Physics I

Lecture 3

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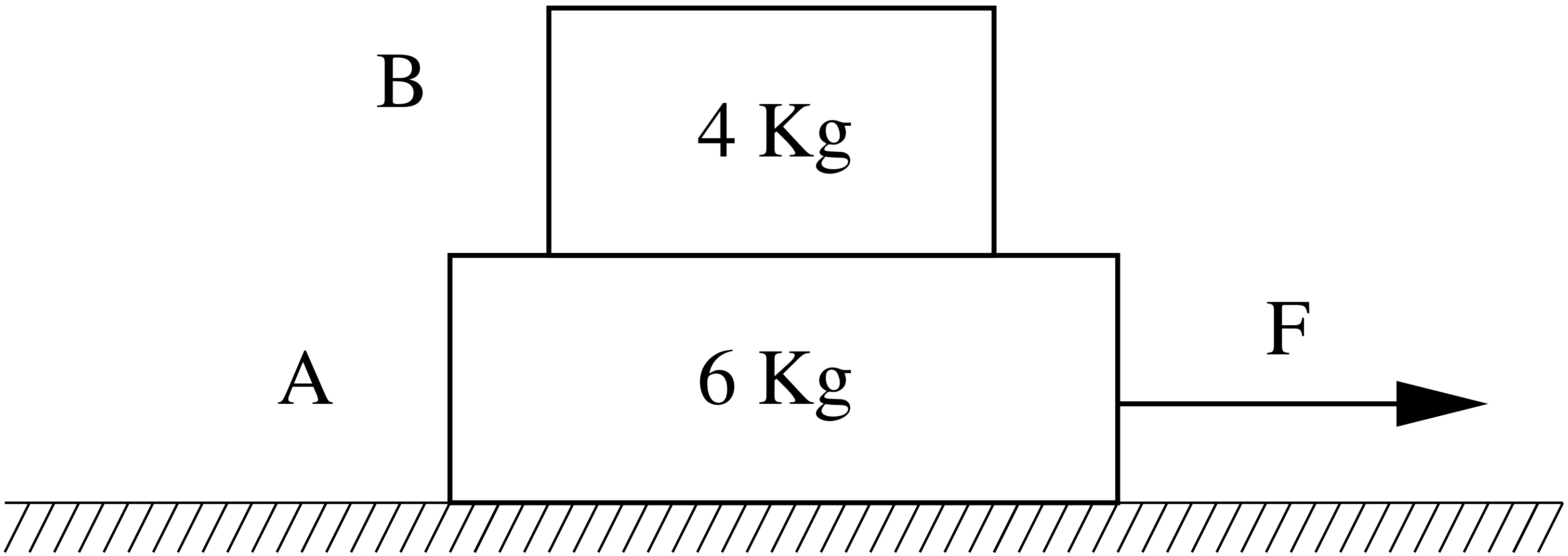
Systems of Particles

- Notion of Particles
- Systems of Particles
- Systems as Particles!
  - Atoms
  - Solids
  - Planetary Systems
  - Galaxies

Internal and External Forces



The System consists of Blocks A and B
Environment has table and earth.

Internal and External Forces

- System:

  Blocks A and B

- Environment:

  Hand, Table and Earth

- Internal Forces:

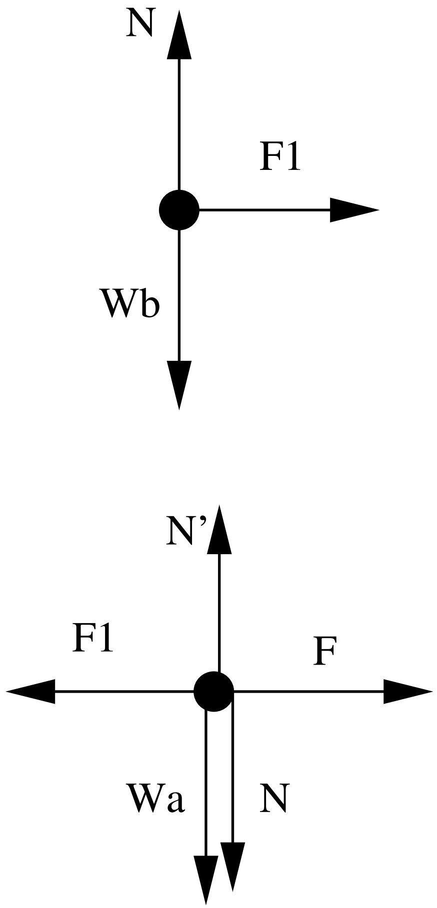
  On A: N and F_1

  On B: $-N$ and $-F_1$

- External Forces:

  On A: W_A, N', and F

  On B: W_B

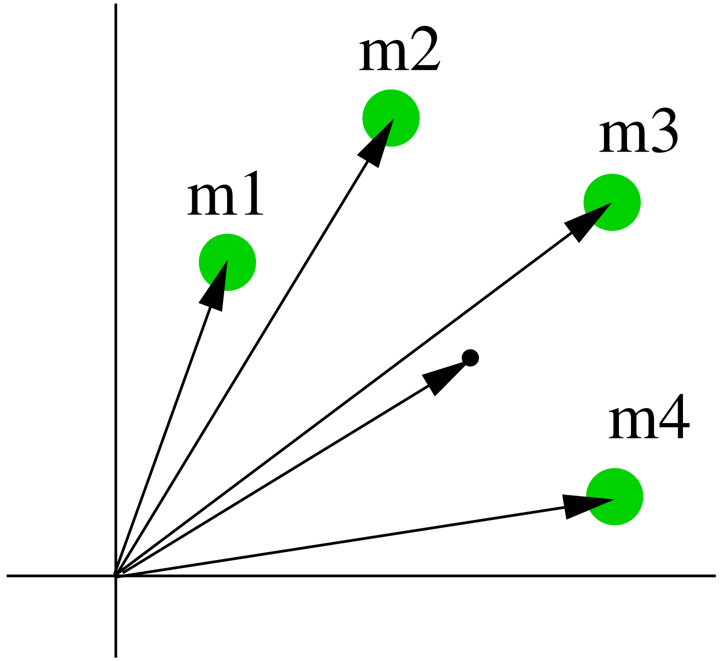


Center of Mass

- A system has n particles with masses and positions given by

$$m_1,\ m_2, \ldots,\ m_n$$

$$\mathbf{r}_1,\ \mathbf{r}_2, \ldots,\ \mathbf{r}_n$$



- Define a **Center of Mass** as

$$\mathbf{R}_{CM} = \frac{1}{M}\left(\sum_i m_i \mathbf{r}_i\right)$$

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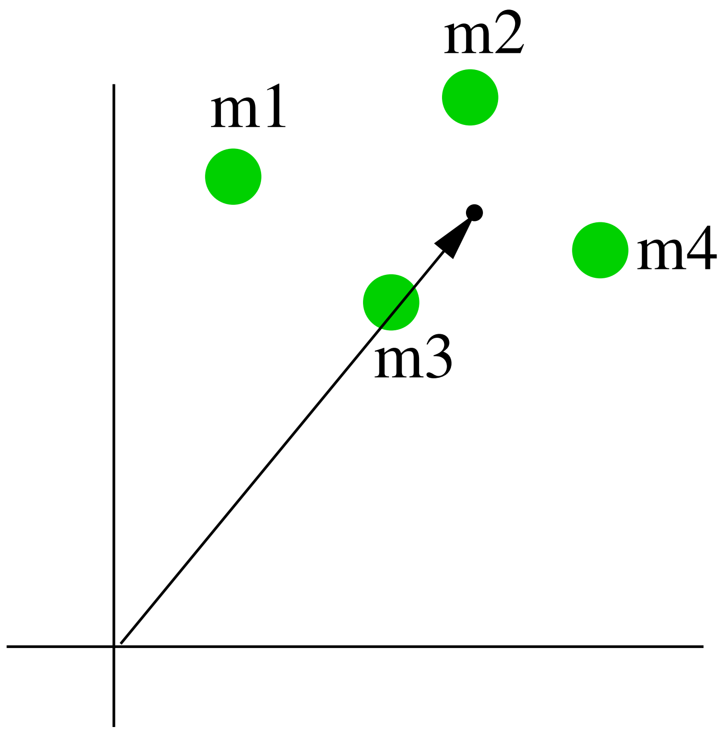


Example

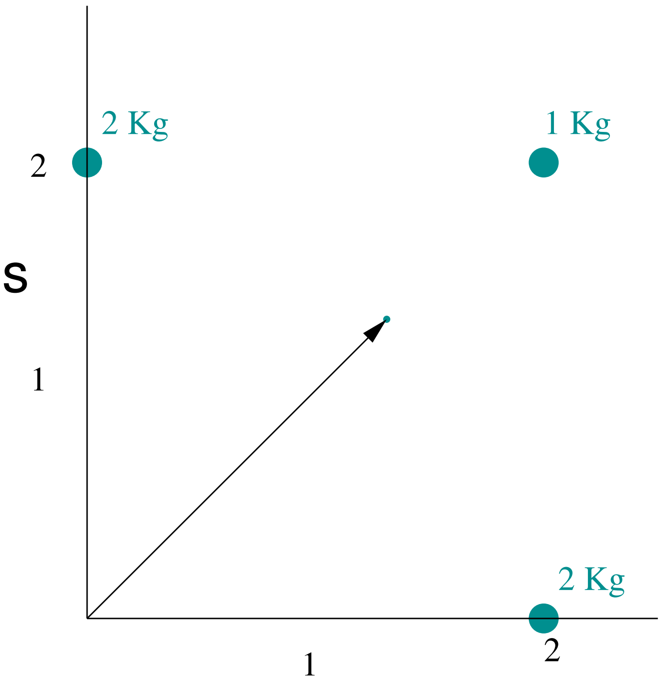
Three masses are kept in a plane as shown in the figure.

$m_1 = 2$ Kg, $m_2 = 2$ Kg and $m_3 = 1$ Kg

$r_1 = 2\mathbf{j}$, $r_2 = 2\mathbf{i}$ and $r_3 = 2\mathbf{i} + 2\mathbf{j}$

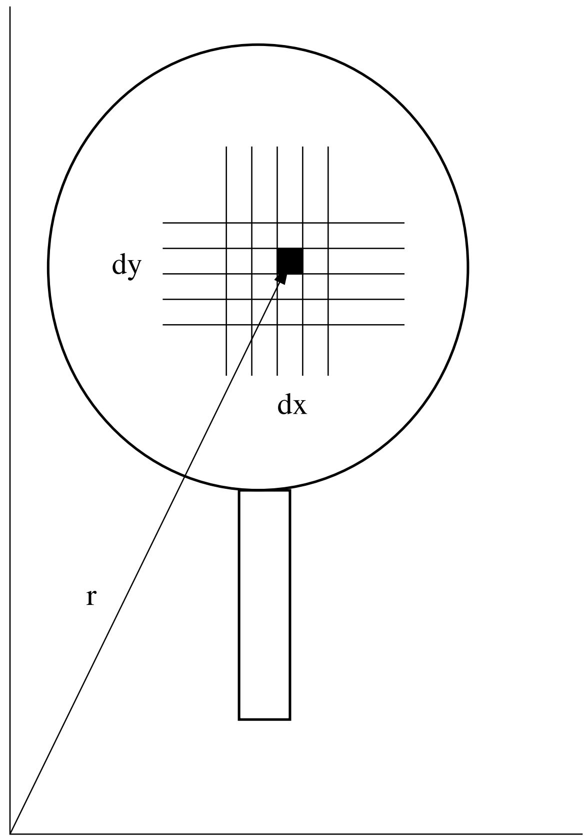
Total Mass is 5 Kg. Then Center of Mass is given by

$$\mathbf{R}_{cm} = \frac{1}{5}(2\mathbf{r_1} + 2\mathbf{r_2} + \mathbf{r_3}) = \frac{6}{5}(\mathbf{i} + \mathbf{j})$$



Planar Continuous Bodies

The density is given by $\rho(\mathbf{r})$. An element at $\mathbf{r}$ and of area $dxdy$ has a mass $dm = \rho(\mathbf{r})dxdy$.



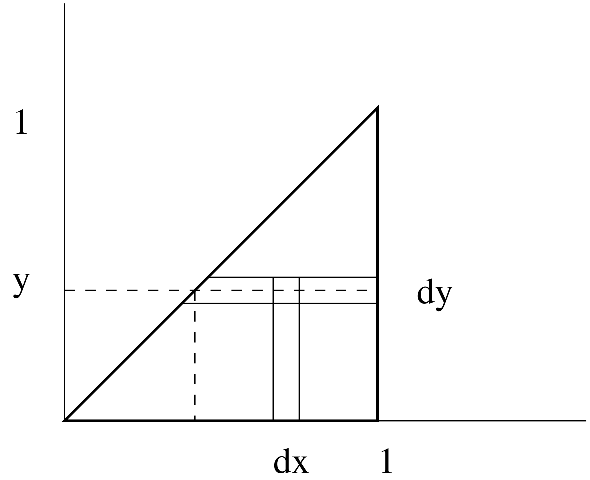
$$\mathbf{R}_{cm} = \frac{1}{M}\sum \mathbf{r}dm = \frac{1}{M}\int \mathbf{r}\rho(\mathbf{r})dxdy$$

And

(1) $$M = \int \rho(\mathbf{r})dxdy$$

Example

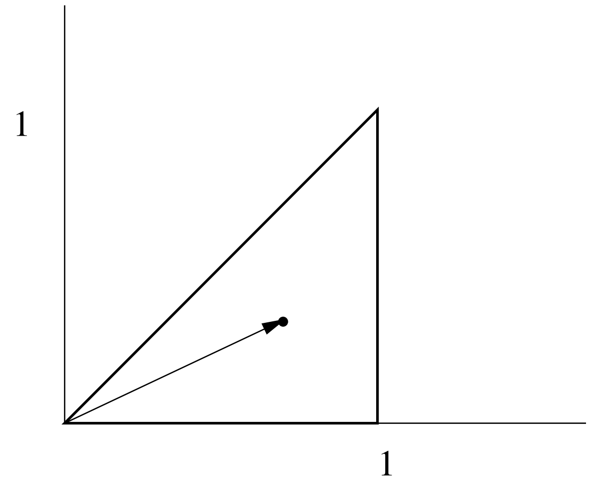
A triangular sheet with uniform density ρ_0 is placed as shown in the figure. Cleary $M = \rho_0/2$



$$\mathbf{R}_{cm} = \frac{1}{M}\int \rho_0(x\mathbf{i} + y\mathbf{j})dxdy$$
$$= 2\left(\int xdxdy\right)\mathbf{i} + 2\left(\int ydxdy\right)\mathbf{j}$$

Example

$$\int_0^1 dy \int_y^1 x\, dx = \int_0^1 dy \left(\frac{1}{2} - \frac{y^2}{2}\right)$$
$$= \frac{2}{3}$$



This gives

$$\mathbf{R}_{cm} = \frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j}$$

Equations of Motion

Now, by definition,

$$
\begin{aligned}
M\mathbf{R}_{cm} &= \sum m_i \mathbf{r}_i \\
M\ddot{\mathbf{R}}_{cm} &= \sum m_i \ddot{\mathbf{r}}_i
\end{aligned}
$$

But for each particle, labled by i,

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i = \mathbf{F}_i^{ext} + \mathbf{F}_i^{int}$$

Hence,

$$M\ddot{\mathbf{R}}_{cm} = \sum \mathbf{F}_i^{ext} + \sum \mathbf{F}_i^{int}$$

Equations of Motion

But by Newton's third law, all internal forces appear in pairs and are equal and opposite. Thus in the following summation all internal forces cancel each other out.

$$\sum_i \mathbf{F}_i^{int} = 0$$

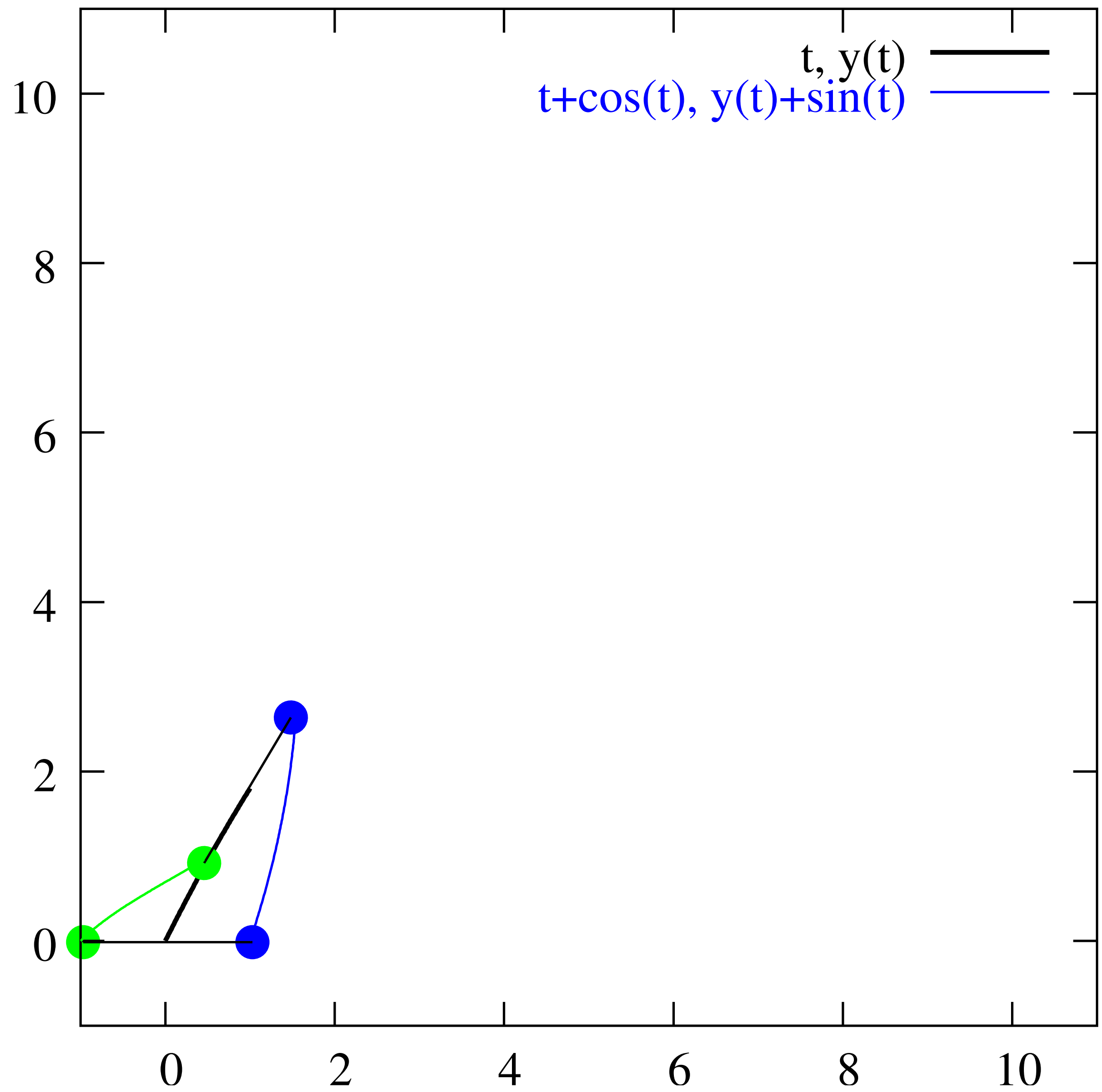
Thus Equation of Motion for Center of Mass of any system

$$M\ddot{\mathbf{R}}_{CM} = \mathbf{F}^{ext} = \sum_i \mathbf{F}_i^{ext}$$

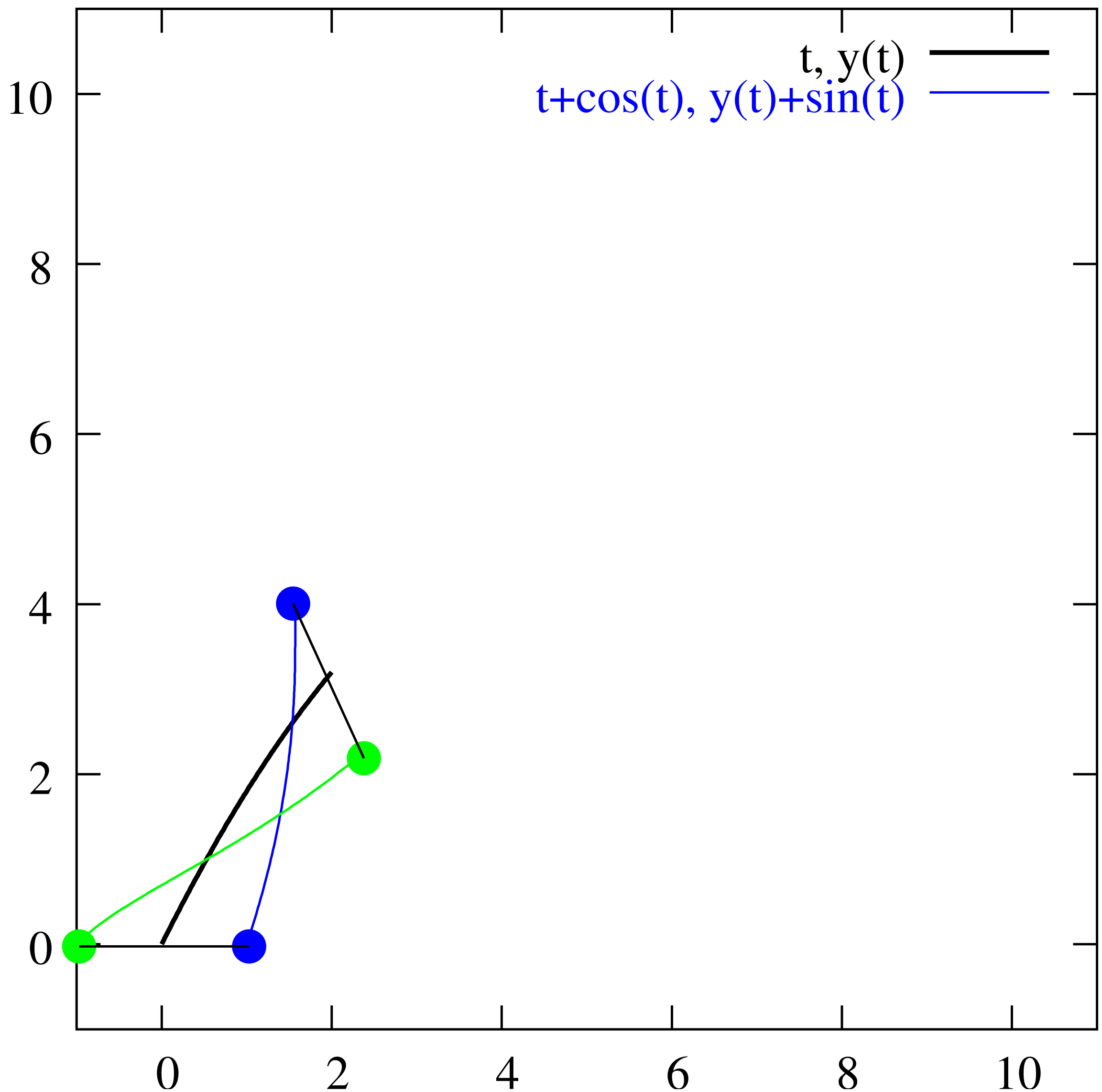
Result

One point $\mathbf{R}_{cm}$ traces the same motion as that of a single particle of mass M under the influence of a force $\mathbf{F}^{ext}$

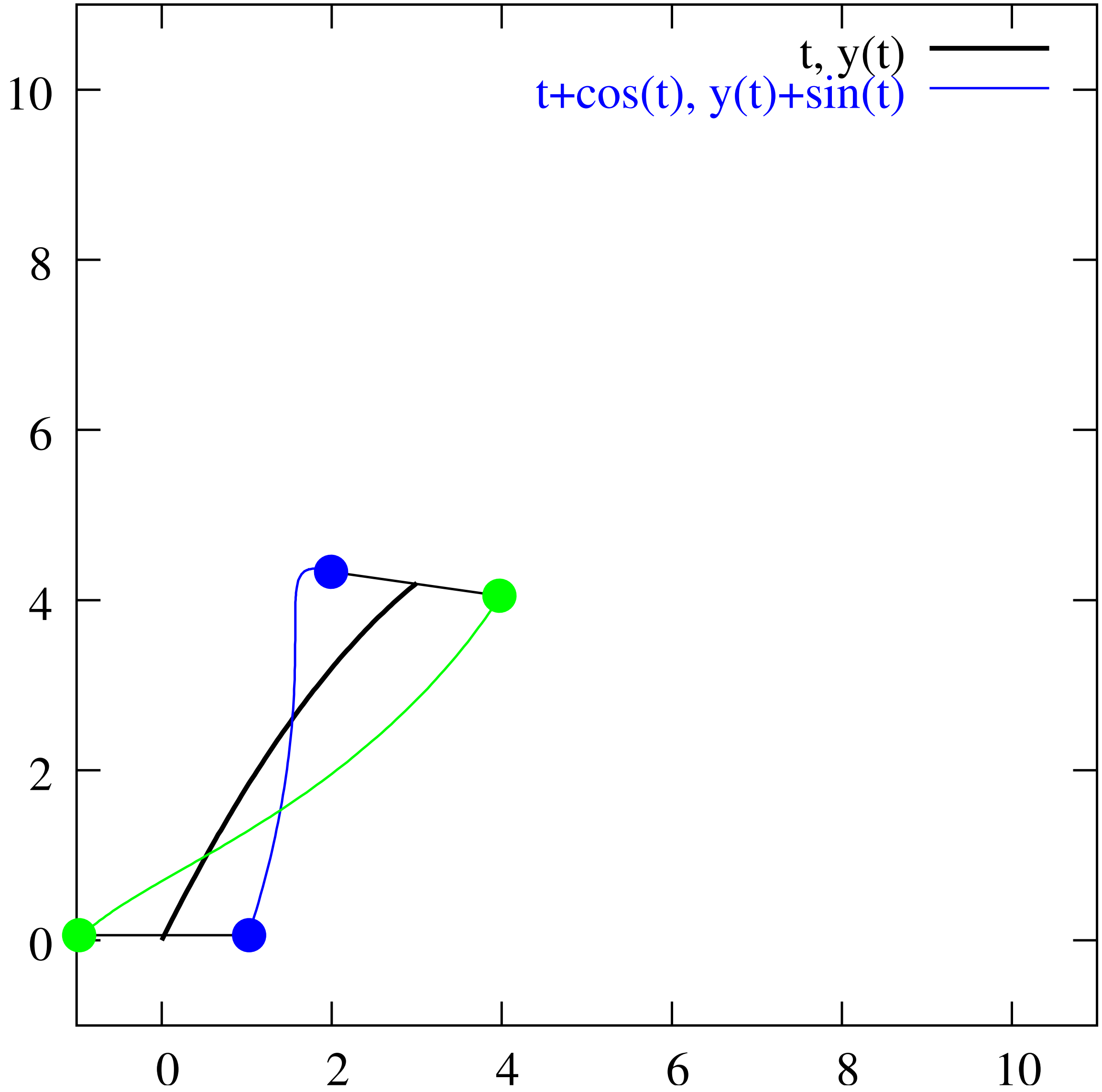
example



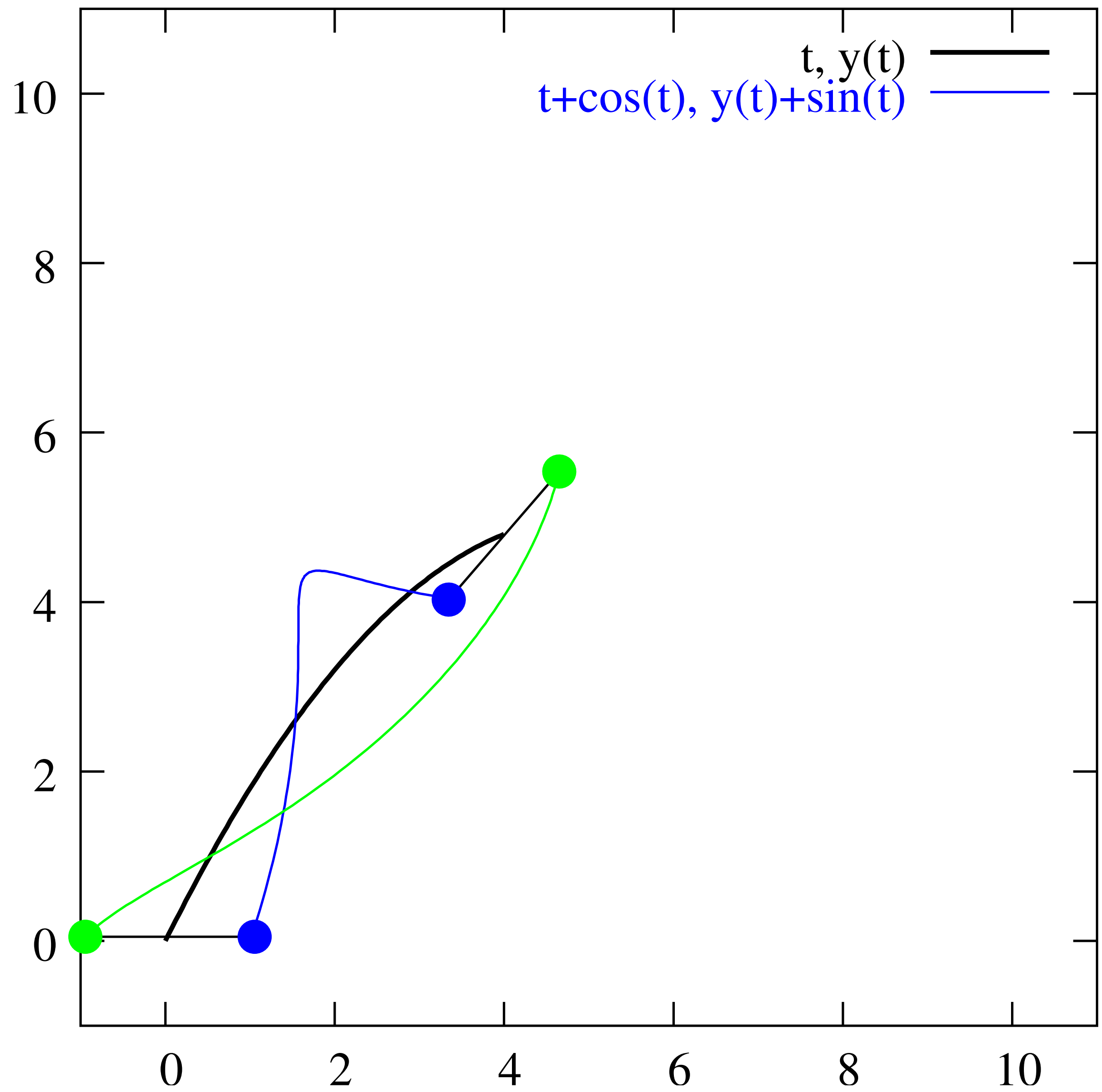
example



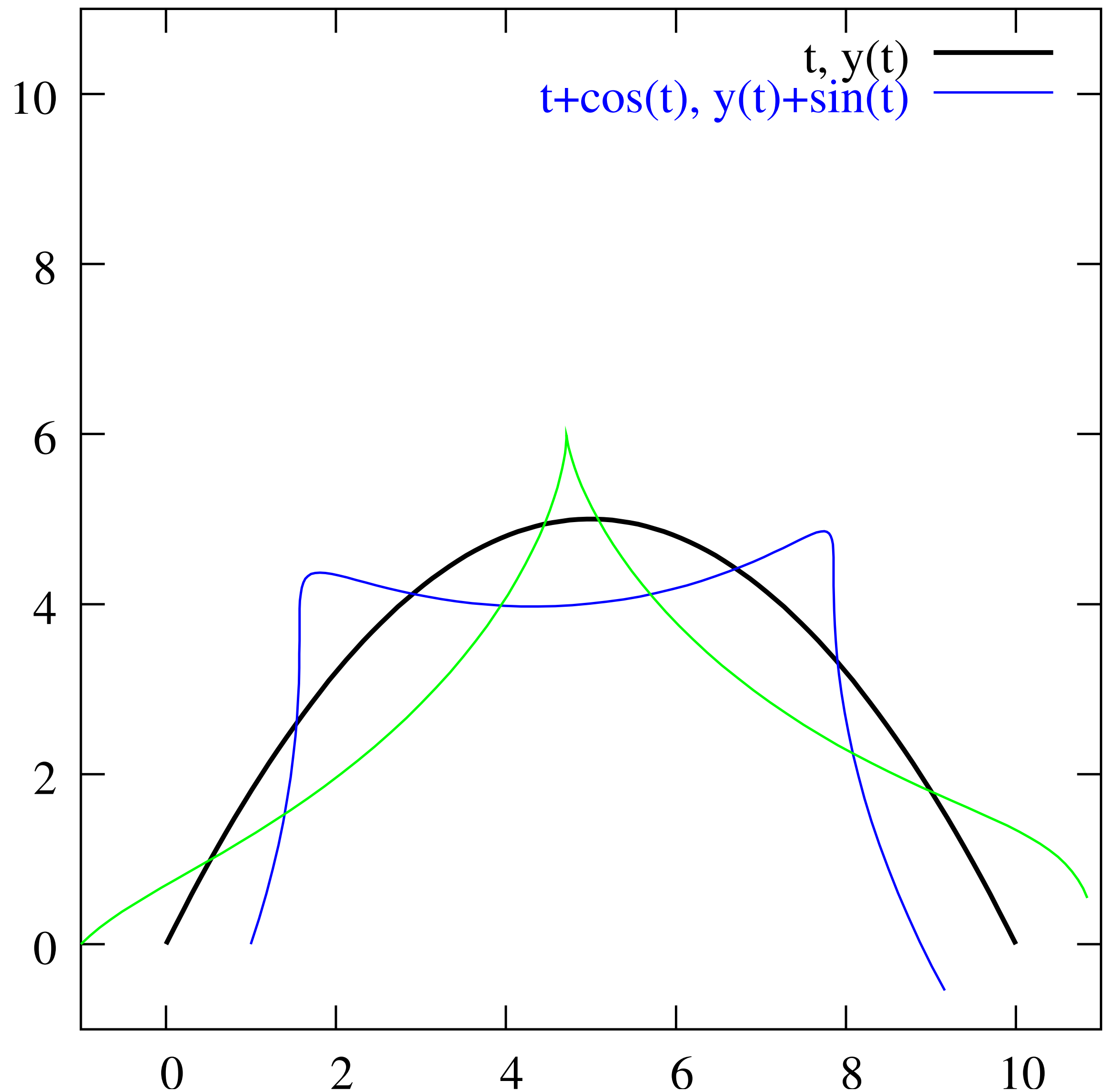
example



example



example



Momentum

Momentum of a particle is defined as $m\mathbf{v}$.

For a system of particles, net momentum is defined as

$$\begin{aligned}\mathbf{P} &= \sum m_i \dot{\mathbf{r}}_i \\ &= M\dot{R}_{cm} \\ &= \mathbf{P}_{cm}\end{aligned}$$

The equation of motion can now be written as

$$\frac{d\mathbf{P}_{cm}}{dt} = M\ddot{\mathbf{R}}_{cm} = F^{ext}$$

Conservation of Momentum

The momentum of a system is conserved if the net external force on the system is zero