Physics I

Lecture 10

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Cross-Roads

Why do we like the principle of relativity so much?

	Mechanics	Electrodynamics	To Do
GR	Yes	No	Look for Ether
GR	Yes	Yes	Failure of Electrodynamics
New?	Yes	Yes	New laws for Mechanics

Other Theories (with and without relativity) did not agree with all experiments that were thought of!

Special Theory of Relativity

Postulates of Special Theory of Relativity

1. The laws of physics are same in all inertial systems. No preferred inertial system exists.
2. The speed of light in free space has the same value in all inertial frames.

Simultaneity

In a galactic quiz competition, Alex and Bob participated in a buzzer round. Quizmaster has to find the first one to hit the buzzer.

- If Alex and Bob were nearby (at same location), we could record the times at which each one of them hit the buzzer and decide the order.
- If they were in different locations (Say, Front and back of a huge spaceship) then we have to use two clocks. But then, the clocks must be synchronized.

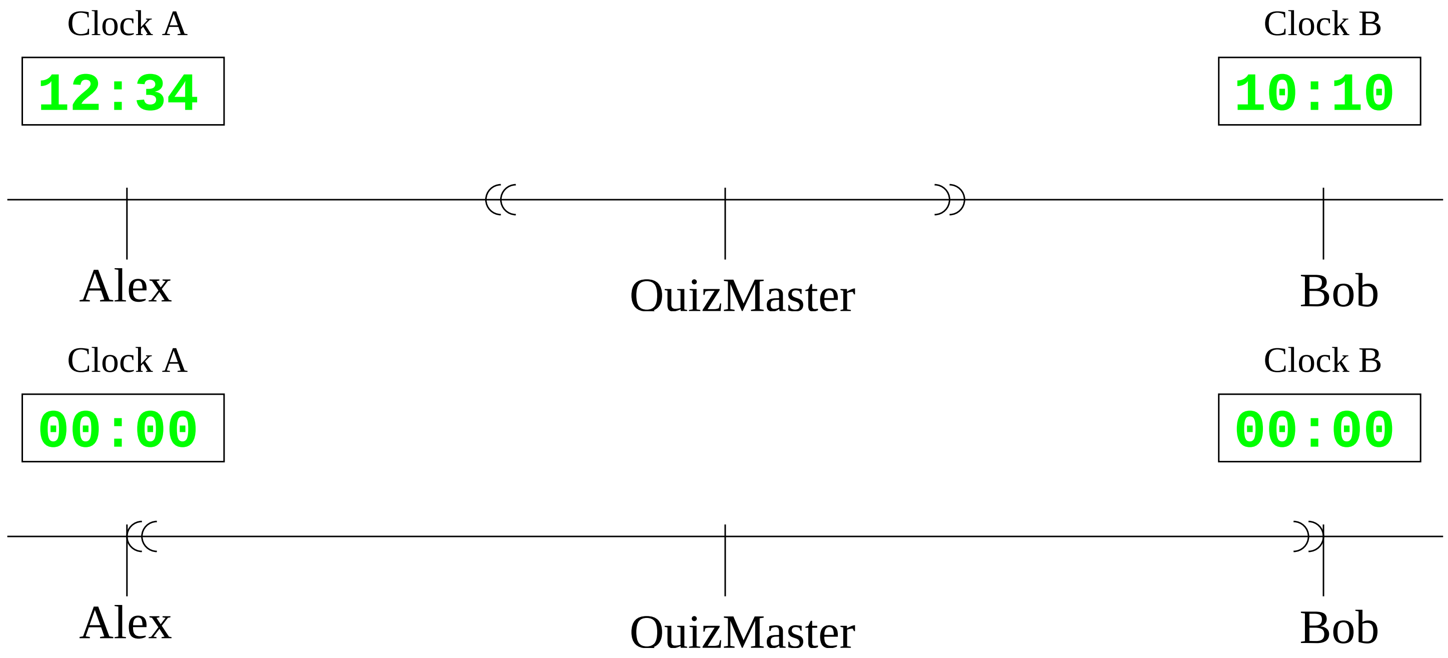
If we have synchronized clocks everywhere, we can chronologically order the events occurring at different places.

Synchronizing Clocks

There are several ways in which we can synchronize the clocks. Here is one procedure.

- First place the clock in their locations (A and B).
- Find the midpoint O.
- Flash a LIGHT signal.
- When A and B receive the light, they set their clocks to predefined time, say 00.

Synchronizing Clocks



This is based on simple fact that the light has same speed in both directions, so it would take exactly same time to reach A and B.

Once we know how to synchronize two clocks, we can sync all clocks.

Simultaneity

In our example, first, we sync the clocks close to Alex and Bob.

Then, Alex hits the buzzer 1300 according to the clock kept close to him

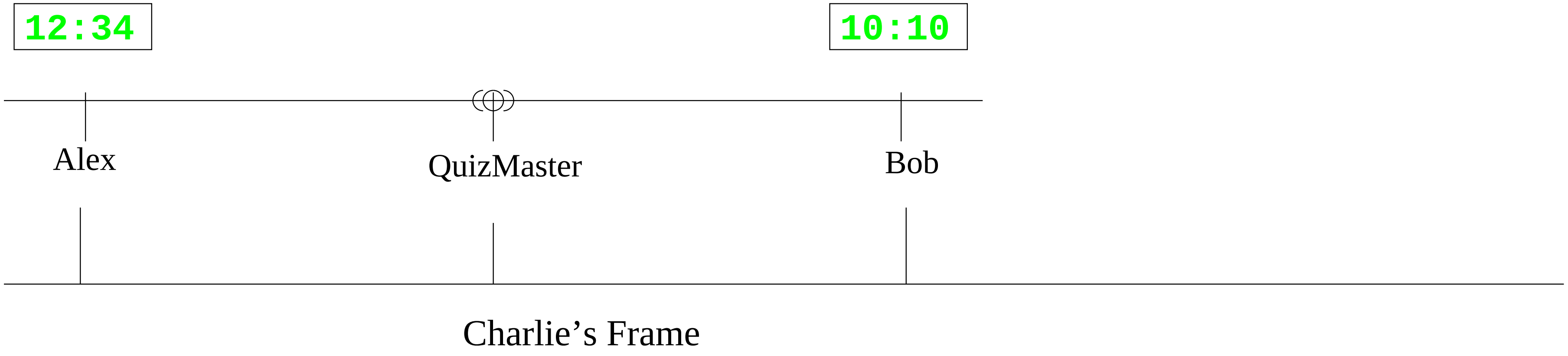
and Bob also hits the buzzer at 1300, but according to the clock with him.

Conclusion: Alex and Bob hit the buzzers *simultaneously*

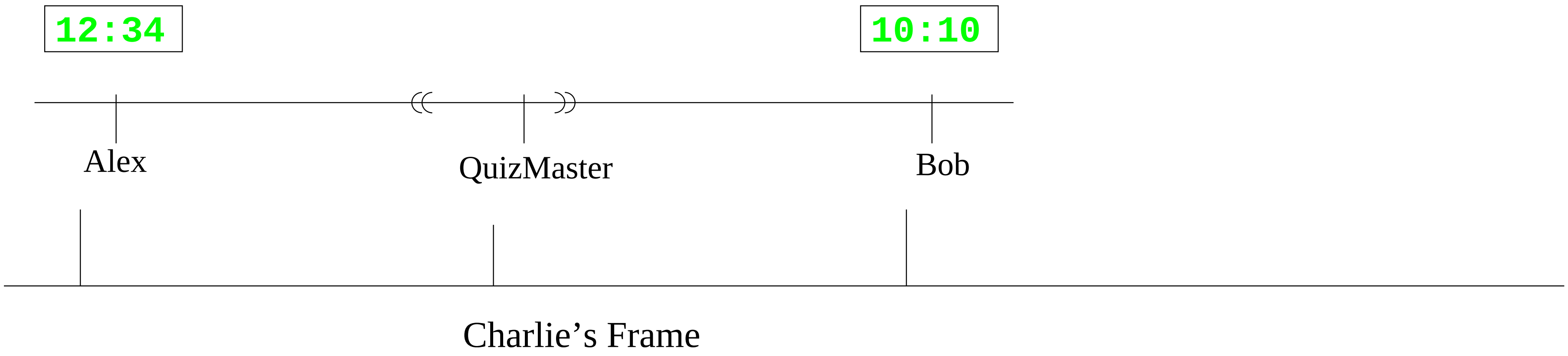
Simultaneity

Now, From another spaceship, Charlie is looks at Quizmaster's spaceship and finds that they are moving to the right with speed v. Charlie looks at their synchronization procedure with some curiosity.

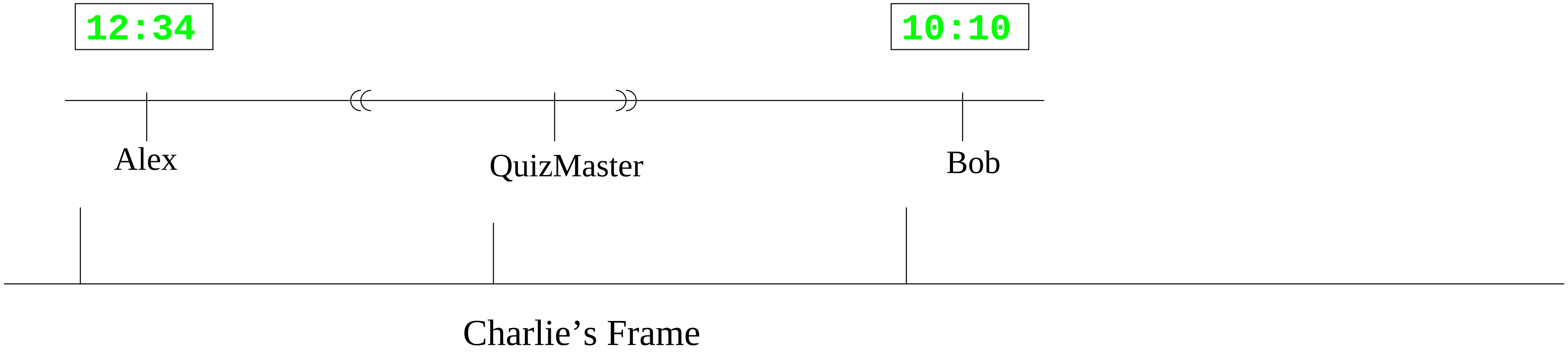
Charlie's View



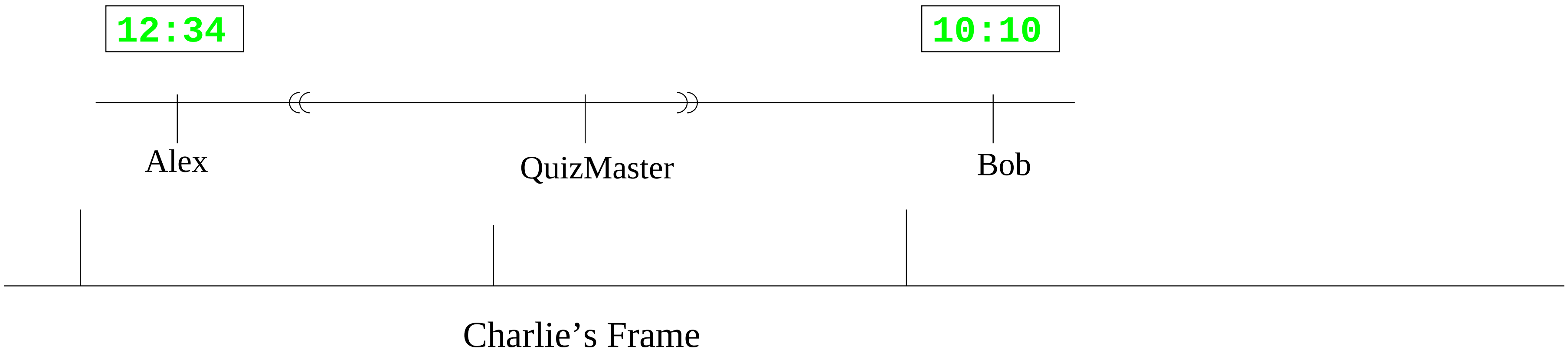
Charlie's View



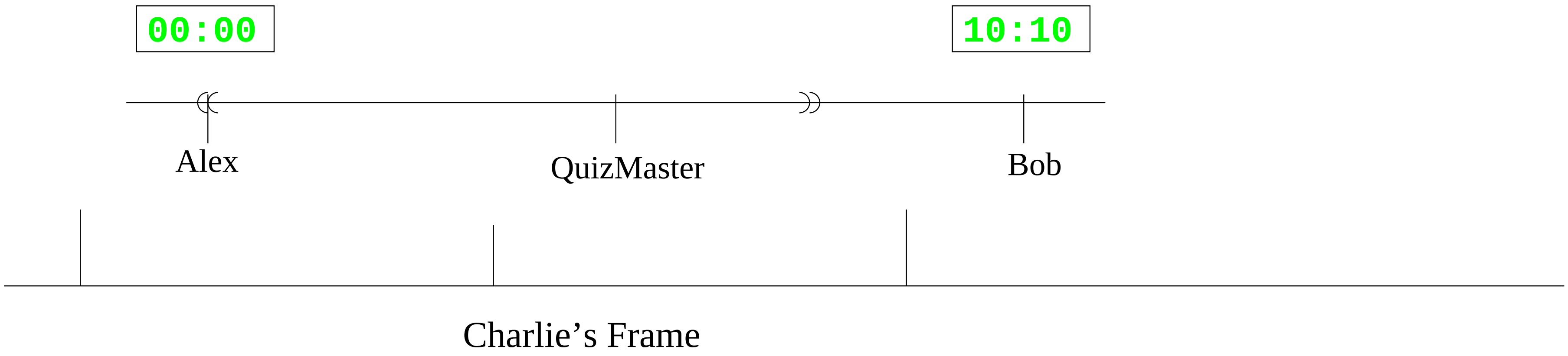
Charlie's View



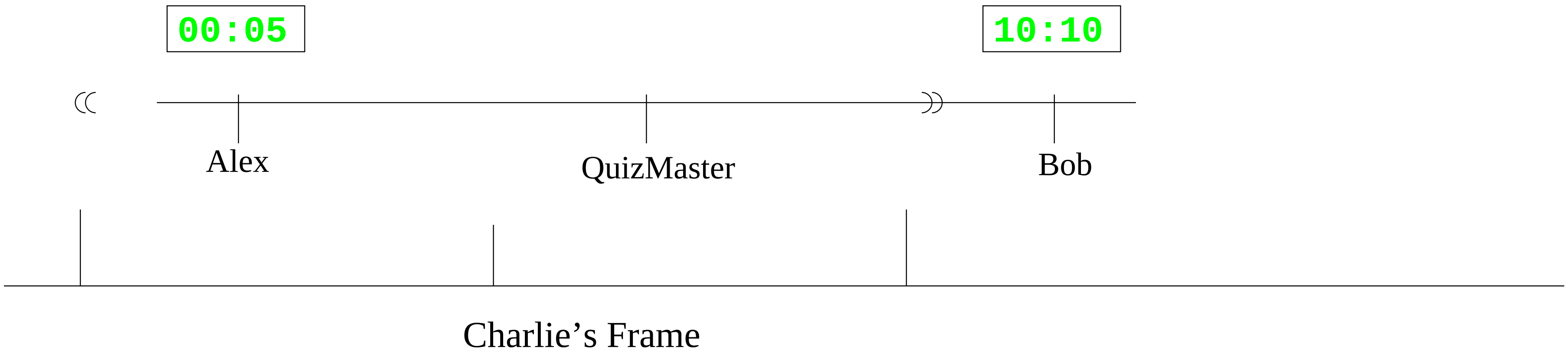
Charlie's View



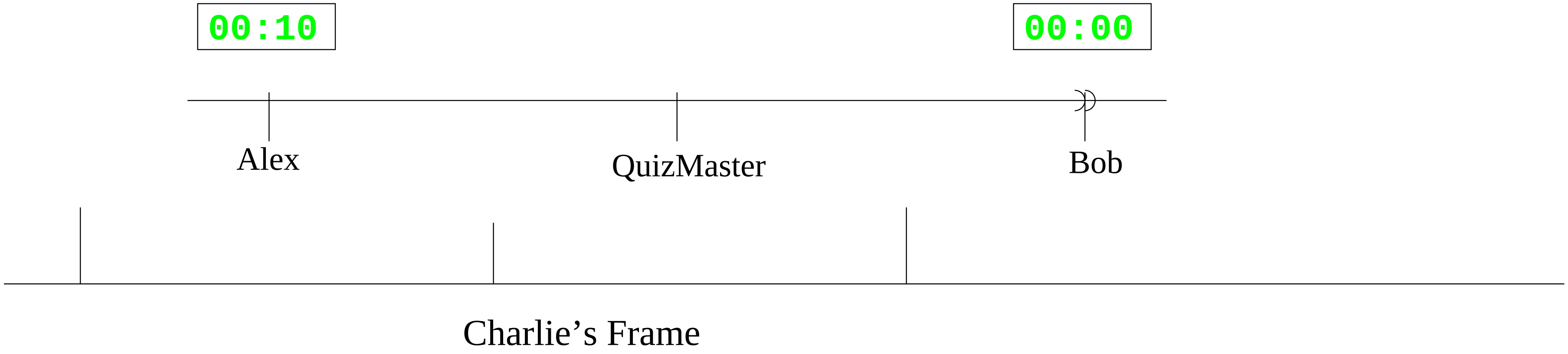
Charlie's View



Charlie's View



Charlie's View



Simultaneity

1. O flashes the signal.
2. Since light is travels with speed 'c', the signal meets, first, A at new position. A sets his clock to 00.
3. Little time later the signal reaches B. B sets his clock to 00.

The clocks A and B are not synchronized according to Charles.

Simultaneity

Then, if Alex hit the buzzer at 1300 according to the clock kept close to him and moving with him and Bob also hit the buzzer at 1300, but according to the clock with him and moving with him.

But Alex's Clock is ahead of Bob's clock.

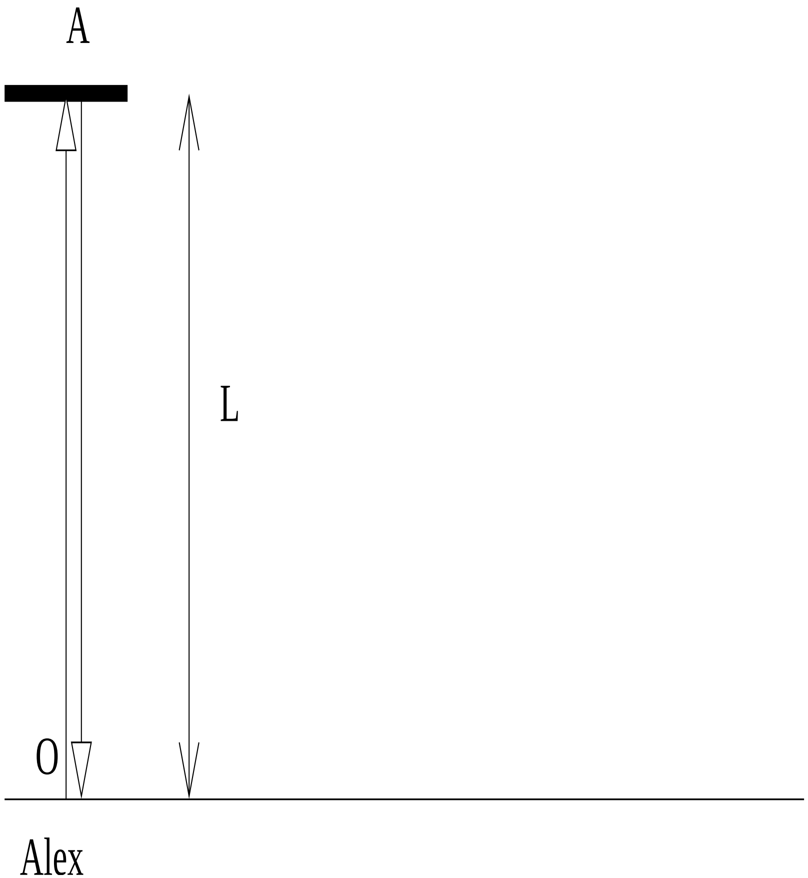
Charlie's Conclusion: Alex hit the buzzer *before* Bob did.

Simultaneity

The idea of simultaneity is not an absolute concept but a relative one. It depends on the reference frame.

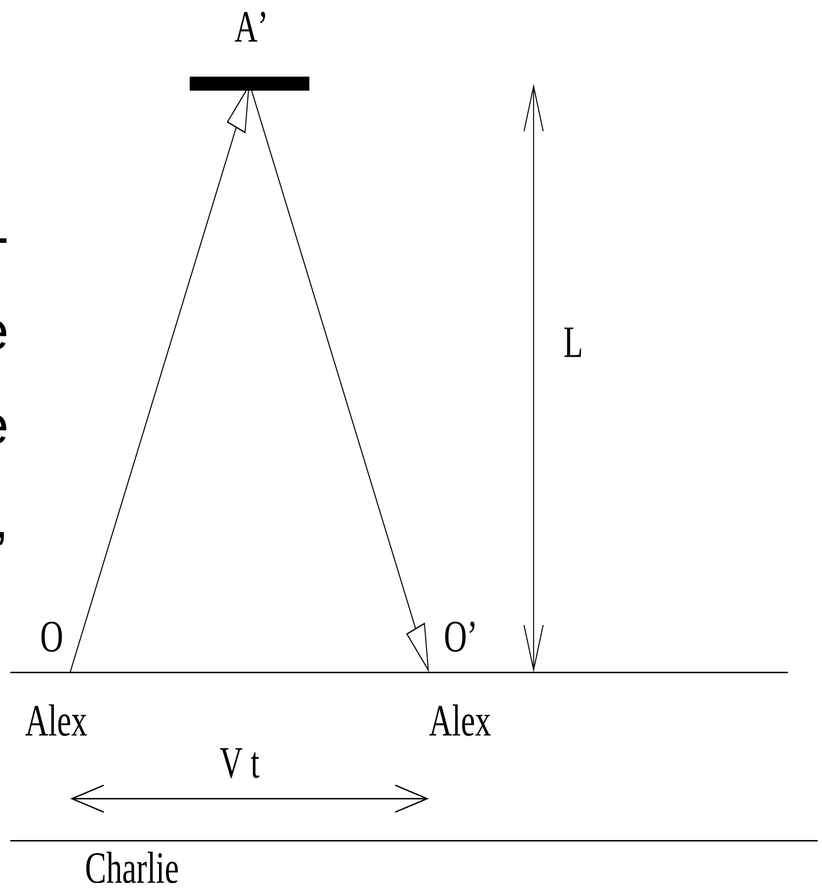
Time Interval

Alex does one experiment in which he measures a time interval by sending a pulse of light to a mirror and receiving it back. He sets up one mirror at $L = 24 \times 10^8$ m. $t = 0$, he emits the pulse, and at 16s later receives the pulse back.



Time Interval

But according to Charlie, Alex is moving with speed 0.6c. The distance the pulse travels is 60×10^8 m. Hence Charlies clock would show 20 sec, while Alex's clock only shows 16 sec!



Conclusion: Moving clocks run slowly!

Lorentz Transformation

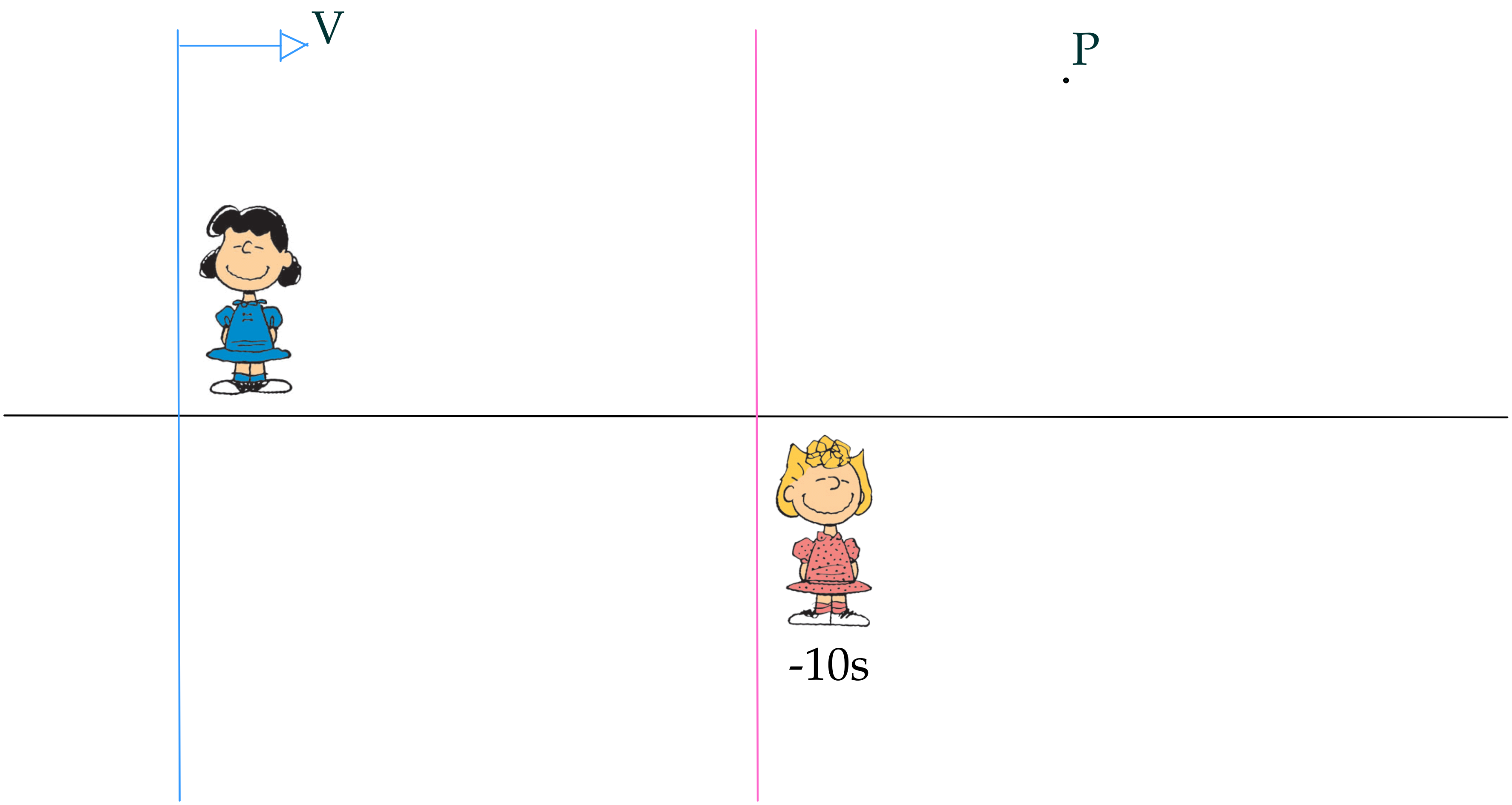
- Consider two frames, S and S'. S' is moving wrt to S with speed v.
- x' axis is always coincident with the x axis.
- At $t = t' = 0$, origin of the two frames coincide.
- "Coordinates" of an event wrt S are (x, y, z, t)
- "Coordinates" of an event wrt S' are (x', y', z', t')

We need the transformation equations that relate these two sets of numbers.

Lorentz Transformations

Sally's View

Lucy is moving to the right with speed v



Lorentz Transformations

Sally's View

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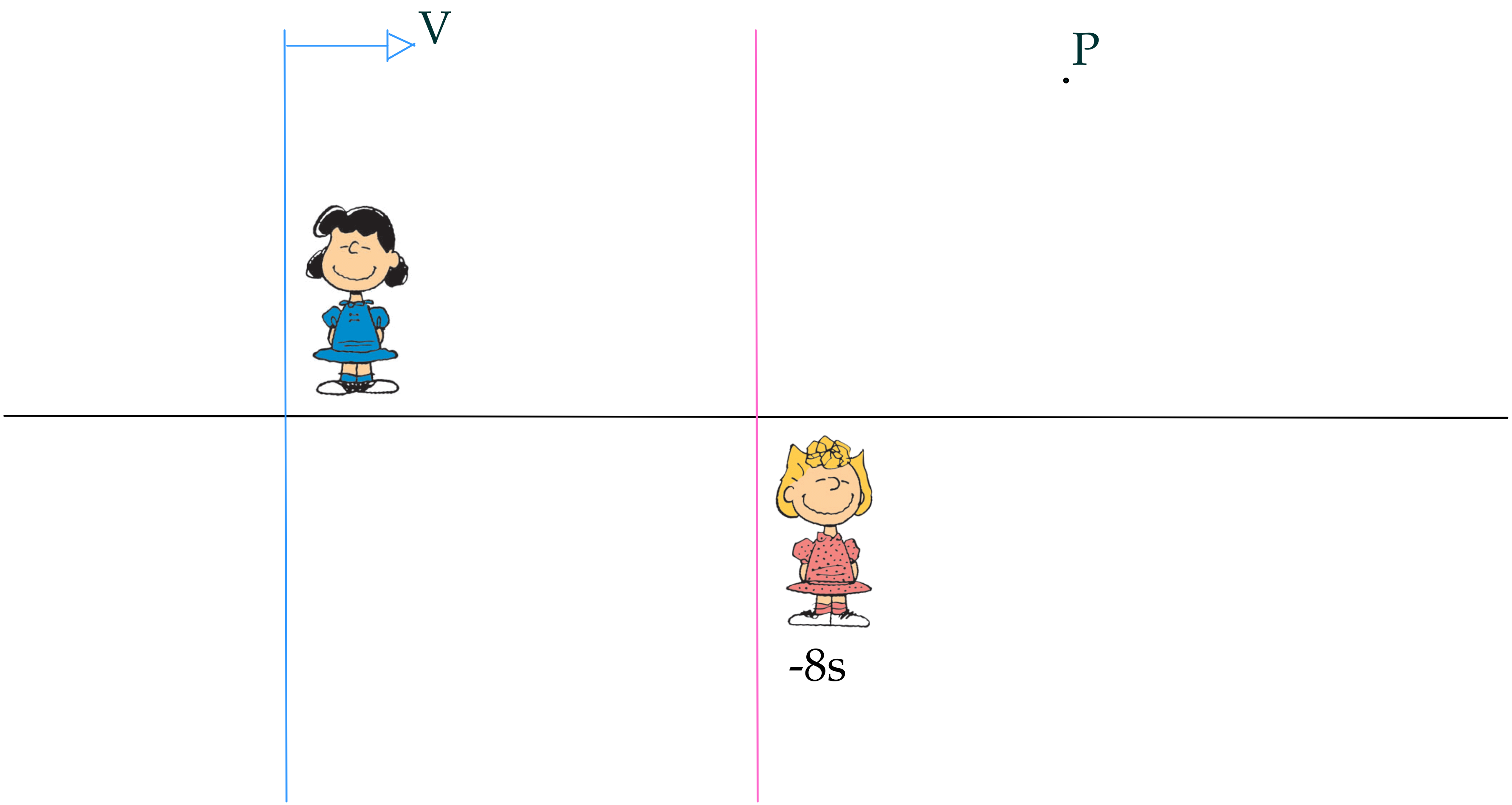

Lorentz Transformations

Sally's View

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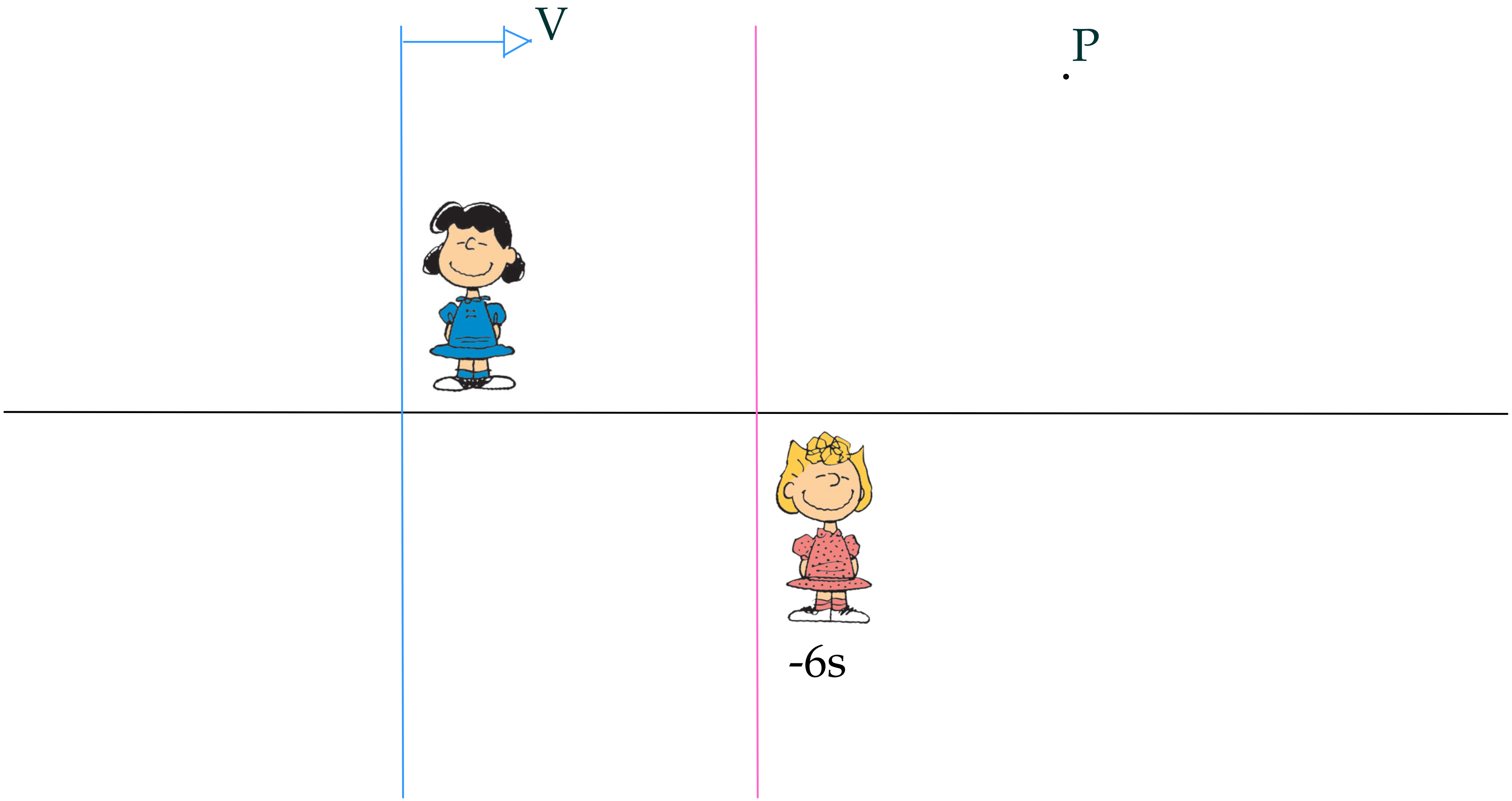

Lorentz Transformations

Sally's View

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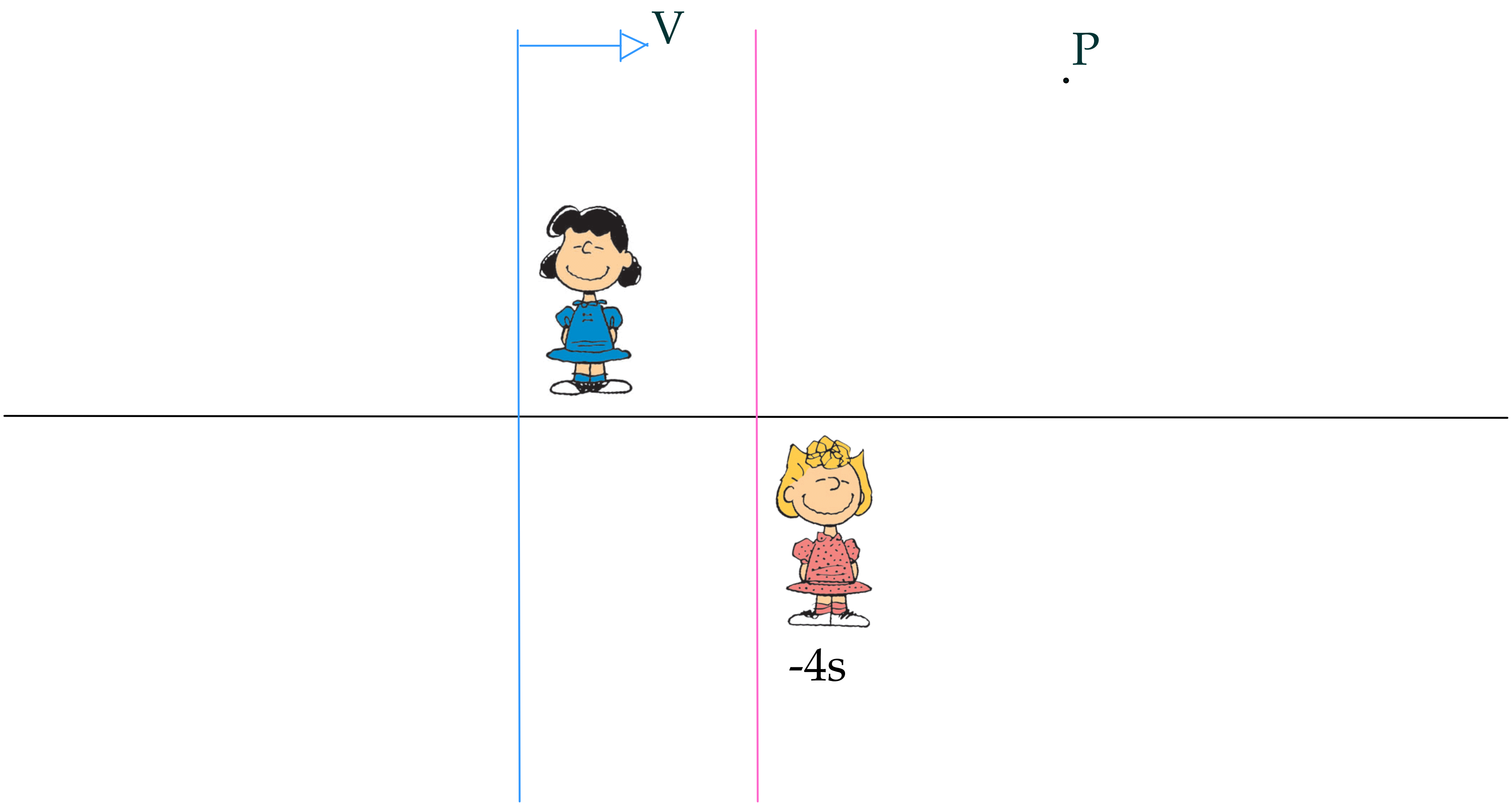

Lorentz Transformations

Sally's View

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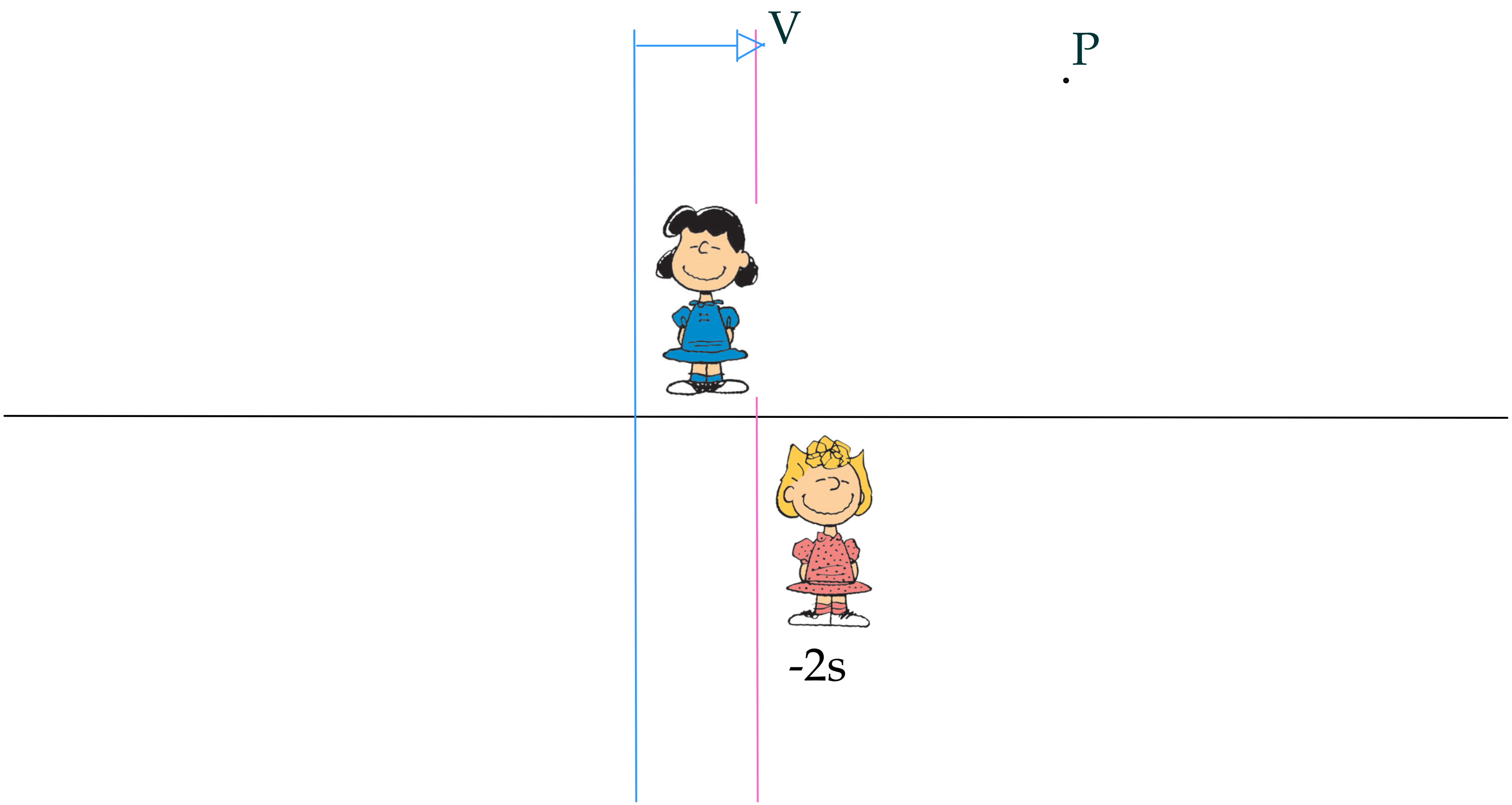
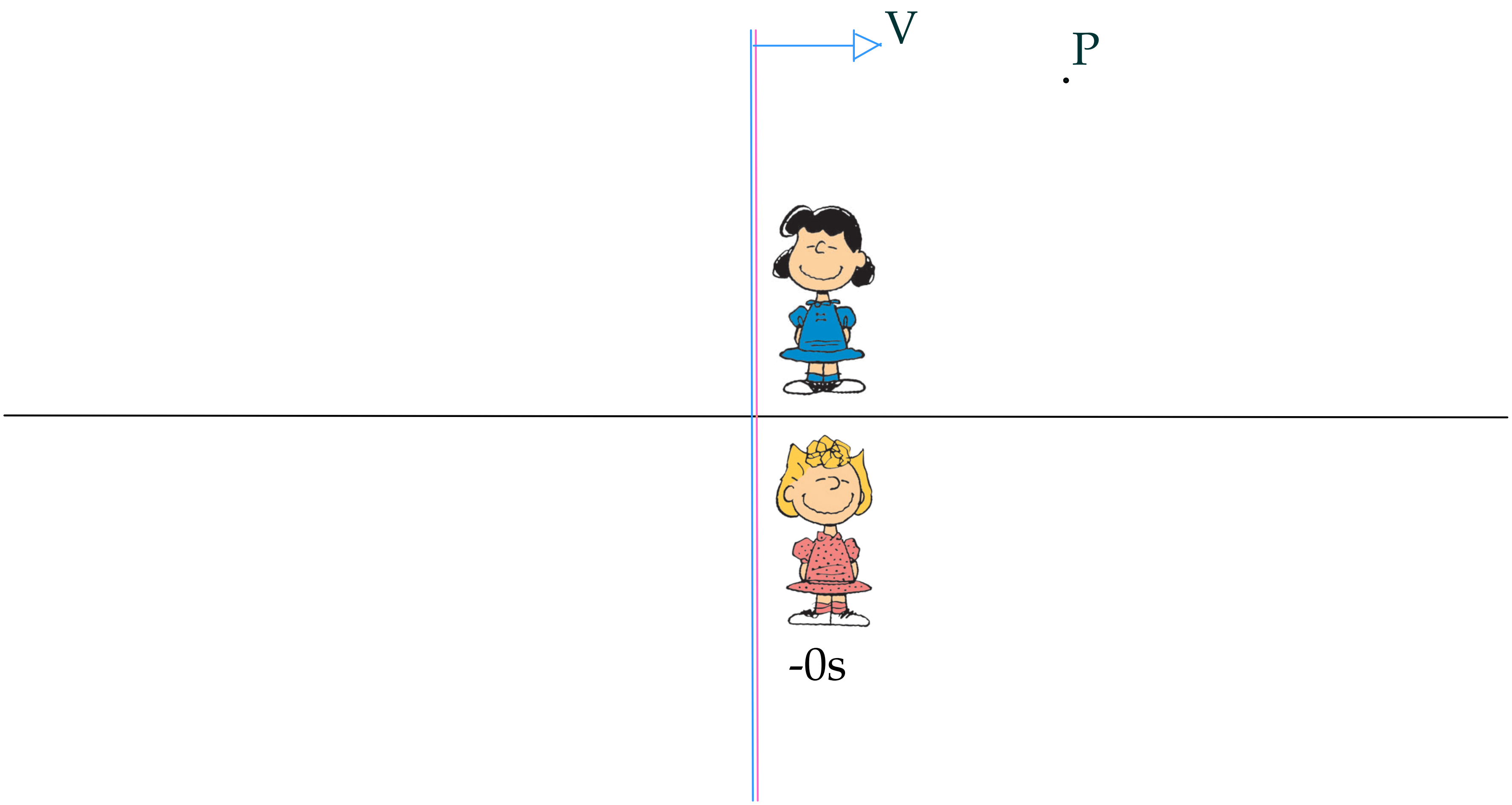


Lorentz Transformations

Sally's View

A light pulse is emitted from common origin



Lorentz Transformations

Sally's View

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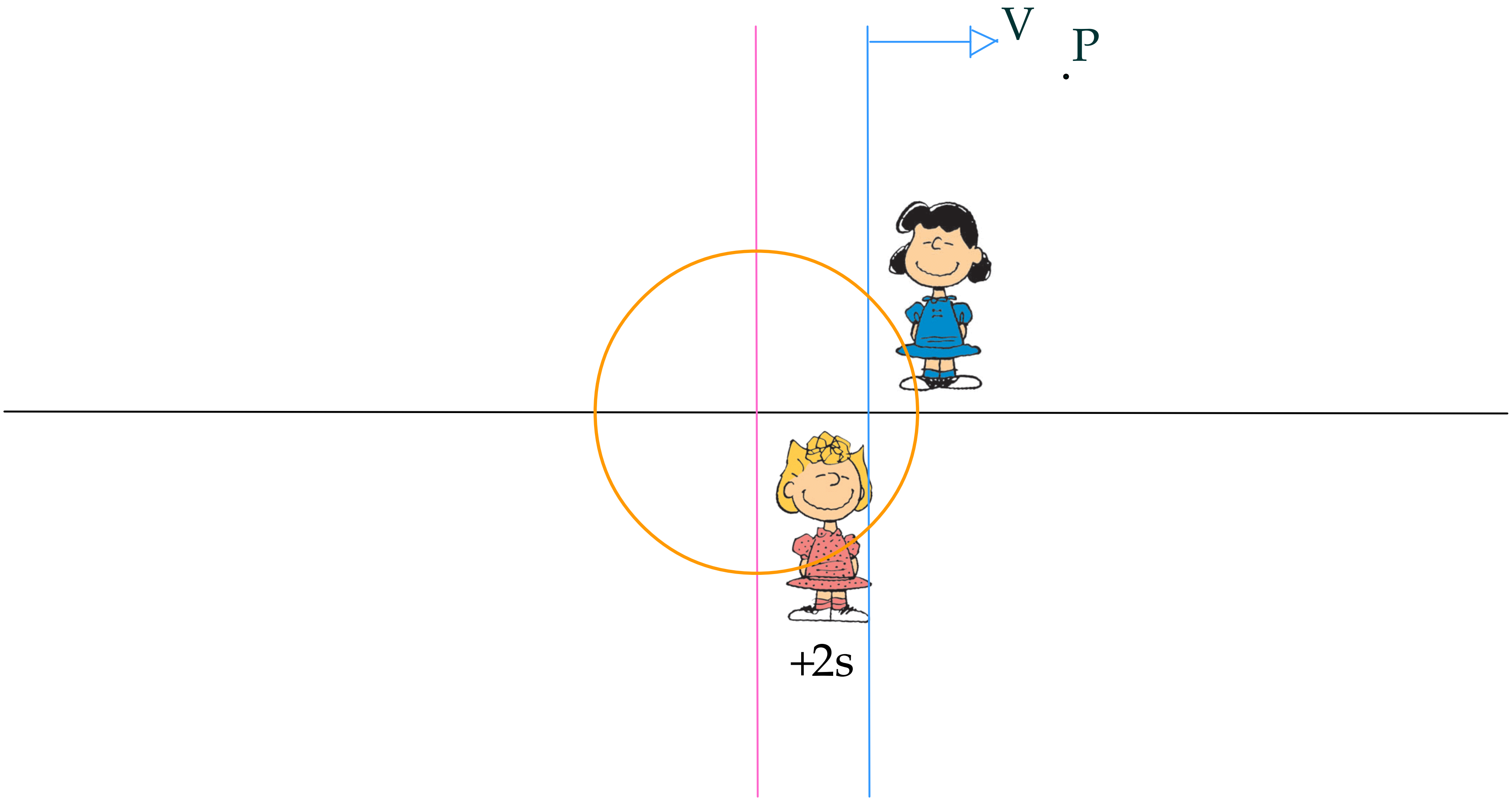

Lorentz Transformations

Sally's View

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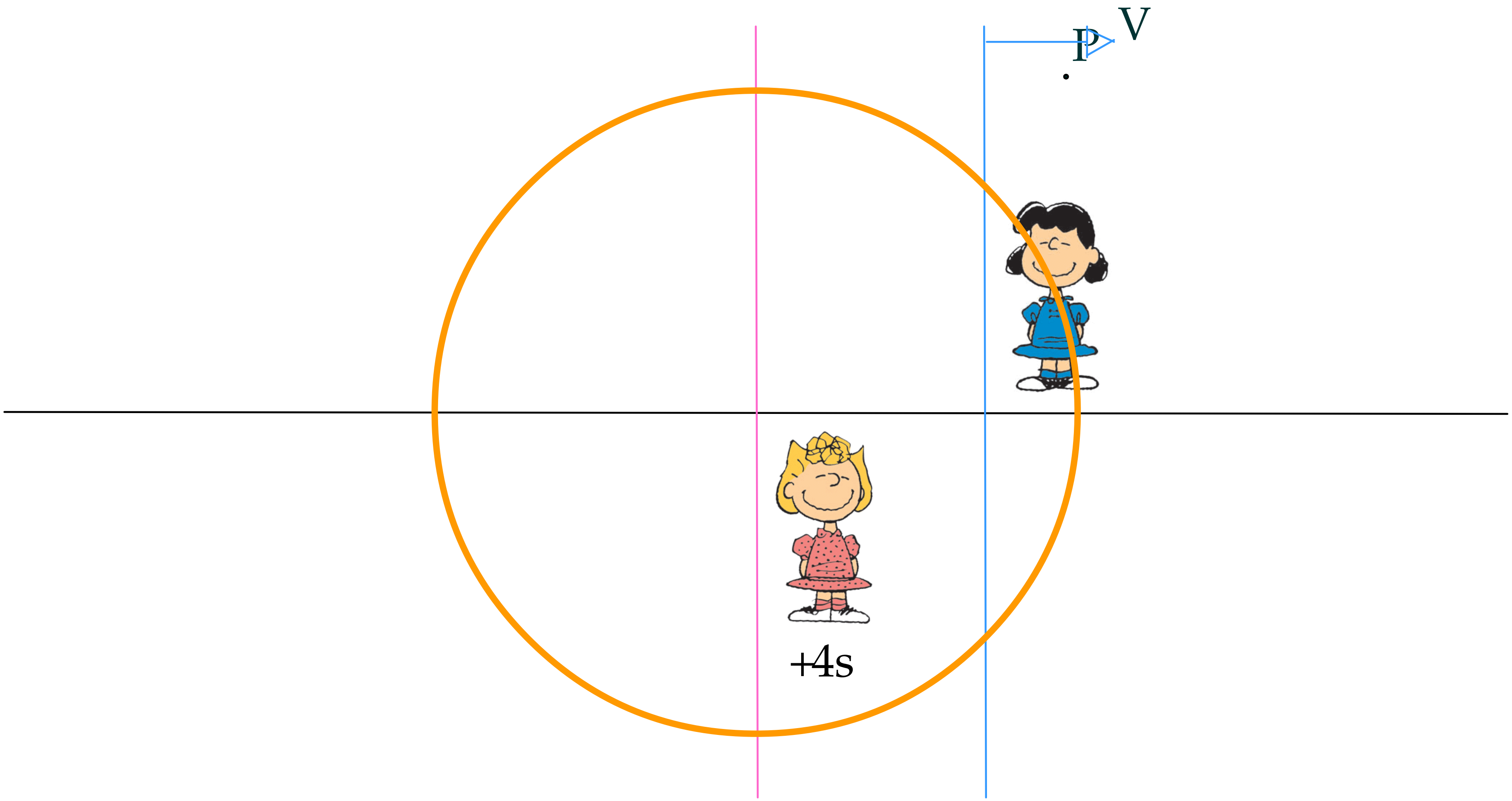
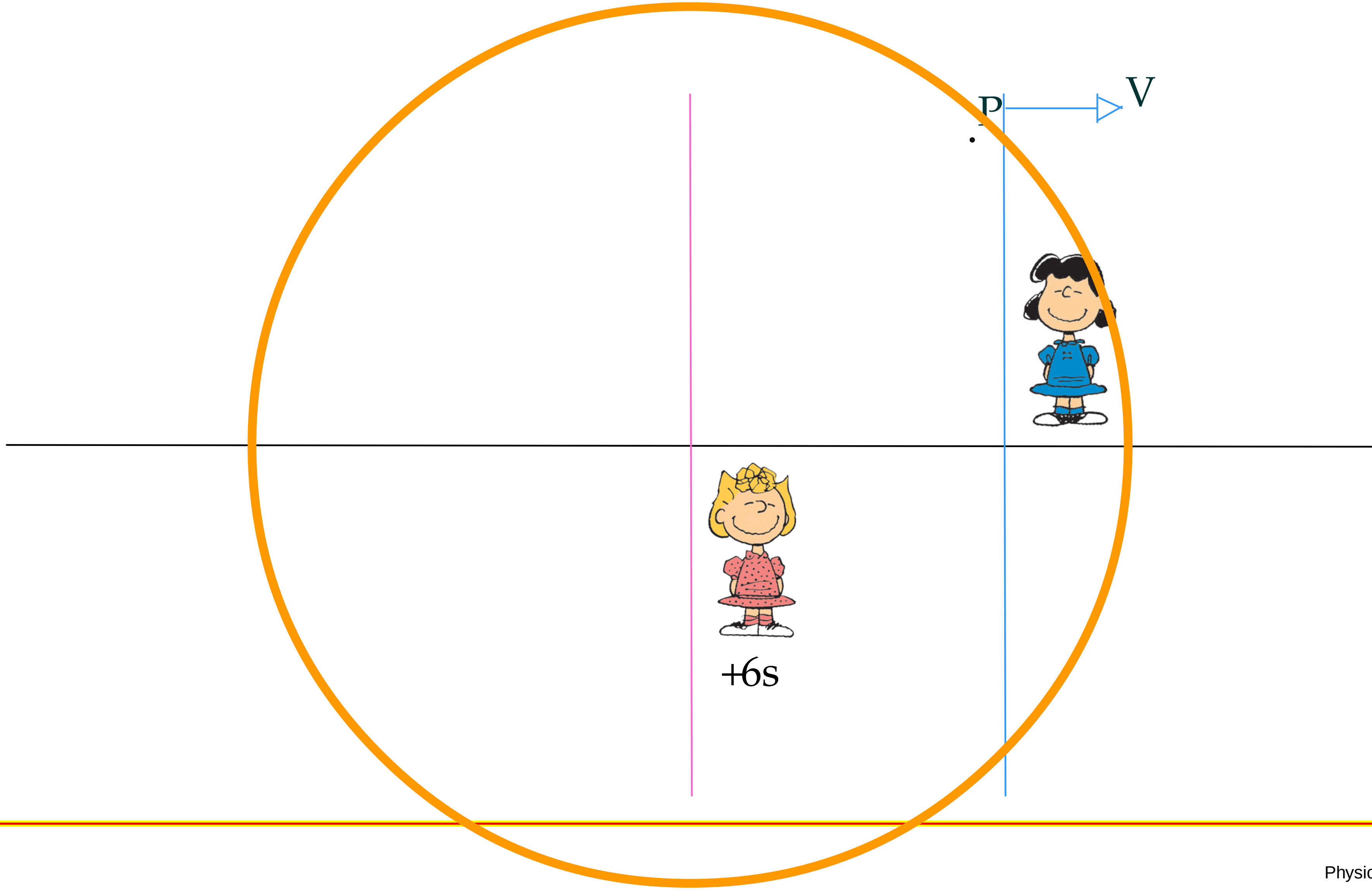


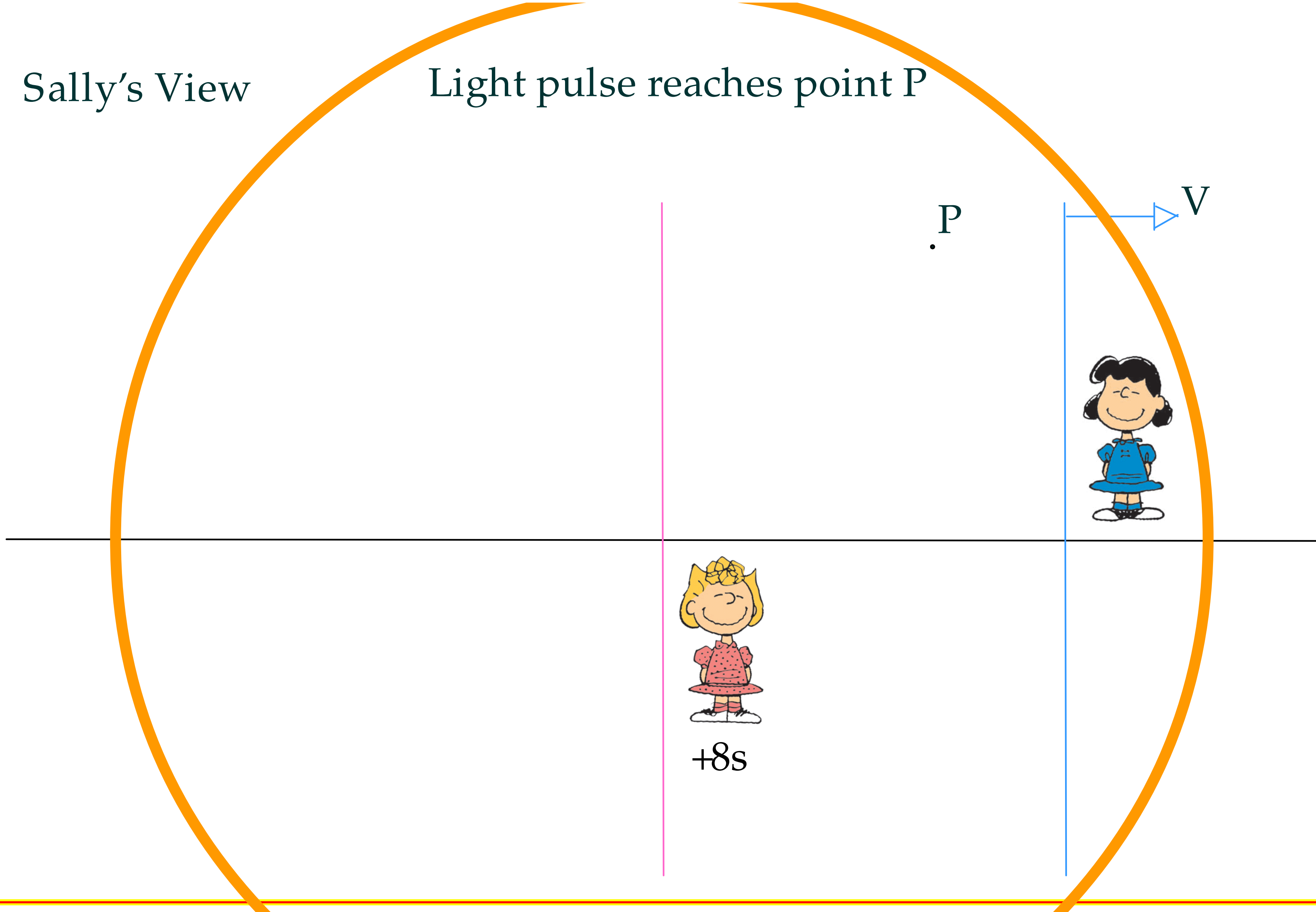
Lorentz Transformations

Sally's View

Light pulse reaches point P



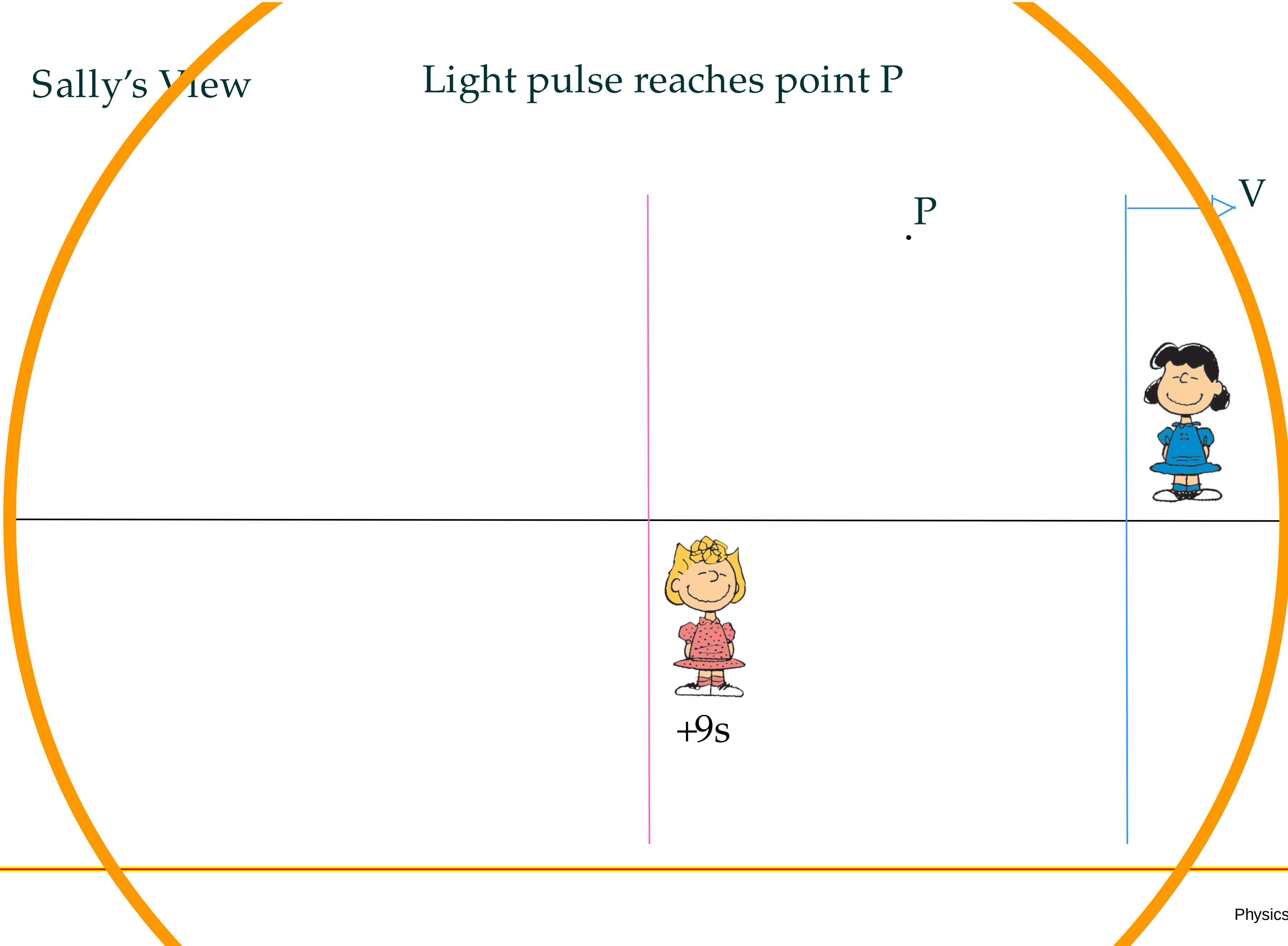
Lorentz Transformations



Lorentz Transformations

Sally's View

Light pulse reaches point P



Lorentz Transformation

Postulates of Special Theory of Relativity

1. The laws of physics are same in all inertial systems. No preferred inertial system exists.
2. The speed of light in free space has the same value in all inertial frames.
3. The space is homogeneous and isotropic.
4. The time is homogeneous.

Lorentz Transformation

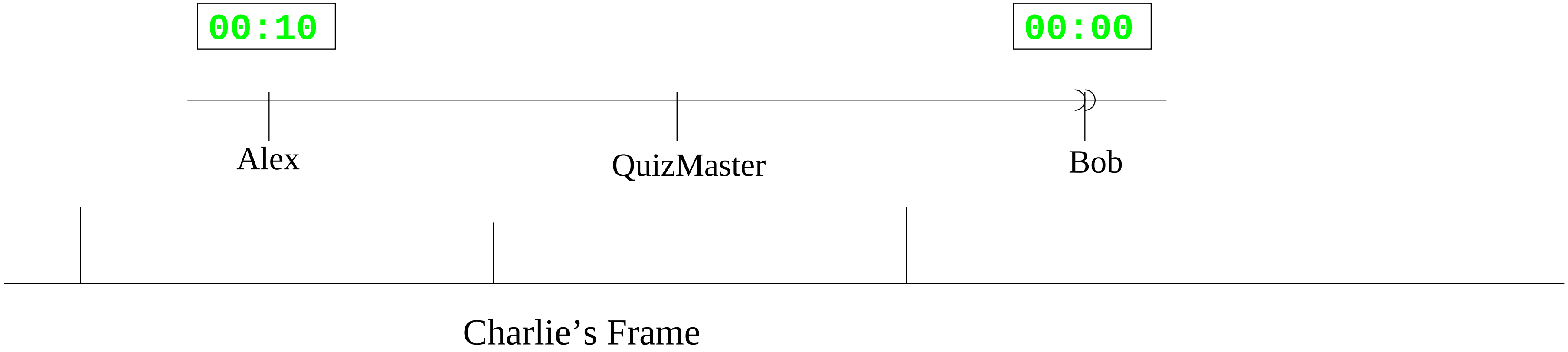
From Quiz Example earlier, we have seen that the time measurement is affected by location of clock(?), t' is not only function of t, but also must be a function of x.

Let us assume that the transformation equations are

$$\begin{aligned} x' &= Ax + Bt \\ y' &= y \\ z' &= z \\ t' &= Cx + Dt \end{aligned}$$

Charlie’s View

The two moving synchronized(by QuizMaster) clocks separated in space look asynchronous to Charlie.



Lorentz Transformation

Event	Lucy's Frame(S')	Sally's Frame(S)
Location of	$(x', 0, 0, t')$	$(x, 0, 0, t)$
Lucy's Origin	$x' = 0$	and $x = vt$

Substituting these coordinates in the equation $x' = Ax + Bt$, we get

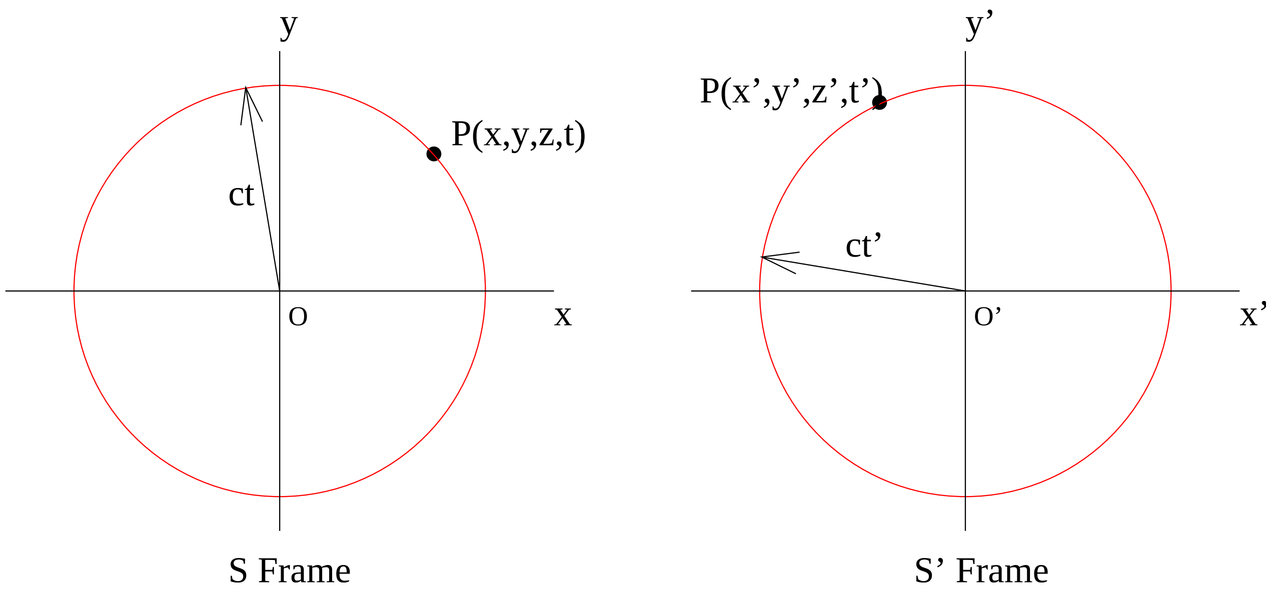
$$\begin{aligned} 0 &= A(vt) + Bt \\ B &= -Av \end{aligned}$$

So the first equation reduces to

(1) $$x' = A(x - vt)$$

Lorentz Transformation

Event	Lucy's Frame(S')	Sally's Frame(S)
Pulse Emission	$(0, 0, 0, 0)$	$(0, 0, 0, 0)$
Pulse Detection at P	(x', y', z', t') $d(O', P) = ct'$	(x, y, z, t) $d(O, P) = ct$



Lorentz Transformation

In S Frame,

$$x^2 + y^2 + z^2 = (ct)^2$$

In S' Frame,

$$\begin{aligned}
x'^2 + y'^2 + z'^2 &= (ct')^2 \\
A^2(x - vt)^2 + y^2 + z^2 &= c^2(Cx + Dt)^2 \\
(A^2 - c^2C^2)x^2 + y^2 + z^2 &- 2(vA^2 + c^2CD)xt \\
&= (c^2D^2 - v^2A^2)t^2
\end{aligned}$$

Lorentz Transformation

Comparing the two equations we get

$$
\begin{aligned}
(A^2 - c^2C^2) &= 1 \\
(vA^2 + c^2CD) &= 0 \\
(c^2D^2 - v^2A^2) &= c^2
\end{aligned}
$$

Solving these three equations, we get

$$
\begin{aligned}
A = D &= 1/\sqrt{1 - v^2/c^2} \\
C &= -\frac{v}{c^2}/\sqrt{1 - v^2/c^2}
\end{aligned}
$$

Lorentz Transformation

Lorentz's Transformations

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

Inverse Lorentz's Transformations

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$
$$y = y'$$
$$z = z'$$
$$t = \frac{t' + (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

Length Contraction

Suppose that a scale is stationery in S' Frame. One end of the rod is at $x' = x'_l = 0$ and other at $x = x'_r = L_0$.

S Frame observer measures the length of this rod by noting the coordinates of both ends, Say x_l and x_r at the same time, say $t = 0$. Then (let $\gamma = (1 - v^2/c^2)^{-1/2}$)

$$\begin{aligned} x'_l &= \gamma x_l - vt \\ x'_r &= \gamma x_r - vt \end{aligned}$$

Hence

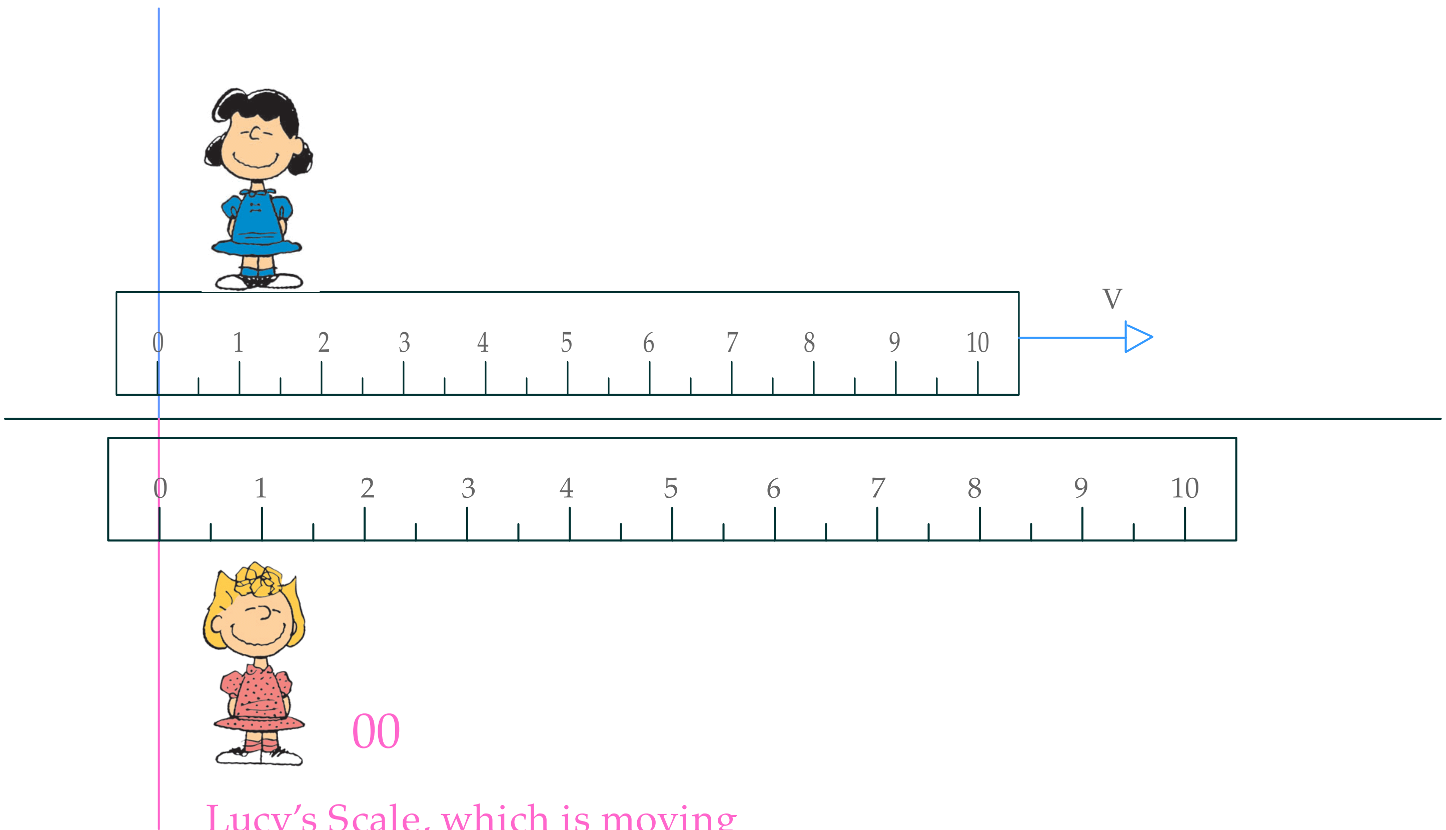
$$L = x_r - x_l = \frac{1}{\gamma}(x'_r - x'_l) = \frac{1}{\gamma} L_0$$

Moving objects look smaller in the direction of motion.

Length Contraction

If Lucy's speed wrt Sally is $0.6c$, then $\frac{1}{\gamma} = 0.8$. Lucy's scale will look shorter by factor of 0.8.

Sally's View



Lucy's Scale, which is moving
looks smaller to Sally

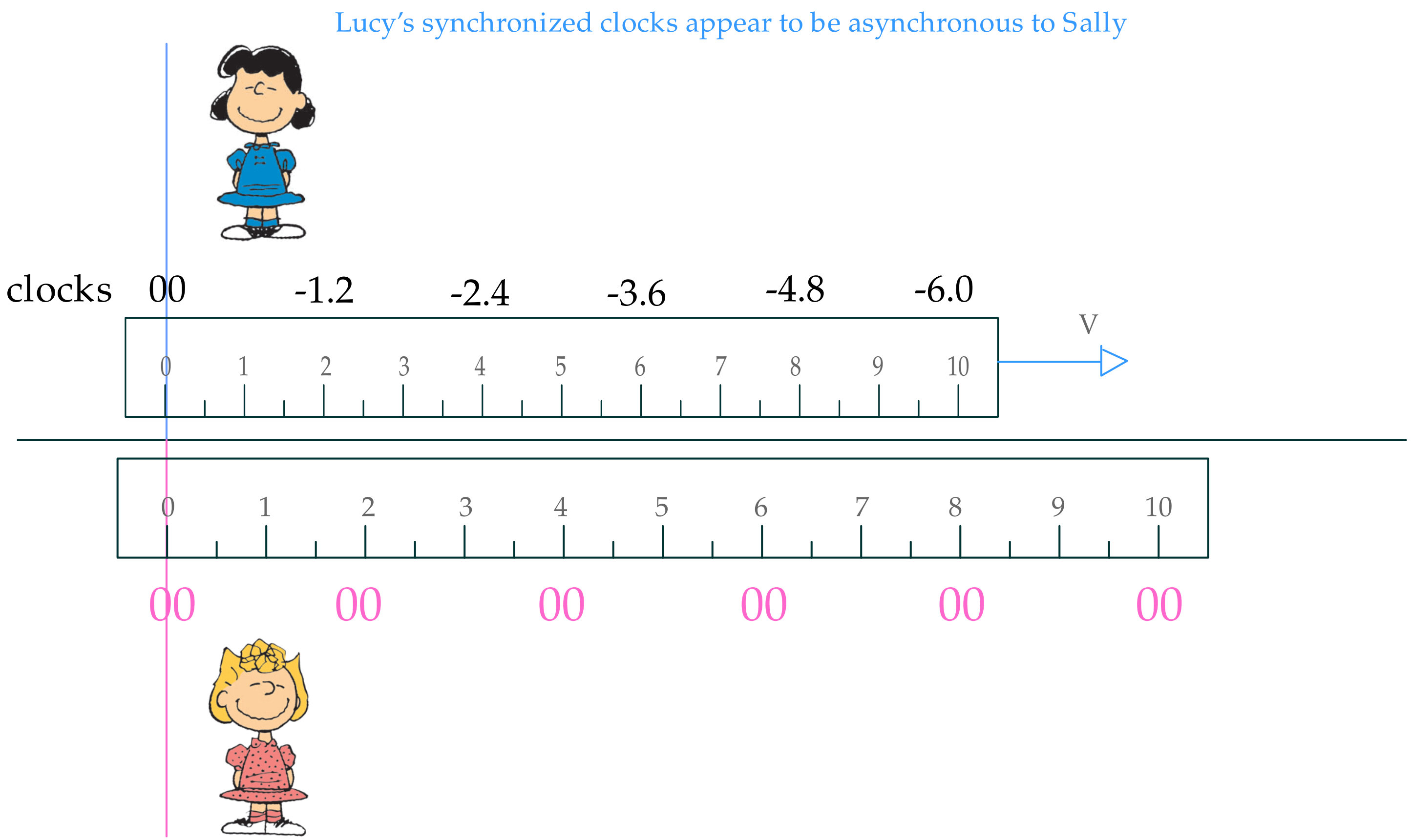
Simultaneity

If S' has kept several synchronized clocks at the unit markers on the scale, those will be asynchronous according to Sally. A clock which is at x' corresponds to $x = x'/\gamma$ at $t = 0$ in Sally's frame. Then

$$t' = \gamma(t - (v/c^2)x) = -\frac{vx'}{c^2}$$

Simultaneity

Sally's View



Time Dilation

A clock, at $x' = 0$, reads $t' = 1$. Correspondingly, by inverse transformation,

$$\begin{aligned} x &= \gamma v t' \\ t &= \gamma t' \end{aligned}$$

since $\gamma > 1$, $t > t'$. This means one second in moving clock seems longer than stationery clock.

Time Dilation

Sally's View