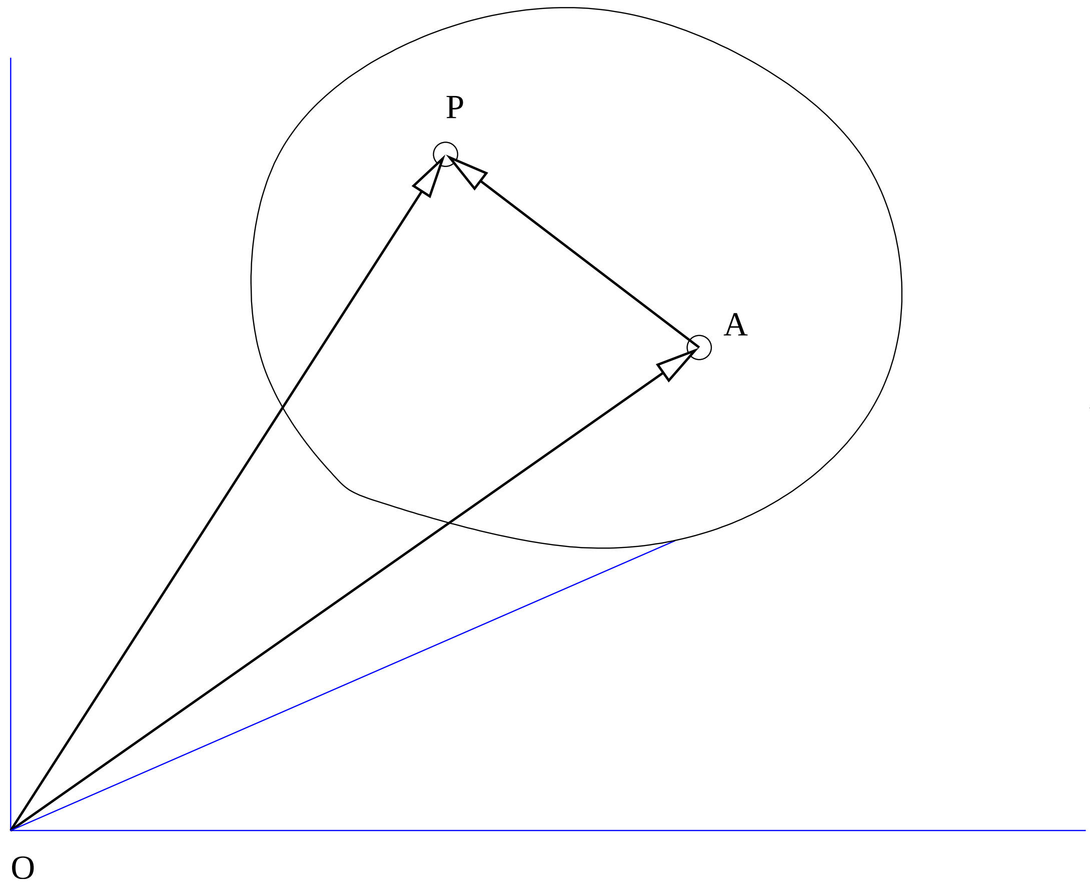
Same is true for red ball observer

For planar bodies in 2D, motion = translation + rotation

Rotation and Translation

- The idea can be generalized to 3D
- All rigid body motion can be split into:
  - A translation of one point of rigid body
  - Rotation of rigid body about that point
- A special case in which rigid body motion is combination of fixed axis rotation + translation of fixed axis keeping it parallel to the some fixed axis in space.

Angular Momentum

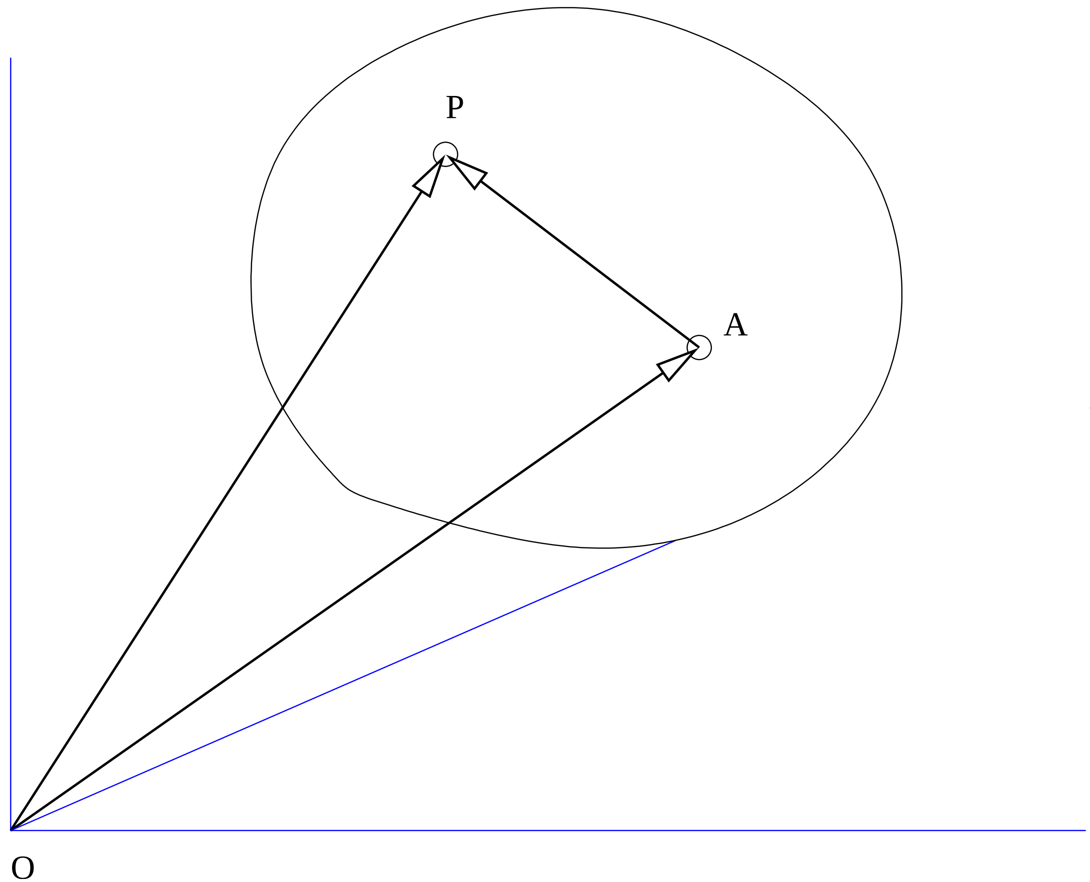


A: CM of body

P: A point of body

$$\vec{r} = \vec{R}_A + \vec{r'}$$
$$\vec{v} = \vec{V}_A + \vec{v'}$$

Angular Momentum



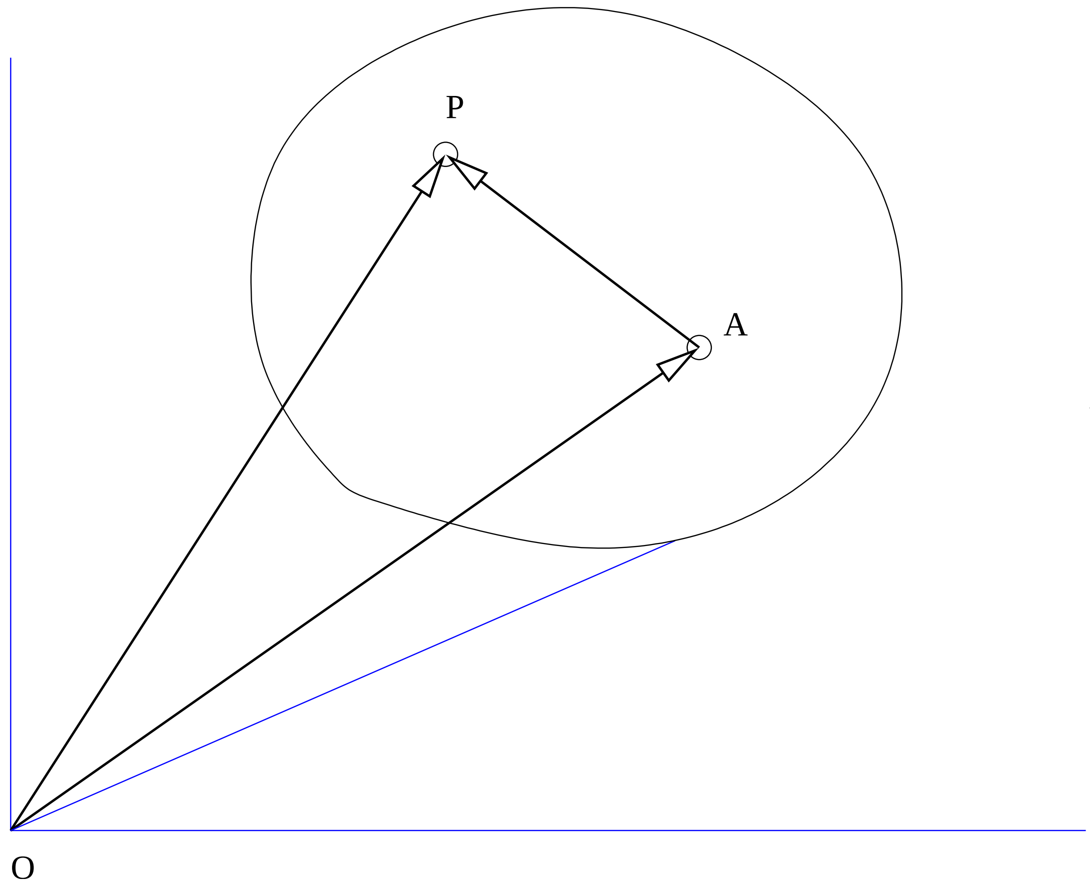
A: CM of body

P: A point of body

Angular Momentum about O: $\vec{L} = \sum m_i \, \vec{r}_i \times \vec{v}_i$

Angular Momentum about A: $\vec{L}_0 = \sum m_i \, \vec{r'}_i \times \vec{v'}_i$

Angular Momentum



A: CM of body

P: A point of body

$$
\begin{aligned}
\vec{L} &= \sum m_i \vec{r}_i \times \vec{v}_i \\
&= \sum m_i \left(\vec{R}_A + \vec{r'}_i \right) \times \left(\vec{V}_A + \vec{v'}_i \right)
\end{aligned}
$$

Angular Momentum

$$
\begin{aligned}
\vec{L} &= \sum m_i \left(\vec{R}_A + \vec{r'}_i\right) \times \left(\vec{V}_A + \vec{v'}_i\right) \\
&= \sum m_i \vec{R}_A \times \vec{V}_A + \sum m_i \vec{r'}_i \times \vec{v'}_i \\
&\quad + \sum m_i \vec{R}_A \times \vec{v}_i + \sum m_i \vec{r'}_i \times \vec{V}_A \\
&= \left(\sum m_i\right) \vec{R}_A \times \vec{V}_A + \sum m_i \vec{r'}_i \times \vec{v'}_i \\
&\quad + \vec{R}_A \times \left(\sum m_i \vec{v}_i\right) + \left(\sum m_i \vec{r'}_i\right) \times \vec{V}_A \\
&= M \vec{R}_A \times \vec{V}_A + \vec{L}_0 \\
&= \vec{L}_{cm} + \vec{L}_0
\end{aligned}
$$

Angular Momentum splits nicely into two terms

Torque

$$\begin{aligned}
\vec{\tau} &= \sum \left(\vec{R}_A + \vec{r'}_i\right) \times \vec{F}_i \\
&= \sum \vec{R}_A \times \vec{F}_i + \sum \vec{r'}_i \times \vec{F}_i \\
&= \vec{R}_A \times \left(\sum \vec{F}_i\right) + \sum \vec{r'}_i \times \vec{F}_i \\
&= \vec{R}_A \times \vec{F} + \vec{\tau}_0
\end{aligned}$$

Torque also appears as two terms. Compare with

$$\begin{aligned}
\frac{d\vec{L}}{dt} &= M\vec{R}_A \times \frac{d\vec{V}_A}{dt} + \frac{d\vec{L}_0}{dt} \\
&= \vec{R}_A \times \vec{F} + \frac{d\vec{L}_0}{dt}
\end{aligned}$$

Rotational Dynamics

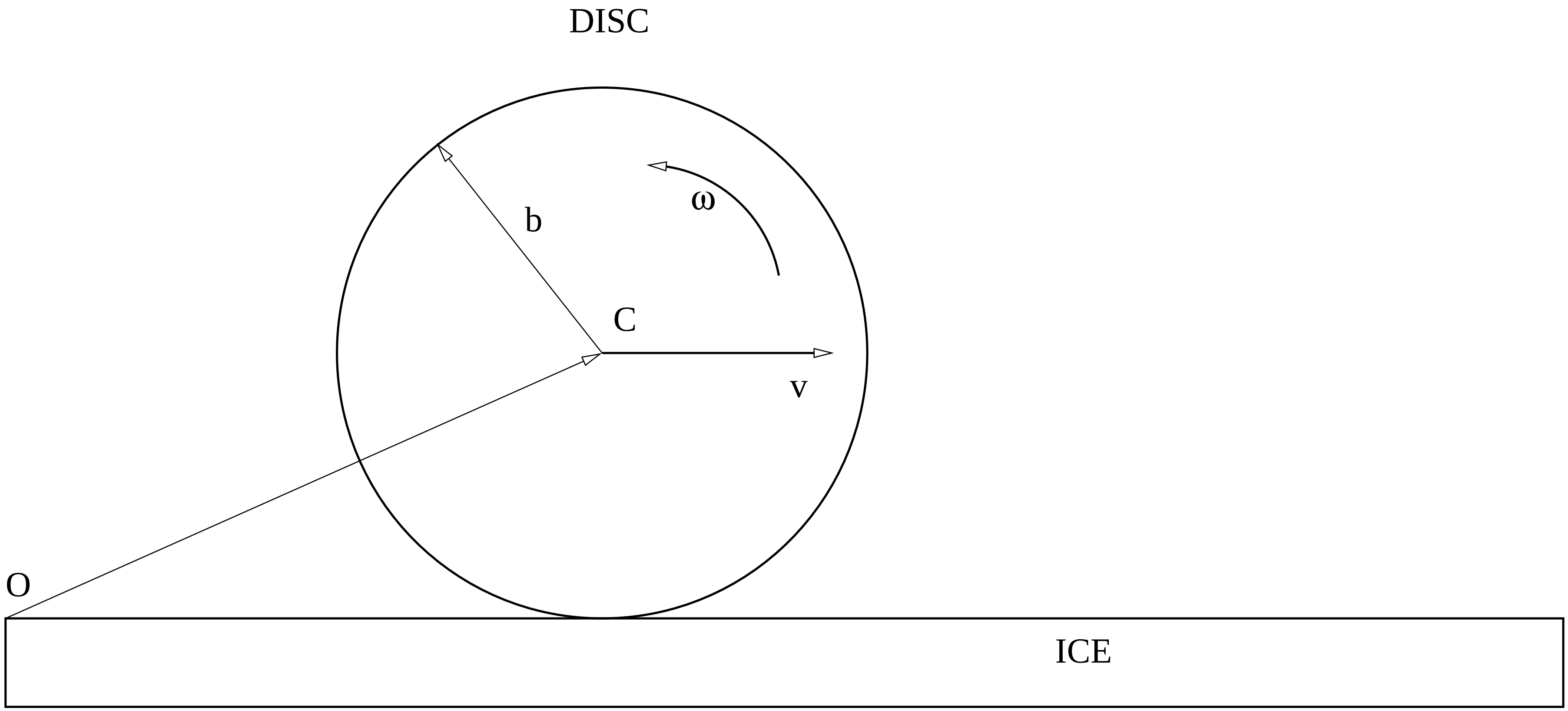
Dynamical Equations

$$\frac{d\vec{P}_{cm}}{dt} = F$$
$$\frac{d\vec{L}_0}{dt} = \vec{\tau}_0$$

If a body moves such that the axis of rotation moves parallel to a fixed axis then we need to consider only the z component of angular momentum.

$$\frac{d\vec{L}_{0z}}{dt} = \vec{\tau}_{0z}$$
$$I^0_{zz}\alpha = \tau_{0z}$$

Drum on Ice



Drum on Ice

- No net force

$$V_{cm} = \text{const}$$

- No Net torque about C

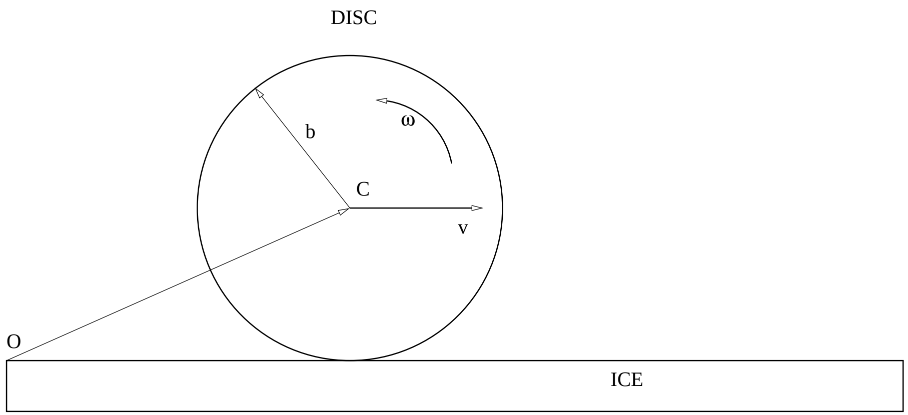
$$\omega = \text{const}$$

- Ang Mom about C

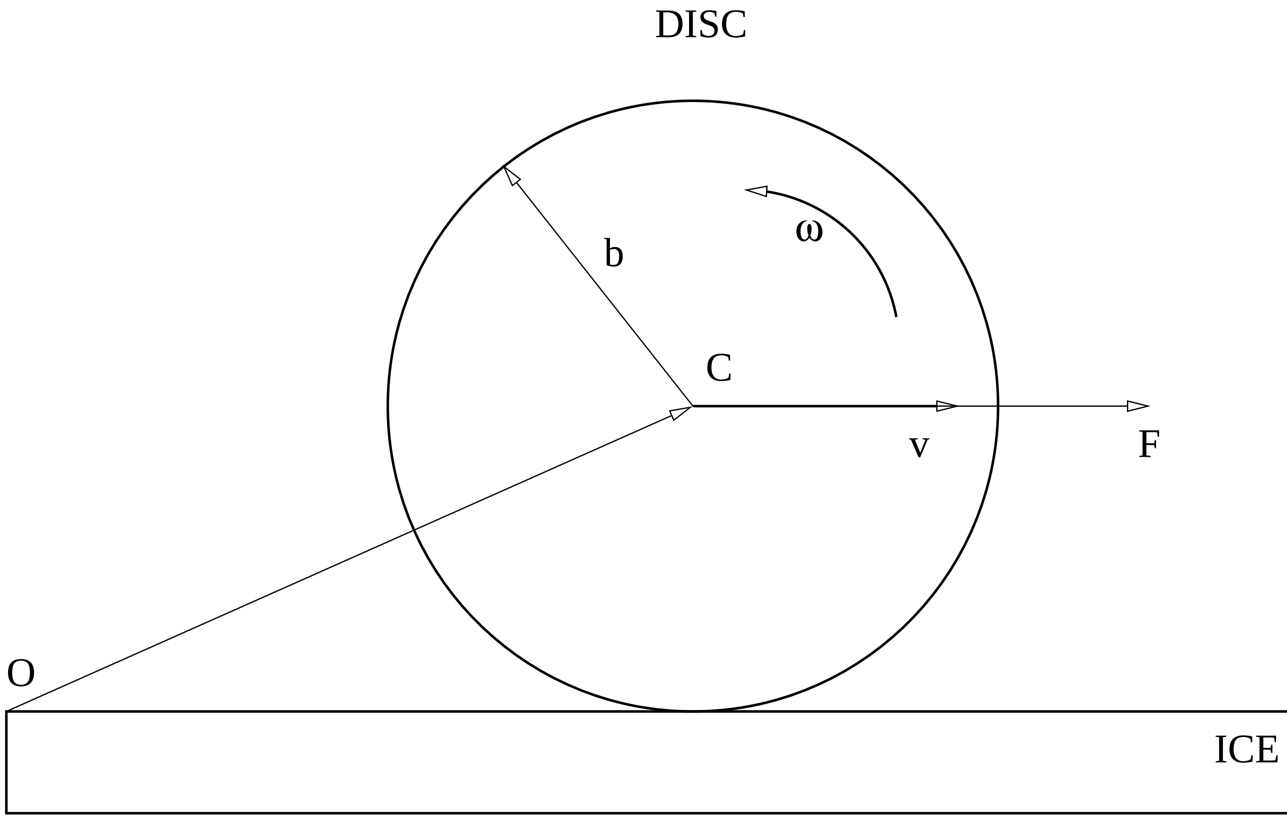
$$L_0 = Mb^2\omega/2$$

- Ang Mom about O

$$L = MbV_{cm} + L_0$$



Drum on Ice



Drum on Ice

- Net force F is constant

$$V_{cm} = (F/M)t$$

- No Net torque about C

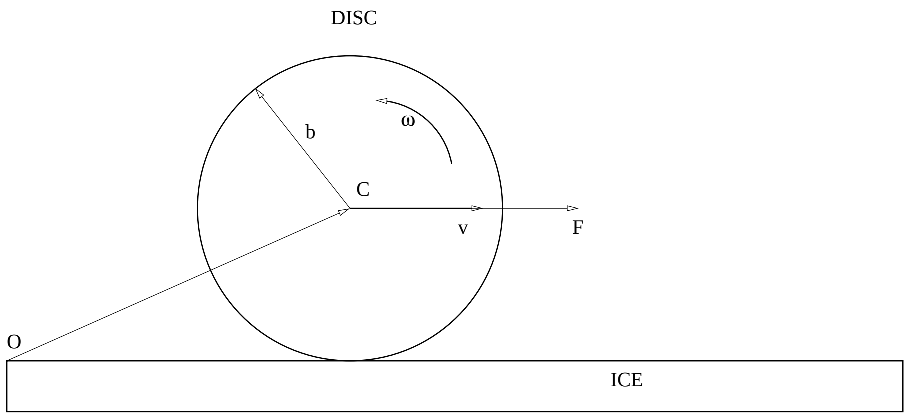
$$\omega = \text{const}$$

- Ang Mom about C

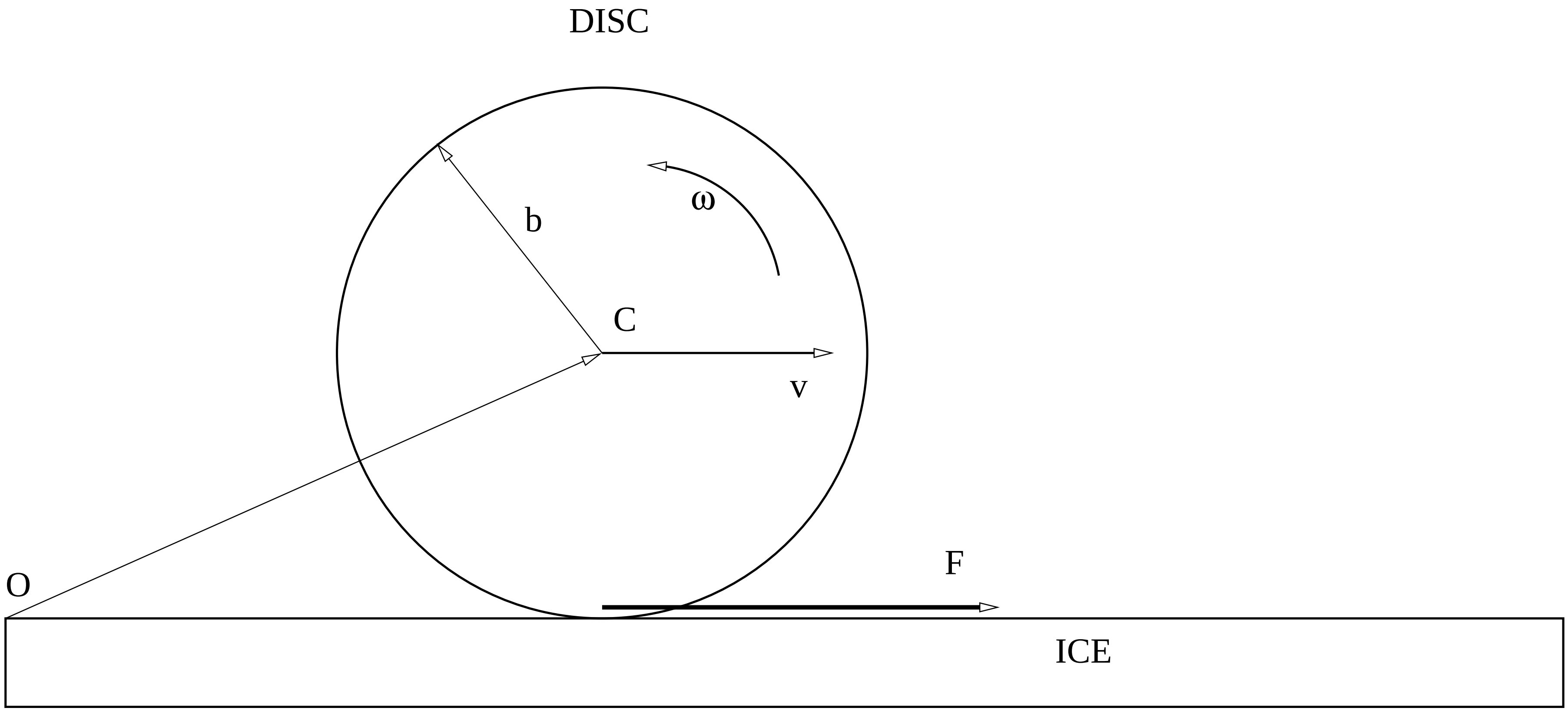
$$L_0 = Mb^2\omega/2$$

- Ang Mom about O

$$L = Fbt + L_0$$



Drum on Ice



Drum on Ice

- Net force F is constant

$$V_{cm} = (F/M)t$$

- Torque about C is bF

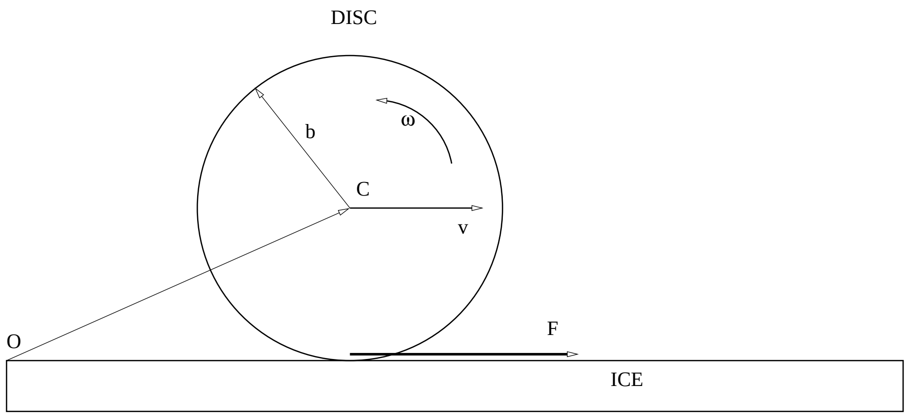
$$\omega = (2F/Mb)t$$

- Ang Mom about C

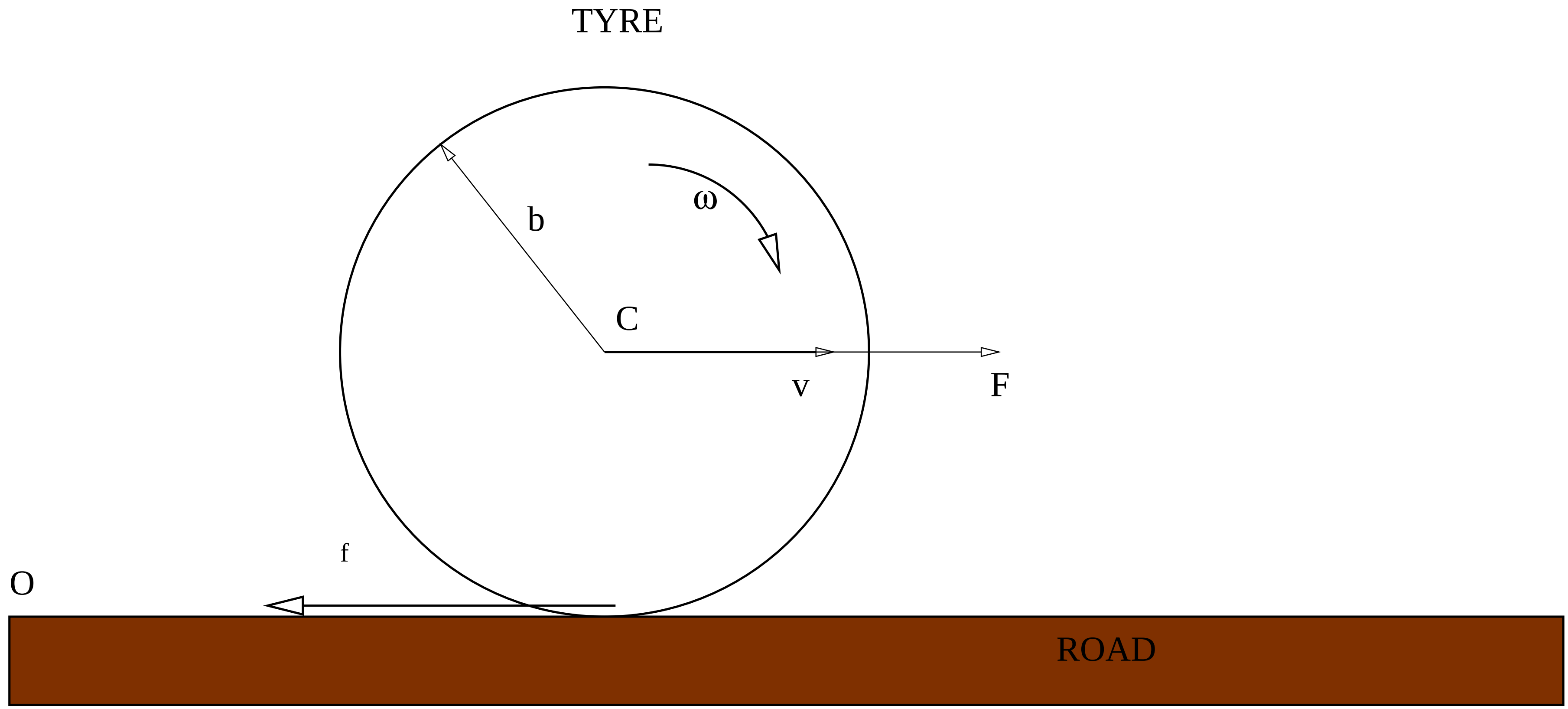
$$L_0 = bFt$$

- Ang Mom about O

$$L = Fbt + L_0$$

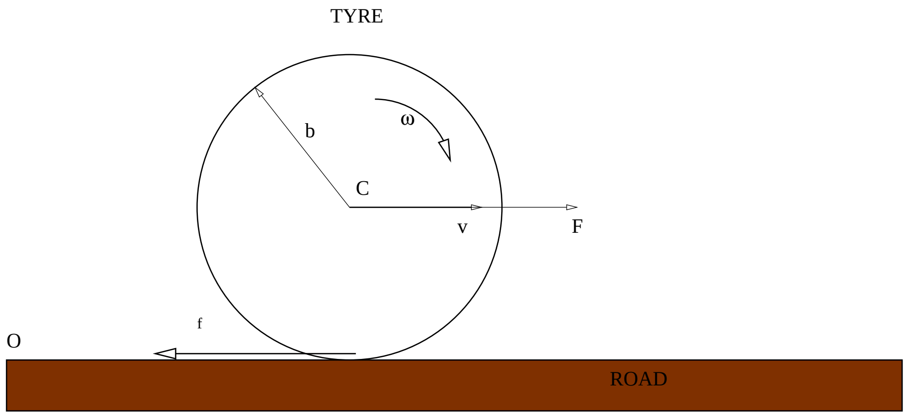


Tyre on Road



Tyre on Road

- Forces on the tyre
  - Force F
  - Frictional Force f
- Torque about C
  - $\tau_0 = bf$



Tyre on Road

The equations $(f < \mu N)$

$$\begin{aligned} Ma_{cm} &= F - f \\ I_0\alpha &= bf \end{aligned}$$

If $f < \mu N$ then there is no slipping, $a = b\alpha$.

$$\begin{aligned} Ma_{cm} &= F - I_0\alpha/b \\ Ma_{cm} + \frac{1}{2}Mb\alpha &= F \\ a_{cm} &= \frac{2F}{3M} \\ \alpha &= \frac{2F}{3Mb} \end{aligned}$$

and $f = F/3$. Clearly $F < 3\mu N$.

Tyre on Road

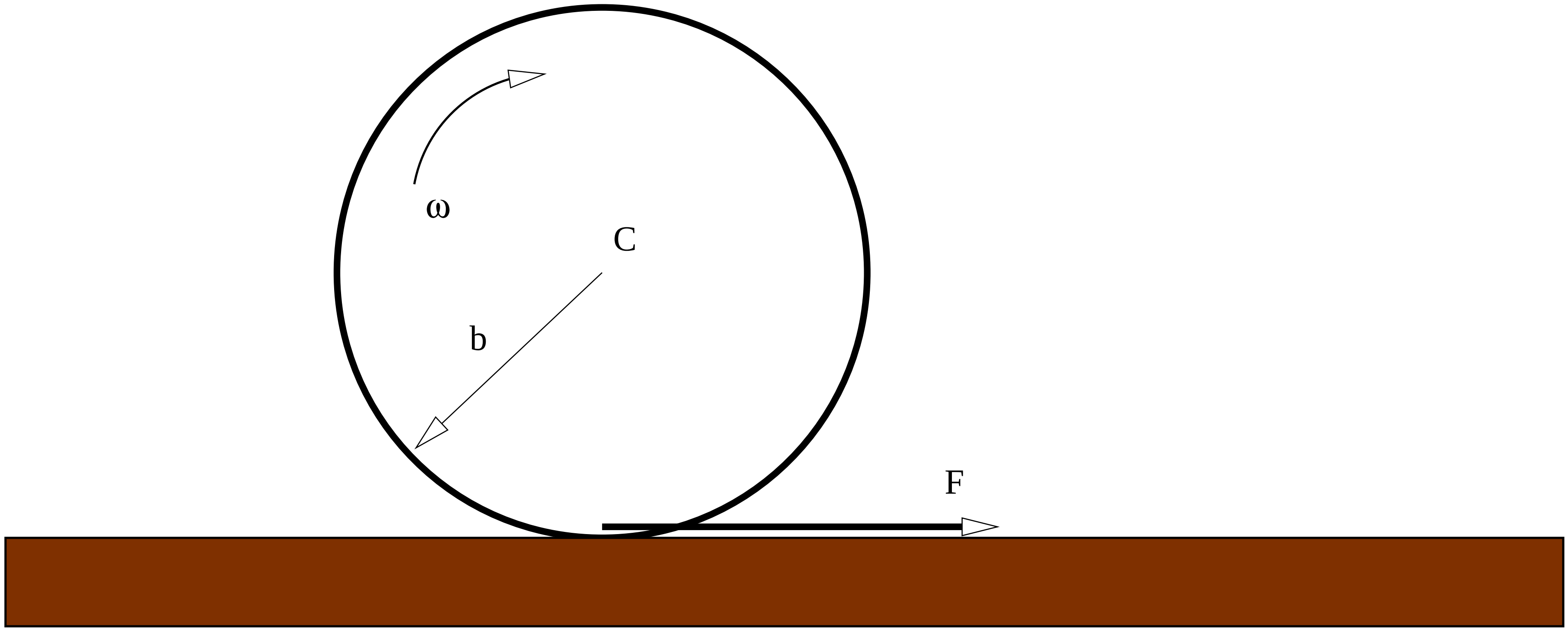
The equations $(f > \mu N)$ that is $F > 3\mu N$

$$\begin{aligned} Ma_{cm} &= F - \mu N \\ I_0\alpha &= b\mu N \end{aligned}$$

In this case tyre slides on the road, there is no relationship between α and a_{cm}

Screeching Start

A wheel that is spinning with speed ω_0 is placed on a rough table (Coefficient of friction μ).



Screeching Start

About C: $I\alpha = bF$. Here $F = \mu M g$.

$$\begin{aligned} \omega(t) &= -\omega_0 + \frac{bF}{I} t \\ &= -\omega_0 + 2\frac{\mu g}{b} t \end{aligned}$$

Also $M a_{cm} = F$. This implies $V_{cm}(t) = \mu g t$. The sliding motion continues till $V_{cm}(T) = b\omega(T)$.

$$T = \frac{\omega_o b}{\mu g}$$