Physics I

Lecture 14

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Simultaneity

Why did we think that the ordering of events was absolute?

A 500 m train passing a station with speed 100 m/s (360 Km/hr).

The driver honks twice in succession at the interval of 5 s. The station-master says the two honks were separated by 500 m.

The driver and guard of signal the station-master at the same time by their watches. According to the station master, the guard signalled first by a time gap of

$$
\begin{aligned}
t_2 - t_1 &= \gamma \frac{v}{c^2}(x'_2 - x'_1) \\
&= 5.6 \times 10^{-13}\mathrm{s}
\end{aligned}
$$

Time Dilation

Charged pions ($m_{\pi^+} = 273 m_e$) decay into muons ($m_{\mu^+} = 273 m_e$) and muon-neutrino.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

- Half life of pions is $\tau_0 = 1.77 \times 10^{-8}$ sec measured in lab when pions were at rest.
- beam of pions with speed $v = 0.99c$
- intensity falls to half at a distance of 39 m.
- But $v\tau_0 = 5.3$ m!
- half life in lab frame now $\tau = \gamma\tau_0 = 1.3 \times 10^-7$ sec.
- Correct distance $v\tau = 39$ m

Time Dilation: Cosmic Rays

Addition of Velocities

Consider usual S and S' frames. A particle has a velocity u' in x' direction wrt S'. What is its velocity as seen by S?

In S' frame $u' = dx'/dt'$ and in S frame $u = dx/dt$

By Lorentz Transformation

$$\begin{aligned} x &= \gamma(x' + vt') \\ t &= \gamma(t' + (v/c^2)x') \end{aligned}$$

Then,

$$\begin{aligned} dx &= \gamma(dx' + vdt') \\ dt &= \gamma(dt' + (v/c^2)dx') \end{aligned}$$

and

$$u = \frac{dx}{dt} = \frac{(dx' + vdt')}{(dt' + (v/c^2)dx')}$$

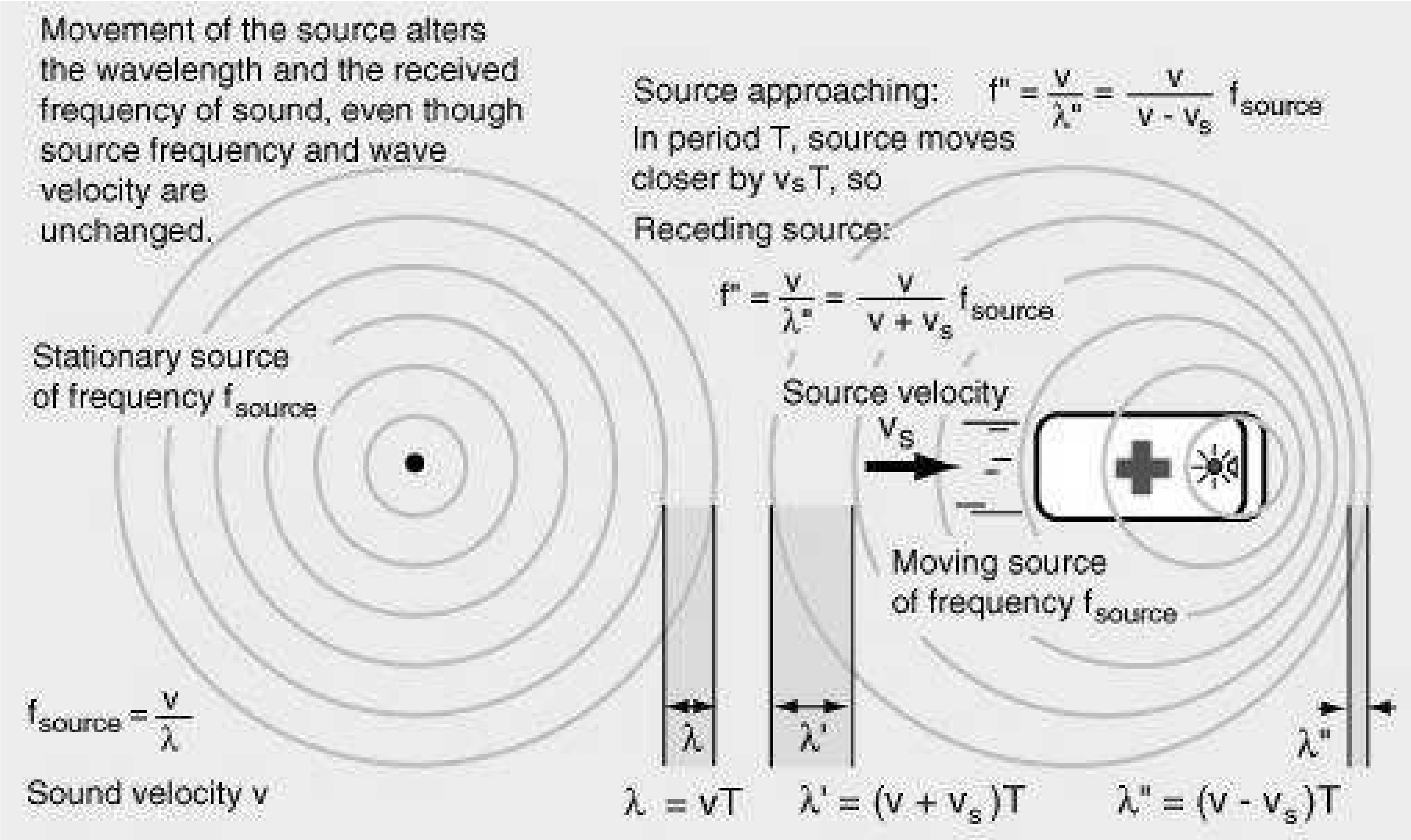
Addition of Velocities

$$u_x = \frac{u'_x + v}{1 + u'v/c^2}$$

- $u' < c$ and $v < c \Rightarrow u < c$.
- $u' = c$ or $v = c \Rightarrow u = c$.
- consistent with constancy of speed of light.
- velocities in y direction are also affected

$$u_y = \frac{u'_y}{\gamma(1 + u'v/c^2)}$$

Doppler Effect



Doppler Effect

- A microphone is stationery in the air
- A speaker is moving towards the microphone with speed v.
- The frequency of the speaker is ν. It emits wave-crests every $T = 1/\nu$ sec.
- First crest arrives at microphone $T_1 = L/c$, (c speed of sound).
- Second crest arrives at microphone $T_2 = T + (L - vT)/c$
- Frequency of detection $\nu_d = 1/(T_2 - T_1) = c\nu/(c - v)$ and is greater than nu.

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Relativistic Doppler Effect

- A observer sees a light source is moving towards him with speed v.
- The frequency of the light is ν_0 when stationery. It emits wave-crests every $T_0 = 1/\nu$ sec when stationery.
- Observer sees the wave-crests emitted ever γT_0 sec.
- First crest arrives at microphone $T_1 = L/c$, (c speed of light).
- Second crest arrives at microphone $T_2 = \gamma T + (L - v\gamma T)/c$
- Frequency of detection

$$\nu_d = \nu\sqrt{\frac{c+v}{c-v}}$$

and is greater than nu.

Relativity and Newton's Laws

Classical dynamical laws are not invariant in relativity.

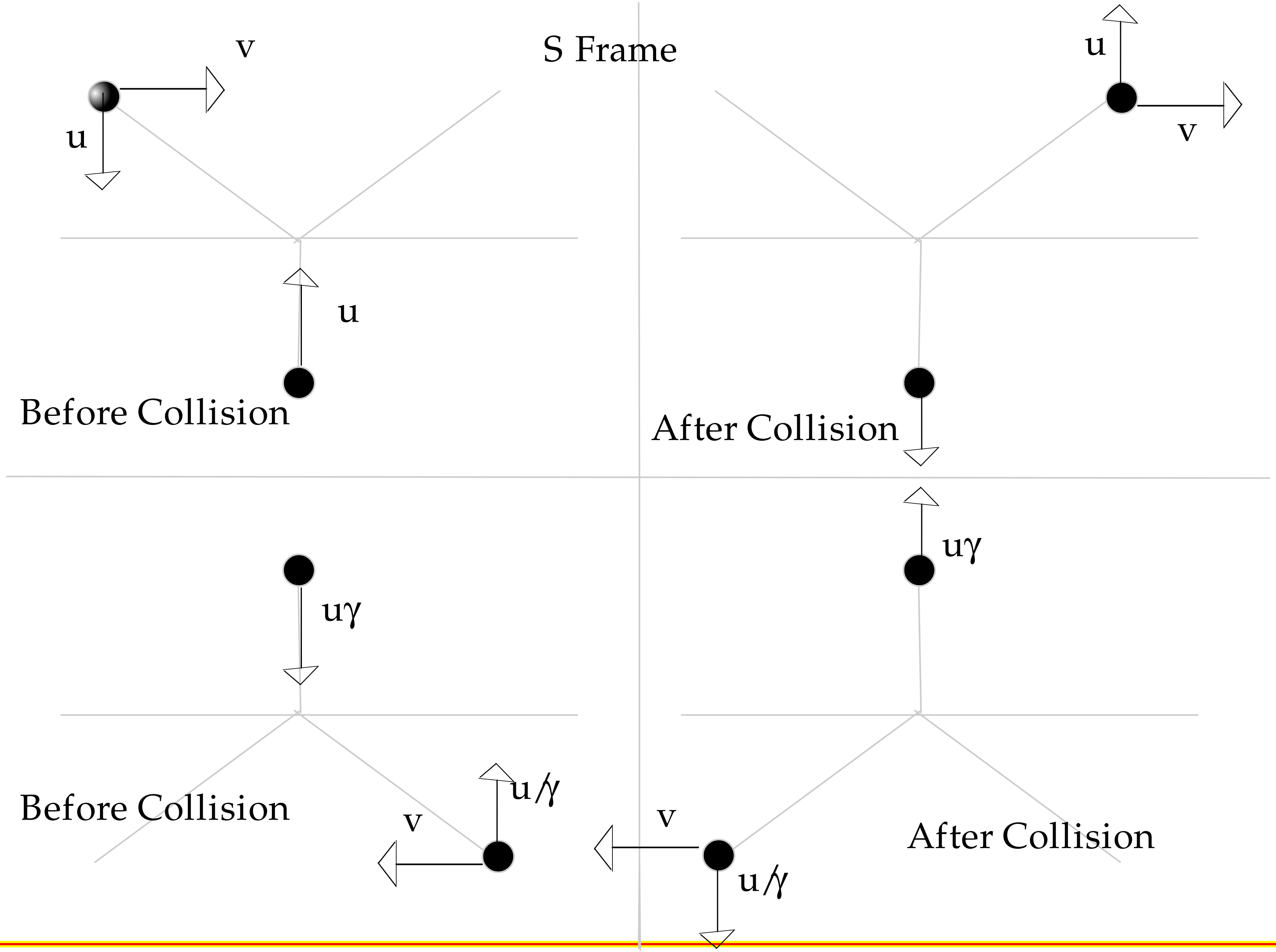
Momentum is defined as $P = mv$ for a particle moving with speed v.

Momentum conservation law states that

If there is no net external force on a system, net momentum of the system is co

We can consider a collision problem for this.

Collision Problem



Collision Problem

S Frame

	Before	After
u_{Ay}	$-u$	u
u_{By}	u	$-u$
Net Momentum	0	0

S' Frame

	Before	After
u_{Ay}	$-\gamma u$	γu
u_{By}	u/γ	$-u\gamma$
Net Momentum	$mu(-\gamma + 1/\gamma)$	$mu(\gamma - 1/\gamma)$

Relativistic Dynamics

- Why do we have to modify Newtonian Mechanics?
- Modify Newtonian Mechanics? Which Laws?
- $F = dp/dt$?
- Conservation Laws! Momentum and Energy.

Momentum

Momentum is defined as

$$P = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

- Momentum Conservation Law is valid in all inertial frames.(Verify in tutorial)
- If we interpret this as mass $\times$ speed, then mass becomes

$$m = \frac{m_0}{\sqrt{1 - u^2/c^2}}$$

  where m_0 is called rest mass.
- In 1907, Bucherer confirmed the dependence of mass on speed of electron.

Kinetic Energy

A particle which was at rest is accelerated to a speed u. We will define the kinetic energy of the particle as

$$\begin{aligned} K &= \int_a^b \frac{dp}{dt} \cdot dr \\ &= \int_a^b \frac{d}{dt}\left(\frac{m_0 u}{\sqrt{1-u^2/c^2}}\right) \cdot u dt \\ &= \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} - m_0 c^2 \\ &= mc^2 - m_0 c^2 \end{aligned}$$

Total Energy

Here Einstein boldly proposes that the total energy of a particle is kinetic energy + rest mass energy which is given by m_0c^2 and the famous formula

$$E = K + m_0c^2 = mc^2$$